Comparing BET and Copulas for Cash Flows CDO's

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This version: Jan 31st 2005

This paper begins with a short explanation of collateralised debt obligations (CDO's) and how investors are using them. We describe the copula approach and the Binomial Expansion Technique (BET) developed by Moody's[1]. The models are then tested on a real contract of a cash flow CDO showing the different aspects of the methodologies and how their assumptions influence the expected and unexpected losses of the deals. We show the sensitivities of the notes (for the BET and copulas) to changes on interest rates, recovery rates, correlation, overcollateralization and average rating of the portfolio. Special attention is given to check those sensitivities on copula approach using S&P and Moody's correlation assumptions. We finish by showing the care that should be exercised by investors when analysing CDO's using the two methodologies.

Keywords: Collateralised Debt Obligations, Copulas, BET, Cash Flow CDO's.

[1] Disclaimer: the implementation (the algorithm) of the BET approach here used is our own interpretation of how the agency model might function. Moody's is in no way responsible for any mistake we might have made when interpreting its algorithm.

1) Introduction

The market for Collateralised Debt Obligation's (CDO's) has grown from around 4 bi USD in 1996 to 137 bi USD by 2001 [1] (see Backman et alli [Backman(1995)] and the references therein). These versatile instruments have been used for example to reduce credit risk from balance sheet (see Lucas [Lucas(2001)]), as an alternative investment strategy to avoid adverse selection in bond investments (see Duffie and Garleanu [Duffie(2001)]) and to improve capital allocation of a financial institution (see Bluhm et alli [Bluhm(2003)]).

There are currently four main models to determine the Expected Loss (EL) and the Unexpected Loss (UL) of a CDO note: the Binomial Expansion Technique (BET) and the Fast Fourier Transform algorithm (FFT) developed by Moody's (see Cifuentes et alli [Cifuentes(1995)] and Debuysscher [Debuysscher (2003)]), the Standard and Poor's approach, that is copula based (see Bergman [Bergman(2001)]), the Fitch approach (see Bund et al [Bund(2003)]) and the copula function approach developed by CreditMetrics (see Mina [Mina(2001)]). For a comparison between BET, S&P and Fitch approaches we refer to Zhou et al. [Zhu(2003)].

The main objective of this paper is to compare the results between the BET and the copula methodology when determining the expected loss of a CDO. For the copulas approach we will be using two different assumptions for the correlation function (one close to what is suggested by S&P and another one close to what is suggested by Moody's). Additionally all the evaluations are made using a real CDO contract[2].

This paper will be structured as follows. In section 2 we give a summary of what CDO's are and some detail on how they might be used. In section 3 we give a detailed description of the BET methodology. In section 4 we give an overview of the copula method. Finally in section 5 we present two CDO contracts and the results of the tests made. In section 6 we give the conclusions.

2) CDO : A Brief Description

In this section we give a brief overview of the CDO structures. We will restrict the overview to the CDO basics that are necessary to understand the BET and copula models explained below. Additionally we refer to Lucas [Lucas(2001)] for more detail on the structure of the contracts, to Hill and Vacca [Hill(1999)] for arbitrage CDO's, to Goodman [Goodman(2002)] for synthetic CDO's, to Falcone [Falcone(1998)] for a rating approach on market value CDO's.

[2] We will not mention the name of the contracts due to confidentiality agreements.

A CDO which belongs to the Asset-Backed Security (ABS) class is set up as a Special Purpose Vehicle (SPV) that invests in a pool of securities to be used as collateral. The interest and principal payments from the collateral pool are allocated to the notes following a prioritisation schedule which is known a priori and is detailed in the prospectus of the CDO (see section 5 for more detail). The collateral can be of very different types of securities: high yield bonds and emerging market debts (CBO's), bank loans (CLO's), ABS/MBS's, CDS's (synthetic CDO's), equity funds or (as more recently) notes of other CDO's .

Depending on the purpose of the CDO issuer there are basically two main classes of CDO's: *balance sheet* and *arbitrage*. In a *balance sheet* CDO a financial institution securitizes loans in its balance sheet with four immediate purposes: a) relieve in capital, b) increase in liquidity of the loans, c) improve performance measurement ratios, and d) transfer risk off balance sheet. In an *arbitrage* CDO one intends to get the difference between the cost of funding and the return on high yield investment. This means that in an arbitrage CDO the yield on the assets has to be higher than the total fees and the cost of funding the instrument. The typical collateral is either high yield bonds or loans.

Depending on the way the collateral pool is managed a CDO may be of two types: *cash flow* and *market values*. In a *cash flow* CDO the manager is not supposed to engage on actively trading the assets in the collateral and there are very strict rules permitting the buying and selling of collateral. Uncertainties in the payments of cash flow CDO's are related to the number and the timing of defaults. In a *market value* CDO on the other hand the payments are determined by gains on the marked to market value of the collateral pool. The gains are mainly determined by the trading performance of the manager. An algorithm for a market value CDO would have to take into account the trading behaviour of the collateral manager. In this paper we will be dealing with *cash flow* CDO's only.

3) BET Methodology

In this section we give a brief description of the BET algorithm. We refer to Cifuentes [Cifuentes(1996)] for a first reference on BET and to Garcia et alli [Garcia(2003)] for a more detailed description and tests of the parameters on the BET algorithm.

The BET methodology is based on the concept of Diversity Score (DS) and it is an application of the binomial formula from probability theory to a simplified version of the portfolio. The idea behind the methodology is to map the portfolio of heterogeneous correlated securities with distinct probabilities of default into a portfolio of homogeneous independent securities each having the same probability of default. The second portfolio is said to be the *idealized* portfolio. Once the DS and the relevant probability of default are determined one applies the binomial formula to give first approximations for the default scenarios.

