Pricing Baskets using Gaussian Copula and BET Methodology: a Market Test

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In this article we show how the copula and the BET methodology can be used to price baskets of credit derivatives. After brief descriptions of both algorithms we use market data to price first to default and first loss contracts on baskets of credit derivatives. Additionally the copula approach is used with Moody and Standard and Poors correlation functions (this makes things simpler to back office to have a quick present value of baskets) and different recovery assumptions. All the tests are made using market data and the results are then compared with prices given by market makers.

1) **Introduction**

Over the last few years the market for credit derivatives has experienced a tremendous growth. Several aspects have contributed to this start of the matter: a) the need of (financial) institutions to hedge the credit exposures in their portfolios; b) the possibilities of those institutions to optimise the allocation of their capital while at the same time fulfilling the requirements of Basle II.

Two classes of credit derivative contracts can be discerned: single name and multiname contracts. In single name instruments the payoff depends on the credit events of one underlying entity. Examples of such securities are credit default swaps (CDS's), single name credit linked notes (CLN's), credit spread options (CSO's), and total return swaps. The most liquid single name credit derivative is the CDS, representing about 40% of the credit derivatives market (see the British Bankers Association report [1]). It provides insurance against the losses due to a default of one entity called the reference entity. In the plain vanilla version of the contract the buyer pays a periodic fee in exchange for a lump payment by the protection seller in case an event of default happened. We refer to Takavoli [2] and Schonbucher [3] for a more detailed description of single name instruments.

In multi name contracts the payoff is contingent on the credit events of a portfolio of entities. Examples are colateralized debt obligations (CDO), first loss and n^{th} to default basket derivatives and

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multi name credit linked notes. In a first to default basket the protection buyer pays a fee in return for a lump payment by the protection seller for the first entity to default in the portfolio. The main reason why one would want to use a first- to-default derivative is that it makes protection on a portfolio cheaper. Consider for instance an investor who wants to hedge the credit exposure on a portfolio of 5 bonds. He/she might feel that for his/her investment horizon the risk on more than one default is rather small and instead of buying protection on each single name in the portfolio he/she might choose to buy protection to one default [2]. As a consequence the losses in the portfolio might be reduced (and consequent improvement in its rating).

In a first loss basket the protection buyer pays a fee to receive protection on a certain amount. Instead of buying protection on the number of entities to default this derivative allows the investor to focus directly on the possible amount lost. In our example, suppose the protection nominal in each name was 10 millions USD. The investor might think that losses exceeding 30 mi USD are unacceptable. He/she might want to buy a first loss basket yielding credit protection up to 30 mi USD. If the recovery rate for each entity is high the investor might finish by having bought full protection. As before a first loss basket might be used to improve the rating of the portfolio.

In this paper we show how to price a first to default and a first loss credit derivative. Our objective is to describe and test (using market data) two different approaches that we have seen being used by market participants. We base our copula algorithm on the paper of Li [4]. In this paper we have tesd some practical aspects for pricing the contracts. First we make use of *market default probabilities* implied by (extracted from) the CDS market for each underlying name. Second we use the copula algorithm with three correlation functions: a) Moodys (for ABS/MBS products); b) Standard and Poors; and c) the one evaluated from equity returns. Third all the data used is taken from the market while the prices are compared with the bid offer given by market makers (the prices were given by market makers on the condition of anonymity). At last we use the BET algorithm to price the same securities and compare both approaches.

The remainder of the paper is organized as follows. In section 2 we define what will be seen as default correlation. How to imply default probabilities from the CDS market is shown in section 3. We describe what copula functions are and why we will be using gaussian copulas in section 4. In section 5 we show how default correlation, marginal default probabilities and copula functions are put together to price first to default and first loss basket derivatives using the copula methodology. In section 6 we briefly describe the BET approach. In section 7 use market data to price a first to default and a first loss using both methodologies and compare it with a bid quote given by a market participant. Conclusions are drawn in section 8.

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[2] If we assume that the entities in the portfolio are not independent then the first to default basket fee will be

2) Default Correlation

When pricing a credit derivative on a portfolio of credits there are at least two important aspects to be taken into account: a) the default correlation among entities, and b) the joint default probability function. Both quantities should somehow be calibrated to market data. In this section we discuss the issues on assigning a default correlation matrix to a basket of entities.

In the credit derivatives literature there are two main approaches currently being used to define the default correlation.

In an approach described e.g. in Lucas [5] and Gupton et al [6] (and preferred by credit rating agencies) one discretizes the period of observation and defines default as a survival or death event in the observed period. In this way, consider one chooses a one year period interval and let p_1 and p_2 be the one year marginal probability of default of companies C_1 and C_2 respectively. Consider p_{12} as the joint default probability for the two companies for the same period. The (one year) default correlation ($Corr(C_1,C_2)$) between C_1 and C_2 is then given by:

$$Corr(C_1, C_2) = \frac{p_{12} - p_1 \cdot p_2}{\sqrt{p_1 \cdot (1 - p_1) \cdot p_2 \cdot (1 - p_2)}}. \qquad (1)$$

In order to evaluate the correlation one will have to define companies by ratings and get data for a certain period of time. Problems such as determination of the period of observation, inclusion or exclusion of companies in the pool (the rating category) during the period of observation and so on make the methodology rather tricky to be used. We refer to Li [4] for more details.

Another approach (the one employed in this paper) exploits the notion of *survival times* commonly used in actuarial analyses and in reliability theory (see Barlow and Proschan [7]). In this approach the survival time is a random variable and it will represent the time up to default of each entity in the basket. The default correlation between two reference entities is defined as the correlation between their survival times.

Consider T_1 and T_2 the survival times (or default times) of the two entities C_1 and C_2. The default correlation between the two companies is then given by:

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cheaper than buying protection in each individual name.

$$Corr(C_1, C_2) = \frac{E(T_1, T_2) - E(T_1) \cdot E(T_2)}{s_{T_1} \cdot s_{T_{12}}}, \qquad (2)$$

where E is the expectation operator and s denotes the standard deviation of the survival time.

We will also assume that default correlation can be evaluated from the equity markets. This is the hypothesis made in CreditMetrics and KMV and follows from the firm value model developed by Merton[3] [8]. I.e. in order to evaluate the correlation between two entities we will take time series of stock returns, standardize (see section 5) and evaluate the correlation between them. We refer to Embrechts et al [9] and Frey et al [10] for alternative ways of evaluating the correlation matrix in a latent variable model.

