Key Formulas and Examples for Finance:

Minimum monthly payment amount of a loan: this formula allows you to calculate your minimum monthly payment amount given the amount of the loan, the yearly interest rate, and the number of payments you are going to make.

$$R = \frac{A\frac{r}{12}\left(1+\frac{r}{12}\right)^n}{\left(1+\frac{r}{12}\right)^n - 1}$$

R=your monthly payment
A=the amount of the loan
r=the yearly interest rate
n=the total number of payments

Example: Sally wants to buy a particular car and has two different dealers offering her different deals:
Dealer A has offered to sell her the car for $14000. With no money down, he has offered her a 5 year loan with 6% interest.

Dealer B will sell her the car for $15000. He wants her to put $1000 down, and he has offered her a three year loan with 4% interest.

Which deal will result in lower monthly payments? More importantly, which is the best deal overall?

Solution:

Dealer A: Since it's a 5 year loan and there are 12 months in a year, Sally would make 60 payments, so n=60. We use 0.06 for the 6% interest rate. The amount of the loan is $14000, so the monthly payment is given by

$$R = \frac{\$14000\left(\frac{0.06}{12}\right)\left(1+\frac{0.06}{12}\right)^{60}}{\left(1+\frac{0.06}{12}\right)^{60} - 1} = \$270.66$$

With this option, Sally would pay a total amount of (60)($270.66)=$16239.55

Dealer B: Since it's a three year loan, and there are 12 months in a year, Sally would make 36 payments. We use 0.04 for the 4% interest rate. Note that since she put $1000 down, the amount of the loan is only $14000. So, her monthly payment is:

$$R = \frac{\$14000\left(\frac{0.04}{12}\right)\left(1+\frac{0.04}{12}\right)^{36}}{\left(1+\frac{0.04}{12}\right)^{36}-1} = \$413.34$$

Over the course of the loan, her monthly payments will total (36)($413.34)=$14880 Added to the $1000 she put down, this calculation gives a total cost of $15880.

Over the course of the loan, Sally will save $359.55 by buying from dealer B. Even though dealer A offers attractive gimmicks like a long term loan, no money down, and a lower sticker price, his offer is not as cost effective as dealer B's.

Calculating number of payments: Of course, you should ***never*** just make the minimum monthly payment on loan unless you are in dire financial straights! Paying extra on a loan will decrease the number of payments you make on the loan, and it will save you money. On a large loan such as a mortgage or business loan, paying extra can save you lots of money! (How does saving roughly $40,000 sound?) After looking at your personal finances, you should decide how much a month you can afford to pay on your loan. Then, use the following formula to calculate the number of payments. Multiply number of payments by the amount of each payment to get the total cost over the course of your loan.

$$n = \frac{\ln(R) - \ln\left(R - A\frac{r}{12}\right)}{\ln\left(1+\frac{r}{12}\right)}$$

n=the number of payments you will make
R=the amount of the monthly payment you've decided to make
A=the amount of the loan
r=the yearly interest rate

Example: Jim and Jane are purchasing a house for $290000. Their fixed yearly interest rate is 5%. They have applied for a 30 year loan.

Let's calculate their minimum monthly payment, and the total cost of their home if they were to make the minimum payment each month.

Answer: If they only pay the minimum, they will make all thirty years of payments. That's a total of 360 payments. We use 0.05 for 5% interest, and $290000 for the amount of the loan. We will plug these values into the formula for the minimum monthly payment amount of a loan.

$$R = \frac{\$290,000\left(\frac{0.05}{12}\right)\left(1+\frac{0.05}{12}\right)^{360}}{\left(1+\frac{0.05}{12}\right)^{360}-1} = \$1,556.78$$

The total amount they will pay for their "$290,000 house" is (360)($1,556.78)=$560,441.77

Now comes the good part! Let's say Jim and Jane decide that if they stop going out to lunch every day, they could pay an extra $100 a month on their mortgage. So, basically they've decided their monthly payment will be $1,656.78. With this value as their monthly payment, let's calculate the number of payments they'll make, and the total cost of their home.

$$n = \frac{\ln\left(1656.78\right)-\ln\left(1656.78-290000\frac{0.05}{12}\right)}{\ln\left(1+\frac{0.05}{12}\right)} = 314.29$$

Now, the total cost of their "290,000" home is (314.29)*($1,656.78)=$520,717.50

If they only make the minimum payment, then over the course of the loan, they would pay $560,441.77. So, by paying an extra $100 a month, their total savings will be $560,441.77-$520,717.5=$39,724.27

Take-home message: even paying a small amount of extra money on your loan can save you a lot over the course of a loan. So, even though you should save *some* money in a bank account, repaying any loans you have is almost always the wisest investment!