The first step is the DS calculation of the collateral portfolio[3]. The goal of the DS measure is to redefine the pool of correlated heterogeneous assets into a pool of DS independent homogeneous assets. The lower the diversification, i.e. the higher the concentration, the lower should be the DS. Diversification is measured on two levels: first at issuer level by dividing every issuer's exposure by the average issuer exposure and capping the resulting ratio to 1. This gives the Equivalent Unit Score (EUS) per issuer. The more issuers with exposures that are larger than the average (high issuer concentration), the lower will be the average EUS of the issuers. Diversification is also measured at the industry level by summing first all EUS of all issuers in a certain industry (this sum is called the Aggregate Industry Equivalent Unit Score or AIEUS) and then scaling down the AIEUS into an industry diversity score whereby the bigger AIEUS are scaled down more (e.g. if AIEUS(industry i)=1 then DS(industry i)=1, if AIEUS(industry j)=20 then DS(industry j)=5). The DS of the pool is equal to the sum of all industry DS. The higher the average number of issuers per industry (high industry concentration), the lower will be the DS. A detailed computation of the DS of a portfolio is shown in Garcia et alli [Garcia(2003)].

The second step consists in determining the expected cash flows from the collateral for each possible number of homogeneous bonds defaulting. To do this one needs to make some assumptions for:

a) the principal payments from the collateral. All homogeneous bonds are assumed to have the same maturity that is derived from the weighted average life (WAL) of the collateral pool. Alternatively one can take into account the expected principal redemption schedule of the collateral pool;

b) the interest payments from the collateral. All homogeneous bonds are assumed to have the same coupon, equal to the weighted average coupon (WAC) of the collateral pool;

c) the recovery rate (WAR) in case of default. A standard approach uses one predetermined rate[4] (e.g. 30%). Alternatively one can use a recovery rate that takes into account the types of exposures (secured, unsecured, subordinate, bonds or loans, etc) of every collateral pool.

Assume that N_T is the total notional of the collateral portfolio. The notional involved in the case of j defaults is given by:

[3] The Diversity Score algorithm described and used in this work, is the one that, from our experience, is the most widespread. There exists another Diversity Score algorithm that takes into account correlation, face value and default probability (based on rating and maturity). We refer to Cifuentes et al [14] for more details on the alternative calculation method.

$$D_j = \frac{N_T}{DS} \cdot j \qquad (1)$$

and the loss on Dj is given by:

$$Loss_j = Dj \cdot (1 - WAR) \qquad (2)$$

d) the distribution of the losses in time. A standard approach assumes six default patterns:

i. 50 10 10 10 10 10
ii. 10 50 10 10 10 10
iii. 10 10 50 10 10 10 (3)
iv. 10 10 10 50 10 10
v. 10 10 10 10 50 10
vi. 10 10 10 10 10 50.

In each of the scenarios above one distributes the losses along 6 years. In i) 50% is lost in the first and 10% in the remaining years. In ii), 10% of the losses occur in the first year, 50% in the second and 10% in the remaining and analogously for iii) up to vi). With the scenarios above one can determine the amount of money left in the collateral.

In the third step one goes through the waterfall and determines the amounts that each note holder will receive (see table 2 in section 5 for a typical waterfall) and how much are their losses in each of the collateral cash flow scenario's generated in step 2. In order to count for interest rate risk a common market practice is to make parallel shifts on the levels of the yield curve. In here we will be considering three possible scenarios: flat; upper 1% and upper 2%.

In the fourth step one uses the weighted average rating factor (WARF) and the weighted average life (WAL) of the collateral pool to determine the probability of default *p* of

[4] This approach uses issue ratings (as opposed to issuer ratings) for determining default probabilities and in doing so different recovery levels for assets with different seniorities are already taken into account.

one idealized bond[5]. With this probability and the binomial formula one can calculate the probability of each of the collateral cash flow scenario's generated in step 2 (0 up to DS idealized bonds defaulting). Assume for example that in one scenario one would have *n* defaults (where $n <= DS$). The probability of *n* defaults (P_n) is given by:

$$P_n = \binom{DS}{n} p^n \cdot (1-p)^{DS-n} \qquad (4)$$

In order to reduce the impact of the errors due to all the simplified assumptions in the model it is a common market practice to multiply the probability of default *p* by a stress factor ? in eq. 4 above. The size of the stress factor would depend on the target rating one wishes to test a tranche for.

The fifth step consists in bringing together the results for the different default scenarios of step 3 with the probabilities calculated in step 4. This leads to a loss distribution and an average ("expected") loss for every scenario.

In a sixth step one compares the losses obtained in step 5 with the "target" loss (this is the idealized loss for a bond with a maturity equal to the average life of the tranche and a rating equal to the target rating), and decides whether or not the tranche passes for the target rating. The rating of the tranche is then equal to the highest possible rating.

It is not straightforward to determine from the above what the expected loss for a certain tranche is, because:

- the level of losses is impacted by the target rating one wishes to test (through the use of different stress factors): EL's are higher for a higher target rating,
- the above gives EL's for different (typically 6x5) scenarios: which one of them, if any is the expected?

In what follows the EL will be calculated as the average of the 6 EL's obtained in step 5 for the flat forward interest rate scenario.

4) Copulas

In the copula approach we generate jointly default times for the entities in the underlying portfolio. The joint distribution of default times must obey two requirements: a) it recovers the marginal distribution of default probabilities for each underlying; b) it recovers the default correlation for the underlying collateral.

[5] This is done by using a table which maps rating factors to default probabilities (see Garcia et alli [Garcia(2003)a]).

In this work we use the copula function approach to generate correlated default times that obey the marginal distributions of the underlying collateral entities. In what follows we will give a brief description of the copula approach. We refer to Nelsen [Nelsen(1999)] for a rather formal description of the copulas; to Frees and Valdez [Frees(1998)] for a survey article and to Li [Li(1999)] and Garcia et alli [Garcia(2003)b] for the use of normal copulas in pricing baskets of credit derivatives.

Consider that the collateral of the CDO is composed of N underlying names and that $P_1(t_1), P_2(t_2), \ldots, P_N(t_N)$ are the marginal distributions of default times for each entity. A copula function C is a function from $(0,1)^N$ to (0,1) such that:

$$C(P_1(t_1), P_2(t_2), \ldots, P_M(t_M)) = P(t_1, t_2, t_3, \ldots, t_M), \tag{5}$$

where P is the joint distribution function of default times. Under some standard technical conditions (e.g. continuity of the marginals) it can be proved that any multivariate distribution can be written in the form of a copula function (see Sklar [Sklar(1996)] for details).