An advantage of the approach here used is that equity data is easily available in comparison with default times. Moreover the use of equity data permits an easy calibration of the input parameter to be used in a gaussian copula function (as will be seen in section 4).

3) **Marginal Default Probabilities and the CDS market**

In a standard credit default swap (CDS) a protection seller receives a fixed periodic (say every quarter) premium in order to give protection in case of default on a certain reference. Consider that the notional involved is N, the recovery rate in case of default is a and the CDS rate is K_T for a T year contract. By a CDS the contract buyer would be paying 0.25 * K_T * N every quarter in order to receive (1 – a) * N in case of default. Below we give a quick explanation for the evaluation of the CDS rate and we refer the interested reader to the work of Martin et al. [11] or Hull and White [12] for more details.

In our approach we consider that the default time follows a Cox process (see Bremau [13] or Lando [14]). In this way if Q(T) is the probability of default occurring at time T, and ?(t) is the instantaneous *hazard rate* of default at time t, then Q(T) is given by:

$$Q(T) = 1 - e^{-\int_0^T l(t)dt}. \qquad (3)$$

[3] Merton model is a so called latent variable model. In it default happens when the latent variable (equity price) of the reference entity reaches book value.

Suppose CDS premiums are paid at a set of dates (every quarter) that we will represent by t_1, t_2, … , t_n. The expectation of the present value (PV) of the cash flows to be paid by the CDS buyer is then given by:

$$PV_{Buyer} = \sum_{i=1}^{n} 0.25 \cdot D(t_i) \cdot K_T \cdot N \cdot (1 - Q(t_i)), \qquad (4)$$

where $D(t_i)$ is the t_i risk free discount factor, and we assume that payments are made quartely and the CDS rate K_T is given yearly.

The PV of the payment the CDS seller may have to make in case of default is given by the discounted value of the non-recovered fraction weighted by the probability of default occurring at each payment date. The PV of the non-recovered part in case of default is given by:

$$PV_{seller} \cong (1-a) \cdot \sum_{i=0}^{n} 0.5 \cdot (D(t_i) + D(t_{i+1})) \cdot N \cdot (1 - Q(t_i)) \cdot q(t_i, t_{i+1}), \qquad (5)$$

where $q(t_i,t_{i+1})$ is the probability of default between t_i and t_{i+1} given that it survived until time t_i and it is related to $Q(t_i)$ in the following recursive way:

$$\begin{aligned} Q(t_{i+1}) &= Q(t_i) + (1 - Q(t_i)) \cdot q(t_i, t_{i+1}) \qquad (6) \\ Q(t_0) &= 0 \end{aligned}$$

In eq. 5 we have made the approximation that the discount factor for a default inter-time period is the arithmetic average of the extremes in the interval.

As we assume no-arbitrage at the beginning of the contract, the two sides (seller and buyer) agree on the equality of their respective PV's and the CDS rate is then given by:

$$K_T \cong \frac{(1-a) \cdot \sum_{i=0}^{N} 0.5 \cdot (D(t_i) + D(t_{i+1})) \cdot (1 - Q(t_i)) \cdot q(t_i, t_{i+1})}{\sum_{i=1}^{N} (1 - Q(t_i)) \cdot D(t_i)} \quad . \qquad (7)$$

Market quotes for CDS's of different maturities are then used in an optimization algorithm to imply default probabilities. We refer to Martin et al [11] for more details.

4) Copula Functions

In order to generate default times of the entities in a basket portfolio we need to determine a joint distribution function. This joint distribution must fulfil two requirements: a) it recovers the uni-dimensional marginal distributions implied from the CDS market; b) it recovers the two dimensional default correlation matrix from the entities in the portfolio[4].

In this work we use copula functions to generate correlated default times that follow given marginal distributions. We will give a very brief description of the use of copulas. We refer to Nelsen [15] for a complete mathematical description, to Frees and Valdez [16] as a very good survey paper, and to Li [4], Schonbucher [17], and Frey et al [18] on the use of copulas in pricing baskets.

In order to understand how the copula is used consider we have a basket on M entities and $P_1(x_1)$, $P_2(x_2)$, …, $P_M(x_M)$ are the marginal distributions of default times implied from the CDS market for each entity (as described in section 3). A copula function C is a function from $(0,1)^M$ to (0,1) such that:

$$C(P_1(x_1), P_2(x_2), \ldots, P_M(x_M)) = P(x_1, x_2, x_3, \ldots, x_M), \tag{8}$$

where P is the joint distribution function of default times. It is well known from Sklar [19] that under some standard technical conditions (e.g. continuity of the marginals) any such a multivariate distribution function can be written in the form of a copula function.

A common market practice is to use the multivariate normal copula that is defined as:

$$C(x_1, x_2, \ldots, x_M) = N_M(N^{-1}(x_1), N^{-1}(x_2), \ldots, N^{-1}(x_M), \Sigma), \tag{9}$$

where N^{-1} is the inverse of the univariate standard normal distribution, and N_M is the multivariate normal distribution with correlation matrix function S.

[4] One should be aware that given a joint distribution function one can determine the marginals and the correlation matrix uniquely but not the other way around. I.e. given the marginals and the correlation matrix there is not a unique solution for the joint distribution.

Most of the market participants we are aware of have chosen the normal copula function as here described for the pricing of basket derivatives. A first (and admittedly academic weak) practical reason for it is that multinormal distributed numbers are usually readily available in any numerical package.

A second practical reason is that using gaussian copulas eliminates the necessity for a calibration to a possible default correlation function. I.e. for any other alternative choice of the copula function one will still have to calibrate its parameters in order to recover the correlation matrix. This would be done by the use of eq. 1 where p_1 and p_2 come from the marginal distributions and p_{12} comes from the copula function with the appropriate parameter to recover the observed correlation. But if one assumes the validity of Merton firm value model, default correlation may be taken from the equity market (time series of stock returns). It can be proven that this correlation is the correlation parameter used in the gaussian copula function and the calibration is done by construction, eliminating the time consuming calibration for correlation. We refer to Li [4] and Gupton et al [6] for more details.

Although mathematical convenience has been the driving force behind the market and our preference for the gaussian copulas, we are well aware of possible limitations on the approach. We refer the interested reader to the work of Marshal et al [20] and Schonbucher [17] for the use of the alternative copulas and to Frey et al [10] for a critic on the use of asset correlations.

