

Final Review in class assignment I: 6.1-8.1. These problems are not actual exam questions. They are intended to give you an idea of which topics to study. Keep in mind, the final will be multiple choice.

6.1/6.2

1. Which of the following matrices are in reduced row echelon form? If a matrix is not in reduced row echelon form, put it in reduced row echelon form by using appropriate row operations. Read off the solution to the corresponding linear system.

$$\begin{bmatrix} 3 & 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & 0 & 4 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 5 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 5 \\ 0 & 2 & 7 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 & 0 & .5 \\ 0 & 1 & 0 & 0 & 7 \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 1 & 0 & 2 \end{bmatrix}$$

2. Write the augmented matrix for each system of equations. Solve using any method you choose. You don't have to use your augmented matrix if you don't want to.

$$\begin{array}{ll} x + y + z = 2 & x + 2y + 3z = 8 \\ 2x + y - z = 5 & 3x - y + 2z = 5 \\ x - y + z = -2 & -2x - 4y - 6z = 5 \end{array} \quad \begin{array}{l} ax + 3y = 4 \\ x + by = c \end{array} \quad \begin{array}{l} \alpha x + \beta y = \eta \\ \delta x + \varepsilon y = \mu \end{array}$$

3. Jane is now twice as old as Sally. Six years ago, Sally's age was three years less than one third of Jane's current age. How old are Jane and Sally?

4. A manufacturing plant has been told to produce items A, B, and C under the following restrictions: 1. The total number of B and C produced must equal half the number of A produced, and 2. the total number of A and C produced must exceed the number of B produced by 500.

Which of the following are allowable production quantities? (Hint: find the general solution)

$$\begin{array}{llll} A = 960 & A = 800 & A = 700 & A = 600 \\ B = 470 & B = 350 & B = 250 & B = 20 \\ C = 10 & C = 50 & C = 60 & C = 4 \end{array}$$

5. Compute the determinant of the following matrices:

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad \begin{bmatrix} x & 2 \\ 3 & a \end{bmatrix} \quad \begin{bmatrix} 1 & 2 & 3 \\ 2 & 0 & 2 \\ 1 & 4 & 3 \end{bmatrix} \quad \begin{bmatrix} a & 1 & 2 \\ 3 & 0 & b \\ 2 & 3 & 4 \end{bmatrix}$$

6.3/6.4

Perform the following matrix operations:

$$1. \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & 3 \\ 1 & 4 & 5 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 4 & 2 & 4 \\ 0 & 7 & 3 \end{bmatrix} \quad 2. \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 5 \end{bmatrix} \begin{bmatrix} 4 & 1 \\ 2 & 3 \\ 8 & 1 \end{bmatrix} \quad 3. \begin{bmatrix} 4 & 1 \\ 2 & 3 \\ 8 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 5 \end{bmatrix}$$

$$4. \begin{bmatrix} a & b \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & c & 1 \\ 3 & 0 & d \end{bmatrix} \quad 5. \begin{bmatrix} a & b \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 2 & c & 1 \\ 3 & 0 & d \end{bmatrix} \quad 6. \begin{bmatrix} 1 & 2 \\ 3 & a \end{bmatrix}^3$$

$$7. \begin{bmatrix} 1 & 2 \\ 3 & a \end{bmatrix}^{-1}$$

8. Which of the following matrices could be used to encrypt a message (assuming you want to decrypt it later)?

$$\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix} \begin{bmatrix} 1 & 5 & 2 \\ 0.25 & 1.25 & 0.5 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix}$$

9. Use your answer from 8 to encrypt the word “here”.

10. Assume the message 53, 79, 43, 34 was encrypted with your answer to 8. Decrypt it.

8.1

11. Which of the following are sets?

$$\{1, 2, 3, 4\} \quad \{\text{hearts, diamonds, spades, clubs}\} \quad \{(1,1), (1,1), (1, 2), (1, 3)\}$$

12. A scholarships committee wants to award a scholarship to a student who has taken calculus and probability, but hasn't taken differential equations or linear algebra. Let C be the set of all students who have taken calculus, P be the set of all students who have taken probability, D be the set of all students who have taken differential equations, and L be the set of all students who have taken linear algebra. Which of the following represent the set of all scholarship candidates? Drawing a Venn diagram would be a good start. This question could also ask for the Venn diagram.

a. $(C \cap P) \cap (D' \cap L')$ b. $(C \cap P) \cup (D' \cap L')$ c. $(C \cap P) \cup (D \cup L)$ d. $(C' \cup P)' \cap (D \cup L)'$

13. Give a "real world" example of ...

- a. two disjoint sets
- b. a set and its complement
- c. two sets with a nonempty intersection

14. A set has 5 elements. How many possible subsets does this set have?

Note: the problems in 8.1 are particularly good, and most shouldn't take long, so working them is a good idea.