A common market practice is to use the multivariate normal copula that is defined as:

$$C(x_1, x_2, \ldots, x_M) = N_M(N^{-1}(x_1), N^{-1}(x_2), \ldots, N^{-1}(x_M), \Sigma), \tag{6}$$

where N^{-1} is the inverse of the univariate standard normal distribution, and N_M is the multivariate normal distribution with correlation matrix function S.

In this work we will be using Moody's data as input for the copula function. The marginal default probabilities will be taken from Moody's idealized Default Rates (DR's) and Moody's average historical default rates while the correlation function is explained later on in section 5 (see tables 3 and 4 for the DR's). The consequences of these choices for the determination of the ratings will be discussed in section 5.

The copula function is used to generate default times. Once the default times are generated the cash flows are passed through the waterfall and the losses on each note are determined. This step is repeated a certain number of times and the EL is determined by an arithmetic average.

5) Results

In this section we show the results of the tests made using the copula and BET approach. Given that we have already discussed the BET approach in more detail elsewhere (see section 3 for details) the main focus will be on the copulas model while we show the results of the BET approach in most of the tables.

This section will be subdivided as follows. In section 5.1 we show the details of the contract used in the tests. In section 5.2 we show the results for the copulas and where appropriate we show the comparison with the BET approach.

5.1) Contract Description

The data given in here are for a structure that has already started. We have used a real contract in which we have changed the collateral amounts. The maturity date of the contract is May 23rd 2011.

In what follows the EL will be evaluated using the following formula:

$$EL = \sum_i CF_i \cdot DF_i \qquad (7)$$

and:

$$DF_i = e^{-(r+s)\cdot t_i} \qquad (8)$$

where *r* and *s* are the risk free and the spread rates respectively. For the case of the fixed rate notes we discount the EL using the coupon of the note while for the floating notes we use Libor plus the margin. For example the notes A (see table 3) will be discounted at Libor + 0.65 while the notes D1 will be discounted at 11.875% (this means that notes with the same seniority might have different EL's (see e.g. the results for notes D1 and D2 in section 5)). The notes issued in the structure with the current outstanding amount are shown in Table 2.

Notes	Notional	Coupon Type	Coupon Spread (%)
A	160,494,728	Floating	0.65
B	37,000,000	Floating	1.0
C	42,623,383	Fixed	8.625
D1	8,005,709	Fixed	11.875
D2	17,632,067	Fixed	12.57
E	28,009,384	Fixed	3.0
Total	293,765,271	-	-

Table 2 Tranche notes of the CDO structure

The fees to be paid in the structure are described in Table 3.

Name	Fixed Amount	Fixed Rate (%)	Calculation Basis
Adm. Expenses Flt	-	0.0175	Collateral Balance
Adm. Expenses Fxd	40,000	-	-
Snr. Coll. Mgmt. Fee	-	0.05	Collateral Balance
Sub Col. Mgmt Fee	-	0.45	Collateral Balance

Table 3 Fees to be paid during the lifetime of the CDO

The hedges existent in the structure are shown in table 4

Type	Notional	Strike (%)	Start Date	Expiry Date	Type
Swap	269,000,000	6.265	23/04/1999	23/05/2004	Payer
Cap	269,000,000	6.25	23/05/2004	22/11/2004	Payer
Cap	266,210,587	6.25	23/11/2004	22/05/2005	Payer
Cap	245,597,044	6.25	23/05/2005	22/11/2005	Payer
Cap	227,863,689	6.25	23/11/2005	22/05/2006	Payer
Cap	182,836,154	6.25	23/05/2006	22/11/2006	Payer
Cap	162,161,261	6.25	23/11/2006	22/05/2007	Payer
Cap	137,639,658	6.25	23/05/2007	22/11/2007	Payer
Cap	105,362,505	6.25	23/11/2007	22/05/2008	Payer
Cap	60,947,263	6.25	23/05/2008	22/11/2008	Payer
Cap	45,039,112	6.25	23/11/2008	22/05/2009	Payer
Cap	29,500,000	6.25	23/05/2009	22/11/2009	Payer
Cap	28,929,107	6.25	23/11/2009	22/11/2010	Payer
Cap	27,041,995	6.25	23/05/2010	23/11/2010	Payer
Cap	26,475,951	6.25	23/11/2010	23/05/2011	Payer

Table 4 Hedges

The ratios used for the overcollateralization (OC) ratio tests are shown in Table 5. Observe that although we have kept the trigger of the tests as being described by ratios it is also common that besides ratios there might be amounts involved (especially for the senior tranches).

Class	Trigger
A/B	1.2
C	1.07

D	1.03

. Table 5 Ratios used in the OC Ratio tests.

The OC tests for the A/B, C and D tranches are given by:

$$OC_{A/B} = \frac{NumeratorOC}{\sum_{i}^{A,B} NotionalNote(i)} \qquad (9)$$

$$OC_{C} = \frac{NumeratorOC}{\sum_{i}^{A,B,C} NotionalNote(i)} \qquad (10)$$

$$OC_{D} = \frac{NumeratorOC}{\sum_{i}^{A,B,C,D} NotionalNote(i)} \qquad (11)$$

The NotionalNote(i) is the notional amount of the note i (A, B, C and D) that has not yet been redeemed. And:

$$NumeratorOC = ColNotional + MarketValueCurrentDefaults \qquad (12)$$

The ColNotional is notional amount of the performing collateral bonds, and MarketValueCurrentDefaults is the estimated value of the defaulted securities (non-performing bonds) in the collateral.

The values for the Interest Coverage (IC) ratio tests we show in table 6 below.

Class	Trigger
A/B	1.2
C	1.07
D	1.03

Table 6 Ratios used in the IC tests.

The IC tests for the A/B, C and D tranches are given by:

$$IC_{A/B} = \frac{NumeratorIC}{\sum_{i}^{A,B} (Interest(i) + DeferredInterest(i))} \qquad (13)$$

$$IC_{C} = \frac{NumeratorIC}{\sum_{i}^{A,B,C} (Interest(i) + DeferredInterest(i))} \qquad (14)$$

$$IC_D = \frac{NumeratorIC}{\sum_i^{A,B,C,D}(Interest(i) + DeferredInterest(i))} \quad (15)$$

and:

$$NumeratorIC = CollateralInterest - Fees \quad (16)$$

The CollateralInterest is the amount of interest received from the collateral; Fees are the fees senior to the interest of the notes; Interest(i) is the amount of interest to be paid to note i (A, B; C and D); DeferredInterest is the amount of interest that has not been paid yet and should eventually be paid when interest proceedings become available.