5) Pricing Baskets: The Copula Algorithm

In this section we show the steps to price a first to default and a first loss basket. We need to price a first to default or a first loss on a basket containing M entities and the expiration time of the baskets is T_{exp}. The necessary data to price the securities are: a) the yield curve; b) market CDS spread curves for each individual name; c) recovery rate (in case of default) for each name; d) the correlation matrix (equity returns) of the entities in the basket.

We will divide the algorithm in two steps: i) we show how to generate default times; ii) we show how to use the default times generated in step i) to evaluate the first to default (in section 5.1) and the first loss derivative (in section 5.2).

The steps to be taken in the generation of the default times are the following:

a) for each name in the basket estimate the default probability distribution function (see e.g. Martin et al. [8]);
b) from a multivariate gaussian distribution with correlation matrix G (correlation matrix of equity returns) generate M numbers. Consider that $X = [x_1, x_2, \ldots, x_M]$ is the vector of generated numbers;

c) for each number generated in step b) and using a standard normal distribution function evaluate the default probabilities p_i such that:

$$p_i = \int_{-\infty}^{x_i} n(y)dy, \qquad (10)$$

where n(y) is the normal density function with average 0 and standard deviation 1.

d) for each generated number in c) and the marginal distributions of default implied in step a) retrieve the default times. I.e. evaluate t_i such that:

$$p_i - F_i(t_i) = 0 \quad . \qquad (11)$$

The vector of points T = [t_1, t_2, …, t_M] are the simulated default times and F_i is the marginal cumulative distribution function for the asset i evaluated in step a).

From Merton model one can evaluate the correlation function used in step b) above from normalized time series of equity returns. Consider we have used the algorithm described above to generate a number N_{sim} of simulations of default times. The simulated default times are going to be used to price a first to default and a first loss basket.

Assume that T_{exp} is the expiration time of the baskets (first to default or first loss), a_i is the recovery rate of the reference entity *i*, and K_T is the basket rate we want to evaluate. In addition suppose that the exposure for entity *i* is N_i, while N_T is the total exposure in the basket. I.e. if entity *i* defaults the protection seller will cover $(1-a_i) * N_i$. The present value of the cash flow paid by the protection buyer in a certain simulation *k* (PV^k_B) is given by (see eq. 4):

$$PV^k{}_B = \sum_j^n D(t_j) \cdot K \cdot N_T. \quad . \qquad (13)$$

The index *n* in the equation above depends on the type of basket: a) for a first to default it represents the smaller between the time of the first default or the time of contract expiration; b) for a first loss it means the smaller between the time when the amount of protection loss is reached and expiration of contract time. Define by PV^k_N the present value of the notional in simulation *k* if it would be paid at every payment date in the contract, i.e.:

$$PV^k{}_N = \sum_j^n D(t_j) \cdot N_T \,, \qquad t_n = \min(t^k{}_d, T_{\exp}) \qquad (14)$$

where t^k_d is the default time for simulation k. In this way:

$$PV^k{}_B = K \cdot PV^k{}_N \quad . \qquad (15)$$

Analogously the present value of the cash flow of the protection seller for a simulation k (PV^k_S) is given by:

$$PV^k{}_S = \begin{cases} (1-a_i) \cdot D(t^k{}_d) \cdot N_i & if \quad t^k{}_d \le T_{\exp} \\ 0 & otherwise \end{cases} \qquad (16)$$

and we assume that at simulation k the default may have occurred with entity i.

5.1) First to Default Basket

For a first to default basket the pricing algorithm is the following:

a) for each simulation k select the minimum time from the vector of simulated default times (the vector T generated at step d) of last section).;

b) for each simulation k evaluate the PV^k_N and the PV^k_S using eq.14 and 16 respectively;

c) evaluate the average of the present value of the fees paid ($<PV_S>$) and of the notional received ($<PV_N>$) in all simulations. I.e.:

$$< PV_A > = \frac{\sum_k^{N_{sim}} PV_A^k}{N_{sim}} \quad . \qquad (17)$$

where A above maybe N (notional) or S (seller);

a) evaluate the first to default basket rate K which is given by:

$$K = \frac{< PV_S >}{< PV_N >} \quad . \qquad (18)$$

5.2) First Loss basket

Consider that N_i is the notional for each entity i (the maximum protection on an individual name) and N_P is the maximum loss in the basket (the total *protection* for which the investor is covered). As an example consider a basket with five names, each with a notional of 10 mi USD (maximum protection per name (N_i)). Assume the recovery rate for each name is 50% and the maximum loss in the basket is 15 mi USD (N_P). Under this simple example a first default would involve the payment of 5 mil USD by the protection seller and the new maximum loss notional in the basket would reduce to 10 mi USD. The contract continues existing until it expires or N_P reaches zero.

For the case of a first loss basket the evaluations of the fees to be paid (by the protection buyer) and the insurance (by the protection seller) depends on the following points: a) notional in each entity (it might differ from entity to entity); b) the recovery factor of each entity; c) the maximum loss covered at the time a default happens; d) the time of the default.

The algorithm used for pricing a first loss basket is the following:

a) evaluate the default times that occur before the expiry of the contract;

b) evaluate the remaining capital protection after each default and check if the remaining protection (N_P) is still higher than zero. In case N_P goes to zero the contract terminates. The present value of the notional at simulation k is given by:

$$PV^k{}_N = \sum_j^n D(t_j) \cdot N_P(t_j), \qquad t_n = \min(t_d, T_{\exp}), \qquad (19)$$

and where $N_P(t)$ is the outstanding protection at time t. The present value of the amount to be paid by the protection seller at simulation k ($PV^k{}_S$) is then:

$$PV^k{}_S = \begin{cases} \sum_{j}^{n_d} D(t_d{}^j) \cdot N^d{}_i & if \quad t^j{}_d \leq T_{\exp} \\ 0 & otherwise, \end{cases} \tag{20}$$

where n_d is the number of defaults, t_d^j is the time of the jth default, $N^d{}_i$ is the amount covered by the t_d^j -default and is given by:

$$N_i{}^d = \begin{cases} (1-a_i)\cdot N_i & if \quad (1-a_i)\cdot N_i < N_P(t_d) \\ N_P(t_d) & otherwise, \end{cases} \tag{21}$$

and assuming that the default occurred with entity i;

c) evaluate the present values of buyer, seller and the basket rate analogously as it is done for the first to default basket (see eq. 17 and eq. 18 respectively).

In the next section we give the results of our simulations.