The characteristics of the pool data are given in tables 7.a and 7.b.

Collateral Amount[6]	257,136,962
DS	34
WARF	2081 (B1)
WAL	5.00
WAC	8.87%
Payment Freq.	Semi-Ann.
Fixed Rate(%)	100
Interest Acc.	8,437,169
Principal Acc.	4,499,582

Table 7a Collateral Information

Rating	**Amount**	**Rating**	**Amount**
Baa2	3	**B1**	11
Baa3	3	**B2**	11
Ba1	12	**B3**	6
Ba2	3	**Caa1**	4
Ba3	5	-	

Table 7b Rating distribution of bonds in the pool

The waterfall structure in the contract is shown in fig. 2.

[6] All the bonds in the collateral are supposed to be fixed rate bonds.

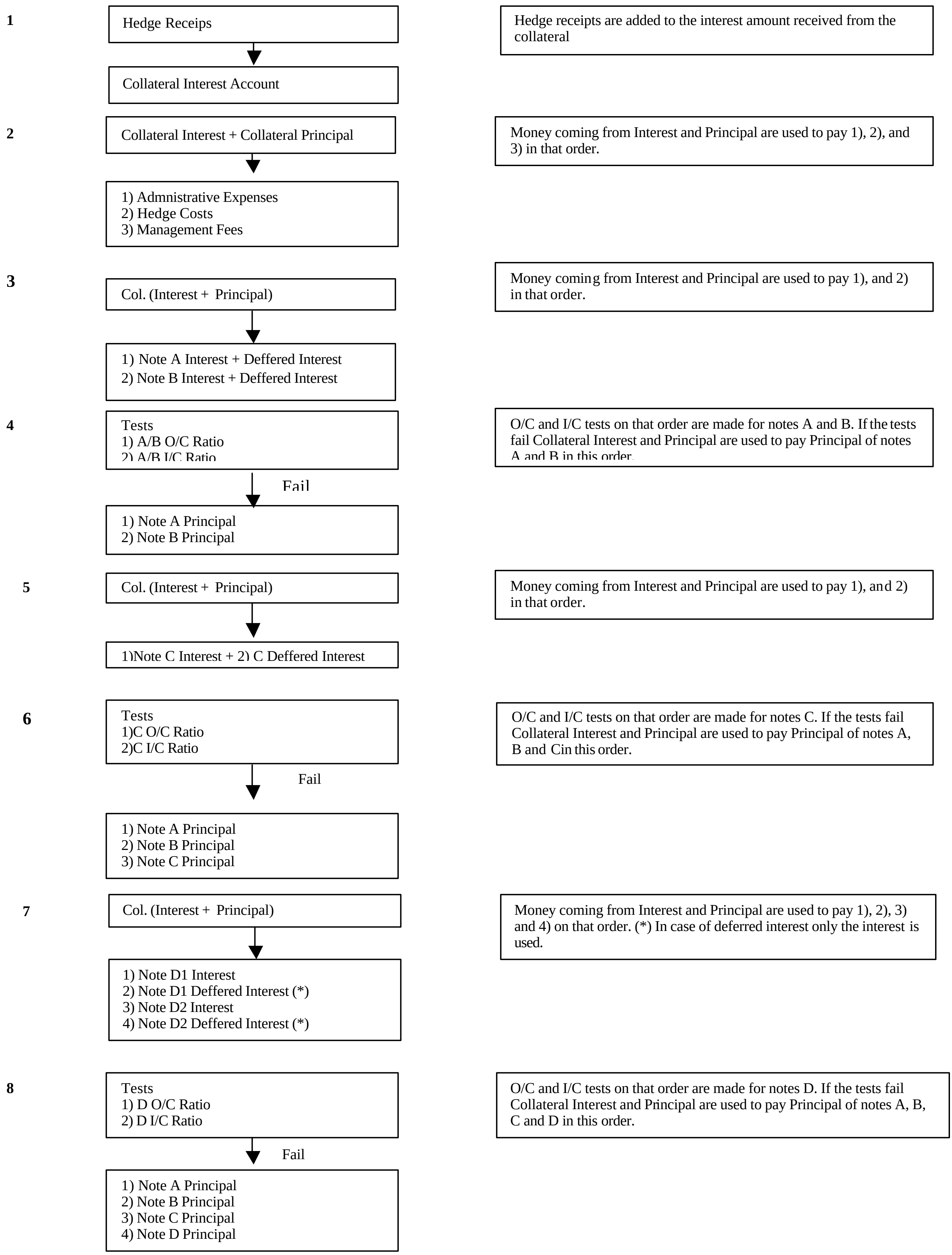

Fig 2 Waterfall structure used in the tests

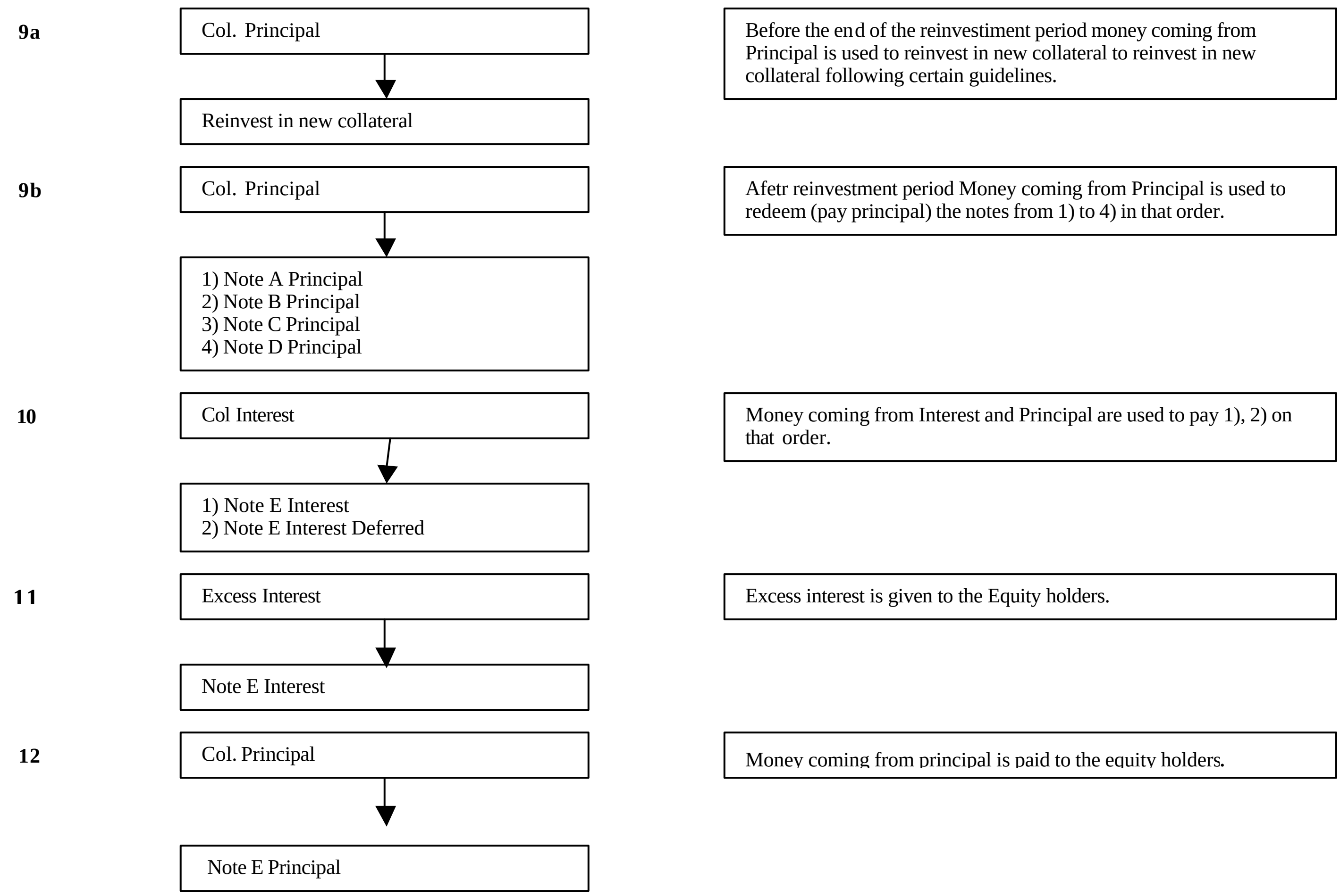

Fig 2 (cont.) Waterfall structure used in the tests

In table 8 we give the stress factors we have used for the probabilities of default and for the recovery rates. As already mentioned the higher the rating the higher the stress factor used.

	Stress Factor	
Rating	**PD**	**Recovery Rate**
AAA	1.50	0.67
AA	1.40	0.67
A	1.31	0.73
BBB	1.23	0.81
BB	1.15	0.89
B	1.00	0.98
CCC	1.00	1.00

Table 8 Stress factors used on the PD's (probabilities of default) and on the RR's (recovery rates) for each target rating.

The yield curve used is shown in table 9.

Time	Rate (%)	Time	Rate (%)
1 W	0.021	**2 Y**	1.454
1 M	0.091	**3 Y**	2.073
2 M	0.182	**4 Y**	2.660
3 M	0.272	**5 Y**	3.079
6 M	0.545	**7 Y**	3.759
9 M	0.817	**10 Y**	4.393
1 Y	1.089	**15 Y**	4.988

Table 9 Yield curve used in the experiment

5.2) The Tests and their Results

In this section we see how changes in different model parameters might affect the results of the different tranches. The EL is calculated using the Present Value (PV) of the coupons and principals as shown in eq. 5.

For the copula model the basic scenario assumes::

a) flat yield curve;

b) 50% recovery rate;

c) the correlation function will be based on two scenarios for intra and inter sector correlations[7]:

   a. a high correlation similar with the one proposed by S&P (see Gilkes ([Gilkes(2002)])) for MBS/ABS with 30% and 10% respectively;

   b. a low correlation similar with one proposed by Moody's (see Moody's ([Moody's(2005)] [8])) with 15% and 3% respectively;

d) the results are computed with 5 runs of 5000 simulations each (and in some cases with 5 runs of 20000 simulations);

For the case of the BET model (as already mentioned) we use the same approach as in Garcia et alli ([Garcia (2003)a]), i.e. when determining the rating of a note the EL has to be lower than the target loss in any of the default scenarios shown in expression 6 (see section 3). Additionally the EL presented in the table is the average of the losses in the 6 default scenarios. For the rest the basic scenario is the same as in a) and b) above.

5.2.1 Parallel Shifts on the Interest Rate Curve

In here we see how parallel shifts in the interest rate curve can impact the EL of the different notes for different model assumptions. The results are shown in Tables 10a and 10b. Generally speaking, and depending on the hedges (swaps and caps) in the structure, the impact of moves on the yield curve can be quite considerable. Observe that all the bonds in the collateral are fixed rate bonds, the A and B notes are floating rate notes while the C down to E are fixed rate notes. This means that once interest rates go up more interest is paid to senior notes letting less excess spread, hurting the lower tranches in the first place. This is indeed what we observe in the results for the shifts in the interest rates varying from –1% up to 2% in steps of 1%.

At this point we can make two observations that will be applicable to most of the results to be shown bellow. The first observation is that comparing the BET and copulas results we see that in general the EL given by the BET is systematically smaller than the one obtained via the copulas. The second observation is that the sensitivity of the different notes for changes in the parameters of the model depends quite considerably on the correlation assumption.

The two observations can be seen in the results on tables 10a and 10b. The results between copula using low (L) correlation and BET is less straightforward: a) for the two senior tranchs both methodologies give identical results; b) for the C tranch BET gives results

[7] In this very recent study Moody's has proposed three possibilities for the correlation parameters. The parameters depend on the following assumptions for inter (%) / intra (%) industry parameters respectivelly: High (20/3), Meddium (15/3) and Low (10/3).

that are consistently higher in any of the scenarios; c) for D1, D2 and E tranches the differences can go from roughly the same to a factor of (roughly) 2 depending on the interest rate scenario. For the high (H) correlation on the table we see higher losses than BET with the exception of the senior tranches. The sensitivity of the note losses with respect to changes in the correlation varies considerably with the interest rate scenario used. See e.g. how the differences in EL between Moody's and S&P correlations vary for the D2 note when one goes from IR –1% and IR + 2%. From the IR – 1% to IR +2% the difference of the EL between the S&P and Moody's correlation varies from a factor of 2 to roughly no difference.