6) The BET Approach

The BET methodology is based on the concept of *diversity score* (DS) and it is an application of the binomial formula from probability theory to a simplified version of the portfolio. We will show one way the market participants use the BET to generate a rating and then a price for a basket structure. We refer to Garcia et alli [23] for a far more detailed study of the BET methodology.

The technique consists in simplifying the portfolio maintaining the same default behaviour of the real one. The basic idea is to map a portfolio of heterogeneous correlated securities with distinct probabilities of defaults into a portfolio of independent securities with the same probability of default. After the mapping is done it is warranted to apply the binomial formula to estimate the expected loss on the portfolio. As adherence of the portfolio to a rating category is determined by its expected loss, one can estimate the rating category of the portfolio once one has evaluated the expected loss. The last step is then to exploit the relation between rating and spread to obtain a valuation of the structure.

We will describe the methodology in somewhat more detail. For simplicity we will assume the collateral portfolio to be composed of bonds issued by different entities.

The first step consists in reducing the portfolio of correlated credits into a portfolio of independent assets. It is achieved by calculating the DS of the portfolio. The DS is assumed to be the number of independent homogeneous assets that gives the same loss distribution as the actual portfolio. It is a function of the default correlation of the names in the portfolio. There are two different ways of calculating the DS of a portfolio. In a first approach one uses a table to map the number of companies in a certain sector to its diversity score (see section 7.2.2 for more details and data on how it is done). In a second approach the DS is calculated from the correlation, the face value, and the probabilities of defaults of the entities in the portfolio, and we refer to Cifuents et al [22] for more details. Here we will be using the first approach as it is more widespread in dealing rooms.

In the second step one calculates the *weighted average rating factor* (WARF). It represents the average rating of the portfolio and as such of the idealized assets. Assume each name *j* in the real portfolio has a rating factor given by RF_j (it is given by the Moodys 10-year estimation of the cumulative probability of default multiplied by 10,000). The WARF is then given by eq. 22 below:

$$WARF = \frac{\sum_{j}^{n} RF_j \cdot N_j}{N}, \qquad (22)$$

where N is the total nominal in the portfolio and N_j is the notional on bond j.

In the third step one uses the WARF and the maturity of the contract to determine the average rating of the generalized bond and its cumulative probability of default. As the generalized bonds are assumed to be independent one can calc ulate the probabilities of 0 up to DS (idealized bond) defaults in the time period of the contract. Assuming that p is the cumulative probability of default the probability P_i of *i* such a bonds defaulting is given by:

$$P_i = \frac{DS!}{i! \cdot (DS-i)!} \cdot p^i \cdot (1-p)^{DS-1}. \qquad (23)$$

In order to reduce the impact of the errors due to all the simplified assumptions in the model it is a common market practice to multiply the probability of default p by a stress factor ?:

$$P_i = \frac{DS!}{i! \cdot (DS - i)!} \cdot (g \cdot p)^i \cdot (1 - g \cdot p)^{DS-1}. \qquad (24)$$

In the fourth step one uses the probabilities evaluated in eq. 24 above and the average recovery rate (it is an input to the algorithm) to calculate expected loss (EL) in the idealized portfolio. Suppose a is the average recovery rate and N_{avg} is the nominal of each idealized bond (given by the total nominal in the portfolio N divided by the diversity score). The EL is then given by:

$$EL = \sum_{i=0}^{DS} (1 - a) \cdot N_{avg} \cdot P_i. \qquad (25)$$

In the fifth step one uses the EL and the maturity of the contract to calculate the default probability and the rating of the portfolio. Consider that P_D would be the cumulative probability of default on the portfolio seen as a whole. The expected loss is related with the default probability of the portfolio by the expression:

$$P_D = \frac{EL}{(1 - a) \cdot N}. \qquad (26)$$

Once the probability of default and the maturity are used to determine the rating of the portfolio we are ready to determine the premium to be paid. In what follows we will show one possible alternative for this step.

The fee to be paid is calculated by mapping the rating with an appropriate spread. This mapping is more an art than a science as the methodology is not unique. There are several ways the mapping can be done and each of those ways might lead to different prices. One possibility is to find in the market curves that link spreads to rating *and* sector. The problem with this approach is that these curves do not exist for every combination of rating and sector. As it will be seen in section 7 for the rating Baa3 we could find curves only for industrials, telecom and utilities. Another possibility is to use curves relating rating (only) to spread. Alternatively one would use curves relating sectors to spread. Those curves are available in the market from large investment banks like Goldman Sachs or Morgan

Stanley (MSCI bond index) to mention only two. One may still decide to take a weighted average of those curves.

For simplicity in this study we will be using the spreads of the industrial sector for the given rating.

7) Empirical Results

7.1) First to Default

Consider a first to default basket contract composed by five reference entities: Boeing, Disney, General Electric, Goldman Sachs, Hewlett Packard Compaq.

The CDS quotes used to imply the default probabilities are shown in table 1.

	CDS Quotes (in bp) for different maturities (in year)					
Company	**1 y**	**2 y**	**3 y**	**4 y**	**5 y**	**6 y**
Boeing	34	45	49	54	57	54
Disney	50	63	67	77	84	92
General Electric	36	42	45	54	60	60
Goldman Sachs	29	37	39	46	50	54
HPQ	78	71	69	63	60	58

Table 1 CDS Quotes in bp taken from a market participant on the date of the evaluation

For pricing purpose we have assumed a standard recovery rate of 20% for every company in the basket. This was the recovery value used by several market participants (in dealing rooms) at the date of pricing (Jan 21st 2003).

The risk free discount curve used for the pricing is reported in table 2.

Date	Discount Factor
21/01/2003	1
21/02/2003	0.998829
21/03/2003	0.997763
21/04/2003	0.996587
21/05/2003	0.995454
23/06/2003	0.994211
21/07/2003	0.993109
21/10/2003	0.989430
21/01/2004	0.985192
21/01/2005	0.959950
23/01/2006	0.923584
21/01/2007	0.882579
22/01/2008	0.838816
21/01/2009	0.795279

Table 2 Risk Free Discount factors used in the pricing of the First to default

The default probability curves (for each name) were implied from the CDS curves of table 1 as described in Martin et al. [11].

In order to generate the joint default probabilities we have used a normal copula function. The covariance matrix parameter used in the normal copula was taken from the equity market and is reported in table 3.