	IR - 1%				**IR**			
	MC (%)		**BET (%)**		**MC (%)**		**BET (%)**	
Tranch	**EL_H**	**EL_L**	**EL**	**Rat.**	**EL_H**	**EL_L**	**EL**	**Rat.**
A	0	0	0	AAA	0	0	0	AAA
B	0	0	0	AAA	0	0	0	AAA
C	0.07	0	0.07	Aa3	0.13	0.01	0.09	A1
D1	2.58	0.62	0.27	Baa1	4.94	1.75	1.24	Baa3
D2	6.07	3.14	3.87	Ba3	10.52	7.02	3.44	B1
E	65.13	69.94	55.12	Caa3	80.22	84.64	77.66	D

Table 10a Expected loss (in % of notional) for the BET and copulas with shifts in the interest rate curves.

	IR + 1%				**IR + 2%**			
	MC (%)		**BET (%)**		**MC (%)**		**BET (%)**	
Tranch	**EL_H**	**EL_L**	**EL**	**Rat.**	**EL_H**	**EL_L**	**EL**	**Rat.**
A	0	0	0	AAA	0	0	0	AAA
B	0	0	0	AAA	0	0	0	AAA
C	0.32	0.02	0.11	Baa1	0.58	0.06	0.38	Baa1
D1	7.41	3.61	2.03	Ba1	11.21	6.55	2.70	B1
D2	16.90	14.38	9.44	B3	26.14	25.12	25.14	Caa2
E	90.55	94.24	92.81	D	96.45	98.42	96.39	D

Table 10b Expected loss (in % of notional) for the BET and copulas with shifts in the interest rate curves.

[8] In Gilkes it is reported that for ABS's the intra- and inter- sector correlations are 30% and 10% respectivelly.

5.2.2) Impact of Changes on the Recovery Rate

In table 11 we can see how changes on the recovery rate (RR) can affect the EL for the different tranches. We present the results for the following three cases of RR's: a) 30%; b) 50% (base scenario); and c) 70%. The lower the RR the higher the expected losses (one gets in case of default of collateral).

In general one expects that the lower the recovery the higher the losses. Moreover the lower the seniority the higher the proportional impact (normally the lower the seniority the lower the amounts of issued notes). For the two senior tranches both BET and the copula (for any of the correlation assumptions used) have the same results of zero EL (the lower tranches have absorbed the losses).

We begin by comparing the results for the two assumptions of correlation in the copulas. As it can be seen on table 11 the results confirm what has been said on the last section. When comparing the results for the two correlation assumptions for the C notes the high correlation presents losses that varies from a factor of 6 (for 30% recovery) to a factor of 10 (for 50% recovery) larger than the low correlation assumption. For the D1 tranch the difference goes from a factor of 12 (with 70% recovery) , to 2.8 (with 50% recovery) and 1.4 with 30% recovery. Finally for the D2 tranch we begin in 3.38 (for 70%) decreasing to 1.5 (for 50%) and again a decrease to 1.13 (for 30%). This means that changes in correlation has an impact on the notes sensitivity to changes in recovery rates.

For the comparison between copulas and BET we see that results depend on the correlation. BET systematically gives lower EL's than copulas for the case of the high correlation assumption, with the exception of the C note and 70% recovery. BET will be more in line for the low correlation assumption for high recovery and high seniority of the notes.

	30%				50%				70%			
	MC (%)		BET (%)		MC (%)		BET (%)		MC (%)		BET (%)	
Tr.	**EL_H**	**EL_L**	**EL**	**Rating**	**EL_H**	**EL_L**	**EL**	**Rating**	**EL_H**	**EL_L**	**EL**	**Rating**
A	0	0	0	Aaa	0	0	0	Aaa	0	0	0	Aaa
B	0.01	0	0	Aaa	0	0	0	Aaa	0	0	0	Aaa
C	1.15	0.19	0.33	Baa1	0.13	0.01	0.09	A1	0	0	0.02	Aa1
D1	12.54	8.04	3.76	Ba2	4.94	1.75	1.24	Baa3	0.60	0.05	0.31	A3
D2	21.02	18.49	11.35	Caa1	10.52	7.02	3.44	B1	2.47	0.73	1.52	Ba1
E	86.35	91.48	86.37	D	80.22	84.64	77.66	D	68.52	70.90	62.84	D

Table 11 Expected loss (in % of notional) for the BET and MC for different assumptions of the recovery rate

5.2.3 Impact of Changes on the Correlation Parameters (copulas Only)

In this section we change the correlation parameters for the copula method. In general one expects that high correlation increases the chances of very extreme scenarios. Such extreme scenarios include situations in which every bond in the collateral goes very good (there is no default) or very bad (plenty of defaults). I.e. high correlation generates scenarios in which no bond defaults or plenty of bonds default. The scenarios with no defaults will impact the equity tranche by reducing its expected loss while the scenarios with plenty of defaults will impact the senior tranches as they will incur in some losses. Indeed this is what is observed in the results in table 12.

Changes in the inter-industry parameters affect the level of correlation among the different industries in the portfolio. Changes in the intra-industry parameter affects how companies in the same industrial sector are correlated with one another. I.e. in a diversified collateral portfolio one would expect that the changes in the inter-industry correlation parameter would have a higher impact (on changes in the EL) than changes in the intra-industry parameter. This is indeed what is observed in table 12. Additionally as observed before an increase in correlation decreases the losses on the equity tranche while increasing the losses in the senior tranches.

	Intra = 30% (%)				**Inter = 10% (%)**			
Tranch	**10%**	**30%**	**50%**	**70%**	**10%**	**30%**	**50%**	**70%**
A	0	0	0.02	0.12	0	0	0	0
B	0	0.05	0.64	1.84	0	0	0	0
C	0.14	1.83	4.27	7.14	0.12	0.14	0.16	0.18
D1	4.62	10.25	11.48	13.21	4.42	4.62	4.80	5.04
D2	10.31	14.68	13.71	14.28	9.99	10.31	10.49	10.58
E	80.33	70.23	55.55	48.85	80.54	80.33	79.86	78.72

Table 12 Impact on the EL (in % of notional) of changes in the Intra and Inter sector correlation parameters.

5.2.4 Impact of Overcollateralization

If overcollateralization decreases there is less proceeds to pay the note holders and then the losses should increase. The losses should affect first the lower tranches and then the senior tranches. Indeed this is what is observed in tables 13a and 13b.

When comparing the two correlation assumptions in the copulas we see again that the sensitivity of the notes are quite different. For a +10% overcollateralization the high correlation assumption gives higher losses for all the notes. With the decrease on overcollateralization the impact of the low correlation for the junior tranches increases significantly. We again see how the correlation assumption can (dramatically) affect the sensitivity of the notes to changes on model parameters (in this case the overcollateralization).

It is interesting to note that for one of the extreme cases analysed (–20%) BET gives losses that are quite in line with the copulas approach. For the other cases one may say that the EL given by BET tends to be lower than the ones given by the copulas method. A word of caution is that the sensitivity of the notes to variation of input parameters varies significantly with the correlation. An example of it can be seen in table 13b for the +10% case in overcollateralization. BET gives higher losses than the low correlation assumption for all the notes but the equity piece.

	-20%				**-10%**			
	MC (%)		**BET (%)**		**MC (%)**		**BET (%)**	
Tranch	**EL_H**	**EL_L**	**EL**	**Rat.**	**EL_H**	**EL_L**	**EL**	**Rat.**
A	0	0	0	Aaa	0	0	0	Aaa
B	0.05	0	0.04	Aa3	0	0	0	Aaa
C	29.77	29.81	27.34	Caa2	2.15	0.72	0.71	Ba1
D1	99.92	99.99	99.97	D	40.61	41.85	14.06	Caa2
D2	99.92	99.99	100	D	71.47	74.62	74.48	D
E	100	100	100	D	100	100	100	D

Table 13a Impact of overcolateralization on the EL (in % of notional) of the notes for the BET and MC approaches

	0%				**+10%**			
	MC (%)		**BET (%)**		**MC (%)**		**BET (%)**	
Tranch	**EL_H**	**EL_L**	**EL**	**Rat.**	**EL_H**	**EL_L**	**EL**	**Rat.**
A	0	0	0	Aaa	0	0	0	Aaa
B	0	0	0	Aaa	0	0	0	Aaa
C	0.13	0.01	0.09	A1	0.01	0	0.05	Aa3
D1	4.94	1.75	1.24	Baa3	0.62	0.06	0.27	Baa1
D2	10.52	7.02	3.44	B1	2.43	0.97	2.62	Ba3
E	80.22	84.64	77.66	D	15.55	12.52	6.10	B2

Table 13b Impact of overcolateralization on the EL (in % of notional) of the notes for the BET and MC approaches

5.2.5 Impact of Changes on the Average Rating of the Collateral Portfolio

Downgrade of the collateral has the impact of increasing the chances of default on the collateral with the consequent increase in the EL. The results here observed are very similar with the ones of the last section. From the two correlation assumptions the sensitivity of the of the low assumption to deterioration of the quality of the collateral is higher. This leads to a faster increase of the losses of the notes when the notch down occurs.

The comparison with the BET also follows approximately the same lines as what we observed in the last sections. The high assumption gives results that are systematically higher than the BET. For the low assumption the C notes presents lower losses while for the remaining notes it presents higher losses than the BET.

	- 2 notches				- 1 notch				Unchanged			
	MC (%)		BET (%)		MC (%)		BET (%)		MC (%)		BET (%)	
Tr.	**EL_H**	**EL_L**	**EL**	**Rat.**	**EL_H**	**EL_L**	**EL**	**Rat.**	**EL_H**	**EL_L**	**EL**	**Rat.**
A	0	0	0	Aaa	0	0	0	Aaa	0	0	0	Aaa
B	0.01	0	0.51	Aa3	0	0	0	Aaa	0	0	0	Aaa
C	3.04	0.94	1.65	Ba1	0.71	0.09	0.54	Baa1	0.13	0.01	0.09	A1
D1	30.60	27.61	7.12	B1	12.55	7.71	4.83	Ba3	4.94	1.75	1.24	Baa3
D2	42.09	43.40	28.45	Caa3	21.78	18.89	12.67	Caa1	10.52	7.02	3.44	B1
E	97.18	99.21	96.63	D	89.96	94.30	90.15	D	80.22	84.64	77.66	D

Table 15 Impact of changes in the rating of the collateral pool on the EL (in % of notional) of the notes.

Rating	1	2	3	4	5	6	7	8	9	10
Aaa	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0001	0.0001
Aa1	0.0000	0.0000	0.0001	0.0002	0.0003	0.0004	0.0005	0.0007	0.0008	0.0010
Aa2	0.0000	0.0001	0.0003	0.0005	0.0007	0.0009	0.0011	0.0014	0.0016	0.0020
Aa3	0.0000	0.0002	0.0006	0.0010	0.0014	0.0018	0.0023	0.0027	0.0033	0.0040
A1	0.0001	0.0004	0.0012	0.0019	0.0026	0.0033	0.0041	0.0048	0.0057	0.0070
A2	0.0001	0.0007	0.0022	0.0035	0.0047	0.0058	0.0071	0.0083	0.0098	0.0120
A3	0.0004	0.0015	0.0036	0.0054	0.0073	0.0091	0.0111	0.0130	0.0152	0.0180
Baa1	0.0009	0.0028	0.0056	0.0083	0.0110	0.0137	0.0167	0.0197	0.0227	0.0260
Baa2	0.0017	0.0047	0.0083	0.0120	0.0158	0.0197	0.0241	0.0285	0.0324	0.0360
Baa3	0.0042	0.0105	0.0171	0.0238	0.0305	0.0370	0.0433	0.0497	0.0557	0.0610
Ba1	0.0087	0.0202	0.0313	0.0420	0.0528	0.0625	0.0706	0.0789	0.0869	0.0940
Ba2	0.0156	0.0347	0.0518	0.0680	0.0841	0.0977	0.1070	0.1166	0.1265	0.1350
Ba3	0.0281	0.0551	0.0787	0.0979	0.1186	0.1349	0.1462	0.1571	0.1671	0.1766
B1	0.0468	0.0838	0.1158	0.1385	0.1612	0.1789	0.1913	0.2023	0.2124	0.2220
B2	0.0716	0.1167	0.1555	0.1813	0.2071	0.2265	0.2401	0.2515	0.2622	0.2720
B3	0.1162	0.1661	0.2103	0.2404	0.2705	0.2920	0.3100	0.3258	0.3378	0.3490
Caa1	0.1738	0.2323	0.2864	0.3248	0.3631	0.3897	0.4139	0.4366	0.4567	0.4770
Caa2	0.2600	0.3250	0.3900	0.4388	0.4875	0.5200	0.5525	0.5850	0.6175	0.6500
Caa3	0.5099	0.5701	0.6245	0.6224	0.6982	0.7211	0.7433	0.7649	0.7858	0.8070
D	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Table 16a Indealized default probabilities up to 10 years calculated from Moody’s tables.