	Boeing	**Disney**	**General Electric**	**Goldman Sachs**	**HPQ**
Boeing	1	0.281696	0.410516	0.122576	0.159480
Disney	0.281696	1	0.445367	0.343534	0.378281
General Electric	0.410516	0.445367	1	0.455073	0.337691
Goldman Sachs	0.122576	0.343534	0.455073	1	0.439621
HPQ	0.159480	0.378281	0.337691	0.439621	1

Table 3 Covariance Matrix from three year time series taken from the equity market

The price for the 5 year first to default basket using the algorithm described in section 5.1 and a bid-offer quotes from a market maker are shown in table 4.

	Basket Rate (bp)
Model Rate	257
Market (Bid-Offer)	205-265

Table 4 CDS Premium for a first to default basket

As it can be seen in table 4 the rate given by the model falls inside the market maker bid-offer spread. Observe that we have used mid quotes to imply default probabilities (table 1). Another observation is that those quotes are not homogeneously liquid. I.e. quotes for the three and five years are far more liquid than any of the others that might have been obtained through interpolation (we have obtained all the spreads used to imply the default probabilities from another market maker). This can explain why the model rate is not closer to the mid of the bid-offer.

7.2) First Loss Basket

Here we will price two first loss baskets: one quoted in dollar and the other in euros. The two portfolios might be seen in table 5.

The contracts offer protection of up to 30 mi euros and 32 mi euros and USD respectively and of up to 10 mi (euros/USD) loss for each name. The portfolio contains 16 and 17 names respectively for the euros and USD contracts. The sector names used for the evaluation of the DS (see section 7.2.2) are included in table 5 (we used the industry classif ication table of Moody's). In section 7.2.1 we present the results of the copula method and in section 7.2.2 for the BET methodology.

Euro Contract			USD Contract		
Entity	**Sector**	**Rating**	**Entity**	**Sector**	**Rating**
AXA SA	Insurance	A1	**Boeing Co.**	Defense	A1
ABN Amro	Banking	Aa3	**Bank of America**	Banking	Aa3
Akzo Nobel	Chemicals	A2	**Citigroup**	Banking	A2
Basf Ag.	Chemicals	Aa3	**Coca Cola**	Food/Beverage	Aa3
Bayer Ag.	Chemicals	A2	**CSC**	Computers/Peripherals	A2
BMW Ag	Autos	A1	**Disney**	Entertainment	Baa1
BP Amoco	Oil	Aa1	**General Electric**	Electrical	Aa3
BAT	Tobacco	A2	**Goldman Sachs**	Securities	Aa3
Carrefour	Retail	A1	**Hewlett Packard**	Computers/Peripherals	A3
Commerzbank	Banking	A1	**IBM**	Computers/Peripherals	A1
Ahold NV	Retail	Baa1	**Lockheed Martin**	Defense	Baa2
Marks & Spencer	Retail	A3	**Lehman Brothers**	Securities	A2
Philips NV	Electronics	A3	**McDonalds Corp.**	Food/Beverage	Aa3
Peugeot	Autos	A2	**Merrill Lynch**	Securities	Aa3
Siemens	Electronics	Aa3	**Morgan Stanley**	Securities	Aa3
Total Fina	Oil	Aa2	**Philipp Morris**	Tobacco	A2
-	-		**Wal Mart**	Retail	Aa2

Table 5 First Loss baskets. The maximum protection per name is 10 mi euros/USD. The total protection is 30 mi euros and 32 mi USD respectively. The sector names were taken from Moody's.

7.2.1) The Copula Method

In here we will price the first loss baskets using the gaussian copula approach as explained in section 5. The yield curves used in the study are shown in table 6. The marginal probabilities of defaults have been implied from the CDS market and the market CDS quotes (as of the date of pricing) are shown in table 7. When using the copula approach we have tested the prices with three different correlation functions: a) taken from Moodys; b) taken from S&P (Standard and Poors); c) evaluated from equity returns.

Date	Discount Factor (euro)	Discount Factor (USD)
28/10/2002	1	1
29/10/2002	0.99991	0.99995
04/11/2002	0.99935	0.99965
28/11/2002	0.99717	0.99842
28/01/2003	0.99181	0.99529
28/04/2003	0.98442	0.99077
28/10/2003	0.96943	0.98086
28/10/2004	0.93524	0.95049
28/10/2005	0.89582	0.91171

Table 6 Discount factors used to price the first loss baskets

For both Moody's and S&P we have used the correlation functions they give for quoting their CDO structures (it is the same used for ABS/MBS structures). Both agencies use correlation based on industry and sectors (inter-industry correlations and intra-industry correlations). We have used an inter-industry correlation of 0 for both cases while an intra-industry of 0.3 and 0.1 for S&P (see Bergman [24]) and Moody's (available upon request) respectively. Bellow we give the relation of companies that belong to the same sector (we have used Moody's industry table to make the groupings):

a) euro basket: (ABN Amro, Commerzbank), (Akzo Nobel, Basf, Bayer), (BMW , Peugeot), (BP Amoco, Total Fina), (Carrefour, Ahold), (Philips, Siemens);

b) USD basket: (Boeing, Lockheed Martin), (Bank of America, Citigroup), (Coca Cola, McDonalds), (CSC, Hewllett Packard, IBM), (Goldman Sachs, Lehman Brothers, Merrill Lynch).

The correlations extracted from the equity returns are shown in tables 8a and 8b for the euros and USD contracts respectively. We have used 3 years of weekly prices to compute the correlation function.

In order to understand the impact of the recovery rate we have computed the prices of the baskets under three different assumptions for the recovery rate: a) 20%; b) 36.2%; and c) 45%. The 36.2% is the average recovery rate for senior unsecured speculative grade bonds for 2001 as reported in Moodys [21]. The 45% recovery is approximately the 1982-2000 average recovery rate for the senior unsecured speculative grade recoveries as reported in the same Moody's [21]. Finally the 20% was a stressed value we got to after talks with market participants at the time of the pricing.

The results of the pricing using different correlations and default assumptions (for the copula approach) are shown in tables 9a and 9b. In the bottom line of the table we have added the bid and offer quotes of a market maker.

We report in tables 10a and 10b the relative errors defined by the difference of the value found and the mid price divided by the mid price.