Rating	1	2	3	4	5	6	7	8	9	10
Aaa	0.0000	0.0002	0.0004	0.0006	0.0015	0.0028	0.0042	0.0054	0.0068	0.0084
Aa1	0.0001	0.0002	0.0004	0.0015	0.0026	0.0040	0.0054	0.0059	0.0084	0.0098
Aa2	0.0002	0.0003	0.0008	0.0026	0.0038	0.0054	0.0068	0.0085	0.0101	0.0114
Aa3	0.0002	0.0004	0.0010	0.0027	0.0040	0.0057	0.0072	0.0090	0.0108	0.0125
A1	0.0002	0.0005	0.0014	0.0030	0.0044	0.0061	0.0078	0.0098	0.0119	0.0141
A2	0.0001	0.0006	0.0019	0.0034	0.0050	0.0068	0.0088	0.0110	0.0138	0.0168
A3	0.0005	0.0016	0.0036	0.0061	0.0085	0.0113	0.0143	0.0174	0.0211	0.0251
Baa1	0.0009	0.0030	0.0058	0.0096	0.0132	0.0173	0.0217	0.0259	0.0308	0.0362
Baa2	0.0015	0.0046	0.0085	0.0141	0.0191	0.0247	0.0309	0.0366	0.0429	0.0500
Baa3	0.0042	0.0116	0.0205	0.0306	0.0404	0.0505	0.0603	0.0702	0.0803	0.0909
Ba1	0.0077	0.0209	0.0362	0.0523	0.0687	0.0847	0.0992	0.1146	0.1298	0.1448
Ba2	0.0121	0.0324	0.0558	0.0792	0.1037	0.1271	0.1475	0.1698	0.1912	0.2119
Ba3	0.0288	0.0591	0.0899	0.1182	0.1464	0.1736	0.1992	0.2235	0.2463	0.2708
B1	0.0458	0.0864	0.1249	0.1581	0.1902	0.2213	0.2522	0.2785	0.3027	0.3312
B2	0.0653	0.1174	0.1646	0.2035	0.2398	0.2754	0.3123	0.3410	0.3668	0.3998
B3	0.1029	0.1633	0.2131	0.2540	0.2889	0.3262	0.3622	0.3951	0.4252	0.4578
Caa1	0.1829	0.2507	0.2951	0.3311	0.3527	0.3850	0.4092	0.4453	0.4800	0.5031
Caa2	0.2496	0.3421	0.4028	0.4518	0.4813	0.5254	0.5584	0.6077	0.6551	0.6865
Caa3	0.3095	0.4244	0.4996	0.5604	0.5970	0.6516	1.0000	1.0000	1.0000	1.0000
D	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Table 17 Average Default Probabilities calculated from Moody's data

6) Conclusions

In this work we have compared the BET with the copula approach to analyse cash flow CDO's. Additionally for the copulas approach we have used different assumptions for the correlation: a) one so called high correlation (close to a S&P proposal); b) a second one called low correlation (close to a recent Moody's proposal). We have tested the sensitivity of the EL of each note for the different assumptions of correlation (copulas) and methodology (copulas versus BET) for variations of the following input parameters: a) interest rates; b) recovery rates; c) overcollateralization; d) average rating of the collateral portfolio. An additional section was dedicated to stressing the values of the correlation used on the copulas approach.

The most important conclusion of this paper is that there is a quite significant variation of the sensitivity of the notes under different assumptions of correlation for variation on the input parameters (e.g. interest rates and state of the collateral). The reason for this complex behaviour is the way correlation impacts the cash flow coming from the collateral: the increase of correlation tends to create more uniform states generating situations in which every note defaults or nobody defaults. The scenarios of no defaults decrease the losses for the junior tranches while the scenarios of high defaults increase the losses of the senior tranches.

A consequence of this change in sensitivity due to correlation is that it becomes quite an art to interpret the results given by the model. In general one might say that BET will give losses that are lower than the copulas with the high correlation assumption while more in line with the low correlation assumption. But there are situations for which BET will give higher losses than both.

We think that BET has a lot of advantages due to its simplicity, quickness in generating its results and less cumbersome in the amount of input from the collateral data. The problem is that the correlation assumed in it is hidden in the DS methodology. I.e. its simplifications might not be enough to catch all the complexities of the instrument. The advantage of copulas relies in its flexibility. The disadvantage relies in its additional complexity: the increases in collateral information and in mathematical complexity and the fact that it might require a large amount of time consuming simulation are just three examples of its additional demands.

Given the constraints imposed by both models we think that when analysing a cash flow CDO it might be a good practice to have both models at hand. The continuation of this study is the analysis of the models for the case of synthetic CDO's.

Acknowledgments
João Garcia would like to express his deeply gratitude to Eric Hermann (Head of Global Risk Management at Dexia Group) for his patience, trust, support and numerous incentives when in the creation of the Credit Methodology in Dexia Group. Without his commitments and management skills none of the codes here used would have been implemented. He would like to thank Ronny Langendries (from the Credit Methodology Dexia Group), Guy Raeves (credit derivatives trader at Dexia Bank Belgium), Geert Gielens (Senior Quantitative Dealer at Dexia Bank) for numerous practical discussions. Finally he would like to thank for the market makers who provided us with their CDO data used in this article (the data was provided on the condition of anonymity).

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