Company	1yr (bp)	3yr (bp)	5yr (bp)
Axa S.A.	121	121	140
ABN Amro	12	29	43
Akzo Nobel	41	54	65
Basf Ag	19	25	33
Bayer Ag.	41	74	85
BMW Ag	22	33	45
BP Amoco	15	18	25
BAT	48	75	85
Carrefour	35	45	55
Commerzbank	116	127	150
Ahold	142	190	200
Marks & Spencer	39	47	55
Philipps NV	83	139	150
Peugeot	35	50	65
Siemens	35	70	75
Total Fina	14	20	25
-	-	-	-

Company	1yr (bp)	3yr (bp)	5yr (bp)
Boeing Co	66	95	110
Bank of America	21	36	48
Citigroup	25	37	50
Coca Cola	25	32	40
CSC	136	155	175
Disney	69	92	115
General Electric	56	70	94
Goldman Sachs	53	71	90
Hewlett Packard	182	161	140
IBM	50	57	70
Lockheed Martin	53	53	60
Lehman Brothers	77	102	92
McDonalds Corp	22	31	40
Merril Lynch	98	115	98
Morgan Stanley	47	64	85
Philip Morris	200	200	200
Wal Mart	22	25	30

Table 7 CDS spreads used in the evaluation of the default probabilities. The left and right tables give the names in the euro and USD contracts respectively. Both contracts give protection of up to 10 mi euros and USD per company respectively. The total loss per contract is 30 mi euros and 32 mi USD respectively.

	(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**	**(7)**	**(8)**	**(9)**	**(10)**	**(11)**	**(12)**	**(13)**	**(14)**	**(15)**	**(16)**
(1)	1.000	0.590	0.365	0.286	0.355	0.416	0.180	-0.078	0.318	0.496	0.335	0.108	0.305	0.450	0.356	0.313
(2)	0.590	1.000	0.387	0.361	0.451	0.498	0.242	0.081	0.381	0.468	0.451	0.299	0.422	0.321	0.446	0.322
(3)	0.365	0.387	1.000	0.498	0.497	0.391	0.179	-0.127	0.142	0.342	0.212	0.178	0.187	0.388	0.219	0.215
(4).	0.286	0.361	0.498	1.000	0.742	0.491	0.290	-0.061	0.173	0.372	0.308	0.113	0.156	0.350	0.264	0.295
(5)	0.355	0.451	0.497	0.742	1.000	0.478	0.229	0	0.200	0.339	0.331	0.142	0.199	0.390	0.300	0.339
(6)	0.416	0.498	0.391	0.491	0.478	1.000	0.249	-0.045	0.268	0.365	0.312	0.241	0.314	0.386	0.409	0.247
(7)	0.180	0.242	0.179	0.290	0.229	0.249	1.000	0.013	0.115	0.222	0.222	0.013	0.159	0.139	0.127	0.593
(8)	-0.078	0.081	-0.127	-0.061	0	-0.045	0.013	1.000	-0.022	0.043	-0.075	0.020	-0.046	-0.089	0.037	-0.027
(9)	0.318	0.381	0.142	0.173	0.200	0.268	0.115	-0.022	1.000	0.153	0.310	0.156	0.270	0.179	0.261	0.385
(10)	0.496	0.468	0.342	0.372	0.339	0.365	0.222	0.043	0.153	1.000	0.144	0.152	0.291	0.356	0.347	0.306
(11)	0.335	0.451	0.212	0.308	0.331	0.312	0.222	-0.075	0.310	0.144	1.000	0.239	0.259	0.201	0.173	0.308
(12)	0.108	0.299	0.178	0.113	0.142	0.241	0.013	0.020	0.156	0.152	0.239	1.000	0.115	0.195	0.120	0.025
(13)	0.305	0.422	0.187	0.156	0.199	0.314	0.159	-0.046	0.270	0.291	0.259	0.115	1.000	0.281	0.648	0.257
(14)	0.450	0.321	0.388	0.350	0.390	0.386	0.139	-0.089	0.179	0.356	0.201	0.195	0.281	1.000	0.283	0.273
(15)	0.356	0.446	0.219	0.264	0.300	0.409	0.127	0.037	0.261	0.347	0.173	0.120	0.648	0.283	1.000	0.256
(16)	0.313	0.322	0.215	0.295	0.339	0.247	0.593	-0.027	0.385	0.306	0.308	0.025	0.257	0.273	0.256	1.000

Table 8a Correlation matrix taken from the equity market (equity returns) for the first loss basket in euro. For reasons of aesthetics names of the companies in the table have been coded in the following way: (1) Axa SA, (2) ABN Amro, (3) Akzo Nobel, (4) Basf Ag, (5) Bayer Ag, (6) BMW Ag, (7) BP Amoco, (8) BAT , (9) Carrefour, (10) Commerzbank, (11) Ahold, (12) Marks & Spencer, (13) Philipps NV, (14) Peugeot, (15) Siemens, (16) Total Fina.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)
(1)	1.000	0.208	0.300	0.211	0.183	0.282	0.411	0.123	0.160	0.079	0.259	0.251	0.211	0.203	0.236	0.157	0.259
(2)	0.208	1.000	0.637	0.219	0.169	0.323	0.454	0.367	0.218	0.231	0.087	0.537	0.268	0.429	0.510	0.106	0.332
(3)	0.300	0.637	1.000	0.285	0.268	0.469	0.609	0.580	0.354	0.417	0.043	0.703	0.269	0.673	0.711	0.113	0.489
(4).	0.211	0.219	0.285	1.000	0.075	0.167	0.251	-0.042	0.026	0.085	0.191	0.082	0.231	-0.004	0.100	0.162	0.310
(5)	0.183	0.169	0.268	0.075	1.000	0.276	0.319	0.232	0.296	0.398	0.067	0.242	0.218	0.275	0.319	0.102	0.186
(6)	0.282	0.323	0.469	0.167	0.276	1.000	0.445	0.344	0.378	0.366	-0.049	0.451	0.276	0.445	0.437	0.009	0.325
(7)	0.411	0.454	0.609	0.251	0.319	0.445	1.000	0.455	0.337	0.391	0.070	0.621	0.243	0.511	0.593	0.208	0.481
(8)	0.123	0.367	0.580	-0.042	0.232	0.344	0.455	1.000	0.440	0.344	0.016	0.783	0.123	0.790	0.829	-0.032	0.373
(9)	0.160	0.218	0.354	0.026	0.296	0.378	0.337	0.440	1.000	0.444	0.036	0.465	0.165	0.424	0.538	0.142	0.216
(10)	0.079	0.231	0.417	0.085	0.398	0.366	0.391	0.344	0.444	1.000	-0.008	0.350	0.208	0.342	0.400	0.164	0.210
(11)	0.259	0.087	0.043	0.191	0.067	-0.049	0.070	0.016	0.036	-0.008	1.000	0.057	0.119	-0.006	0.073	0.217	0.214
(12)	0.251	0.537	0.703	0.082	0.242	0.451	0.621	0.783	0.465	0.350	0.057	1.000	0.217	0.796	0.866	0.027	0.447
(13)	0.211	0.268	0.269	0.231	0.218	0.276	0.243	0.123	0.165	0.208	0.119	0.217	1.000	0.095	0.263	0.223	0.199
(14)	0.203	0.429	0.673	-0.004	0.275	0.445	0.511	0.790	0.424	0.342	-0.006	0.796	0.095	1.000	0.815	-0.009	0.333
(15)	0.236	0.510	0.711	0.100	0.319	0.437	0.593	0.829	0.538	0.400	0.0731	0.866	0.263	0.815	1.000	0.046	0.455
(16)	0.157	0.106	0.113	0.162	0.102	0.009	0.208	-0.032	0.142	0.164	0.217	0.027	0.223	-0.009	0.046	1.000	0.116
(17)	0.259	0.332	0.489	0.310	0.186	0.325	0.481	0.373	0.216	0.210	0.214	0.447	0.199	0.333	0.455	0.116	1.000

Table 8b Correlation matrix taken from the equity market (equity returns) for the first loss basket in euro. For reasons of aesthetics names of the companies in the table have been coded in the following way: (1) Boeing Co, (2) Bank of America, (3) Citigroup, (4) Coca Cola, (5) CSC, (6) Disney, (7) GE, (8) Goldman Sachs, (9) HP, (10) IBM, (11) Lockheed Martin, (12) Lehman Brothers, (13) McDonalds Corp. (14) Merril Lynch, (15) Morgan Stanley, (16) Philipp Morris, (17) Wal Mart.

Recovery Rate (%)	Moody's Corr. (bp)	S&P Corr. (bp)	Eq. Returns Corr (bp)
36.2	460	467	437
20.0	460	460	432
45.0	543	550	530
Market Quote (bp)	**303 (bid) - 403 (mid) - 503 (offer)**		

Table 9a Results for the pricing of the euro first loss basket using different correlation and recovery rate assumptions.

Recovery Rate (%)	Moody's Corr. (bp)	S&P Corr. (bp)	Eq. Returns Corr (bp)
36.2	505	507	470
20.0	510	504	450
45.0	501	501	471
Market Quote (bp)	**387 (bid) - 487 (mid) - 587 (offer)**		

Table 9b Results for the pricing of the USD first loss basket using different correlation and recovery rate assumptions.

Recovery Rate (%)	Moody's Corr. (%)	S&P Corr. (%)	Eq. Returns Corr (%)
36.2	12.4	13.7	7.7
20.0	12.4	12.4	6.7
45.0	25.8	26.7	24.0
Market Quote (bp)	**303 (bid) - 403 (mid) - 503 (offer)**		

Table 10a Rela tive errors for the prices given in table 9a. We take the computed quote subtract the market mid price and divide it by the market price.

Recovery Rate (%)	Moody's Corr. (%)	S&P Corr. (%)	Eq. Returns Corr (%)
36.2	3.7	4.1	-3.5
20.0	4.7	3.5	-7.6
45.0	2.9	2.9	-3.3
Market Quote (bp)	**303 (bid) - 403 (mid) - 503 (offer)**		

Table 10b Relative errors for the prices given in table 9b. We take the computed price subtract the market mid price and divide it by the market price.

7.2.2) BET Methodology

In the first step of the BET methodology one needs to evaluate the DS of the portfolio. For each portfolio we group the companies that belong to the same sector[5]. The *aggregate industry equivalent unity score* (AIEUS) of each sector is computed in two steps: a) for each company in the sector divide its exposure by the average firm exposure in the portfolio; b) add those values in a) for the companies in the sector. We then use the mapping table 11 to get the DS of each group and by adding them up we get the DS of the portfolio. The computations are shown in Table 12.

In the next step one computes the WARF of the portfolio (see eq. 22 in section 6). The rating factors used in the computation are seen in table 13. From the WARF we have the average rating of the portfolio and via interpolation one computes the average default probability of the idealized bond. The results of those computations for both portfolios are shown in table 14.

Once all the necessary parameters are computed one gets the rating of the portfolio (see table 14) as described in section 6. As already mentioned in section 6 we will be using the spreads given by ratings in the industrial sector. I.e. we will take the spread of the Baa3 rating (the rating of both portfolios) for the industrial sector. We should still mention that the other two possible sectors (with Baa3 rating) for which spreads were available in market were the phones and utilities and both of them had spreads higher than the one given by the industrials (with Baa3 rating).

[5] The number of companies in the same sector is called "equivalent industry unit score".

AIEUS	DS
0.0000	0.0000
0.0500	0.1000
0.1500	0.2000
0.2500	0.3000
0.3500	0.4000
0.4500	0.5000
0.5500 0.6500	0.6000 0.7000
0.7500	0.8000
0.8500	0.9000
0.9500	1.0000
1.0500	1.0500
1.1500	1.1000
1.2500	1.1500
1.3500	1.2000
1.4500	1.2500
1.5500	1.3000
1.6500	1.3500
1.7500	1.4000
1.8500	1.4500
1.9500	1.5000
2.0500	1.5500

AIEUS	DS
2.15	1.6000
2.25	1.6500
2.35	1.7000
2.45	1.7500
2.55	1.8000
2.65	1.8500
2.75 2.85	1.9000 1.9500
2.95	2.0000
3.05	2.0333
3.15	2.0667
3.25	2.1000
3.35	2.1333
3.45	2.1667
3.55	2.2000
3.65	2.2333
3.75	2.2667
3.85	2.3000
3.95	2.3333
4.05	2.3667
4.15	2.4000
4.25	2.4333

Table 11 Diversity Score Conversion table (AIEUS : Aggregate Industry Equivalent Unity Score).

Euro Denominated Contract			**USD denominated Contract**		
Sector	**AIEUS**	**DS**	**Sector**	**AIEUS**	**DS**
Insurance	1	1	**Ind. Aerospace Defense**	2	1.5
Banking	2	1.5	**Banking**	2	1.5
Industrial:Chem.	3	2	**Ind. Food**	1	1
Industrial: Autos	2	1.5	**Ind. Electric**	1	1
Industrial: Oil	2	1.5	**Ind. Computers**	4	2
Industrial:Tobacco	1	1	**Ind. Entertainment**	1	1
Ind. Retail	3	2	**Securities**	3	2
Ind. Electronics	2	1.5	**Ind. Restaurants**	1	1
-	-	-	**Ind. Tobacco**	1	1
-	-	-	**Ind. Retail**	1	1
Total	16	**12**	**Total**	17	**13**

Table 12 Computation of the Diversity Scores for the Euro and USD contracts.

As it can be seen in table 14 the spreads (over government) for industrials Baa3 are 219.17 bp and 507.69 bp for US and euro respectively. The quotes have been taken from the Bloomberg professional curves for the date 28 Oct 2002.

		Cumulative Default Probabilities in time (%)									
Rating	**Rating Factor**	**1 yr**	**2 yr**	**3 yr**	**4 yr**	**5 yr**	**6 yr**	**7 yr**	**8 yr**	**9 yr**	**10 yr**
Aaa	1	0.0005	0.0002	0.0007	0.0018	0.0029	0.0040	0.0052	0.0066	0.0082	0.0100
Aa1	10	0.0006	0.0030	0.0100	0.0210	0.0310	0.0420	0.0540	0.0670	0.0820	0.1000
Aa2	20	0.0014	0.0080	0.0260	0.0470	0.0680	0.0890	0.1110	0.1350	0.1640	0.2000
Aa3	40	0.0030	0.0190	0.0590	0.1010	0.1420	0.1830	0.2270	0.2720	0.3200	0.4000
A1	70	0.0058	0.0370	0.1170	0.1890	0.2610	0.3300	0.4060	0.4800	0.5730	0.7000
A2	120	0.0109	0.0700	0.2220	0.3450	0.4670	0.5830	0.7100	0.8290	0.9820	1.2000
A3	180	0.0389	0.1500	0.3600	0.5400	0.7300	0.9100	1.1100	1.3000	1.5200	1.8000
Baa1	260	0.0900	0.2800	0.5600	0.8300	1.1000	1.3700	1.6700	1.9700	2.2700	2.6000
Baa2	360	0.1700	0.4700	0.8300	1.2000	1.5800	1.9700	2.4100	2.8500	3.2400	3.6000
Baa3	610	0.4200	1.0500	1.7100	2.3800	3.0500	3.7000	4.3300	4.9700	5.5700	6.1000
Ba1	940	0.8700	2.0200	3.1300	4.2000	5.2800	6.2500	7.0600	7.8900	8.6900	9.4000
Ba2	1350	1.5600	3.4700	5.1800	6.8000	8.4100	9.7700	10.700	11.660	12.650	13.500
Ba3	1780	2.8100	5.5100	7.8700	9.7900	11.860	13.490	14.620	15.710	16.710	17.800
B1	2220	4.6800	8.3800	11.580	13.850	16.120	17.890	19.130	20.230	21.240	22.200
B2	2720	7.1600	11.670	15.550	18.130	20.710	22.650	24.010	25.150	26.220	27.200
B3	3490	11.620	16.610	21.030	24.040	27.050	29.200	31.000	32.500	33.780	34.900
Caa1	4770	17.382	23.234	28.639	32.479	36.314	38.967	41.385	43.657	45.672	47.629
Caa2	6500	26.000	32.500	39.000	43.880	48.750	52.000	55.250	58.500	61.750	65.000
Caa3	8070	50.990	57.009	62.450	66.240	69.821	72.111	74.330	76.485	78.581	80.623

Table 13 Indealized Moody's default probabilities and rating factors.

	Euro Portfolio	USD Portfolio
DS	12	13
WARF	101.18	95.63
Indealized Def. Prob. (%)	0.3667	0.3894
Stressed Def. Prob. (%)	0.4509	0.4790
Recovery Rate (%)	36.20	36.20
Evaluated Rating	**Baa3**	**Baa3**
Model Price (bp)	**508**	**219**
Market Price (mid)	**403**	**487**
Relative Error (%)	**20.7**	**-55.0**

Table 14 Portfolio parameters computed using the BET Methodology.

As can be seen from table 14 the BET approach gives too large relative errors in the quotes for both instruments. For both contracts it falls outside the bid offer spread of the instruments (303/503 for the euro and 387/587 for the USD).

If the market were to use the BET methodology without any adjustment whatsoever from the results in table 14 it is clear that the approach should not at all be used in practice.

There are several problems with the BET approach. One for example is its use of historical probabilities of default that may not match the ones implied in the CDS market. This can cause problems as the quotes given might not reflect the cost of the hedge as given in the CDS market. An adjustment to it is done by stressing the default probability of the idealized bond. I.e. one multiplies the idealized default probability by a certain factor, as it has been done in here. A second source of errors is due to the discretization process. As we have seen the portfolio default probability implied by the expected loss determines the rating that will be used to evaluate the spread. The sensitivity to the rating might be very high. As an example if the rating for the USD portfolio would be one notch better (B1) the spread would have been 500 bp.

In order to count for the problems mentioned above the user of the methodology might make its own adjustments after checking the rating obtained. The user might want to compare the spread obtained with the one for a rating one notch up and down. A possibility is for example to give as bid the spread for the one notch up while as offer the spread of one notch down. Or alternatively one can take as mid price the average between the spreads for the one notch up and the one down.

8) Conclusions

In this work we used and compared different methodologies to price first to default (FTD) and first loss contracts of portfolios of credits. All the data used in the tests was taken from the market and the model prices were tested with the bid offer spread of two market makers (one for the FTD and another for the first loss).

For the FTD contract we have used the gaussian copula function approach with the correlation being calculated from the equity market. The recovery rate used was 20% and the model price (257 bp) is inside the bid offer of a market maker (205 – 265 bp).

For the first loss basket we have used the gaussian copula method and the BET methodology and the results are compared with bid offer quotes of a market maker on two different baskets. Additionally for the gaussian copula we have used three different recoveries assumptions and for each case we have tested the results of using different correlation assumptions (Moody's, S&P and correlation taken from equity returns). We have observed that the gaussian copula with the correlation calculated from the equity returns gives the best results. It should be noted however that if one uses the Moody's or S&P correlations (for structured products) one is still inside the bid offer. Moreover the results of the copula approach are far better than the ones obtained via the BET methodology.

Our conclusion from this study is that the gaussian copula approach with the equity return correlation and the appropriate recovery rate is capable of capturing the mid prices for the two first loss and the first loss basket instruments priced in this study. A continuation of this study is under way when a more extensive test of the impact of the correlation and recovery rates assumptions including the copula - BET models for the rating of CDO's will be made.

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