

Exam III review: This assignment will not be collected for a grade. It is simply for your exam preparation. Exam questions will test the same concepts at a similar level of difficulty, but will not necessarily come from this sheet. However, as on the last exam, one question will come directly from this sheet. Problems from your book are also an excellent review, and often are a source of exam questions.

Sequences:

Which of the following sequences converge? If a sequence converges, say what it converges to. These are MC candidates, and it is fine to use your calculator to investigate convergence. Another question which could be asked is “write out the first 5 terms of each sequence”.

$$1. \left\{ \frac{100n^2 + 1}{n^3 + e^\pi} \right\} \quad 2. \left\{ \frac{e^n}{n!} \right\} \quad 3. \left\{ \sqrt[n]{n} \right\}$$

Write the general term for each of the following sequences and determine if they converge:

1. 1, 8, 243, 16384, ...
2. 1.5, 3, 4.5, 6, 7.5, ...
3. $e, \frac{e^2}{2}, \frac{e^3}{6}, \frac{e^4}{24}, \frac{e^5}{120}, \dots$

Series: Which of the following series converge? This is a MC candidate. You should be able to tell by inspection that some of these series do not converge. (Hint: what has to be true for a series to even have a chance to converge?) Others may require you to do some algebra/arithmetic in order to determine convergence.

$$1. \sum_{n=0}^{\infty} \frac{n}{2} \quad 2. \sum_{n=0}^{\infty} (5^{n+1})(3^{-2n}) \quad 3. \sum_{n=0}^{\pi} \frac{\pi}{2^n} \quad 4. \sum_{n=0}^{\pi} \frac{n!}{2^n} \quad 5. \sum_{n=5}^{\infty} (e^n)(\pi^{-n-1})$$

Which of the above series are geometric? For each geometric series, re-write it so that it

is in the form $\sum_{n=k}^{\infty} ar^n$ in order to determine convergence. This question is a non MC candidate.

Taylor polynomials:

Find the 5th Taylor polynomial for the following functions at the given point. Could be MC or non MC depending on the difficulty. (You should check your answer by graphing the Taylor polynomial and the original function; near a, they should be similar, at a they should be equal.)

1. $\sin(x)$ at $a = \frac{\pi}{2}$
2. $e^{\cos(x)}$ at $a = 1$
2. $x^{-1} + \ln(x+1)$ at $a = 0$

Trig derivatives and integrals (single variable):

These could be non MC or MC depending on difficulty. Indefinite integrals could change to definite integrals.

1. $\frac{d}{dx}(\sin(x)\cos(x))$
2. $\frac{d}{dx}(\sec(\tan(\ln(\cos(x))))))$
3. $\frac{d}{dx}(\csc^2(x))(\ln(x))$
4. $3 \cdot \frac{d}{dx}\left(\frac{\tan(x)}{\sin(x)}\right)(\sec^2(x))$
5. $\frac{d}{dx}\left(\frac{\sin(1+x)}{1+\sin(x)}\right)$
6. $\int (2x) \csc(x^2) \cot(x^2) dx$

7. $\int \sin^3(x) \cos(x) dx$
8. $\int \sec^5(x) \tan(x) dx$
9. $\int \sec^2(x) \tan(x) dx$

10. $f(x, y) = \tan(e^{2y}) + x \sin(y)$

Trig review material (non MC):

Evaluate all six trig functions for:

1. $\frac{\pi}{3}$
2. $\frac{5\pi}{6}$
3. $\frac{7\pi}{4}$

Verify the following identities. Be organized and systematic. Manipulate one side until it looks like the other. No credit will be awarded for “getting the right answer” since you already know what the “answer” is. It is better to stop if you are stuck than to make a trig or algebra mistake.

1. $\frac{\tan(x)}{1+\sec(x)} + \frac{1+\sec(x)}{\tan(x)} = 2 \csc(x)$
2. $\frac{\cos^2 x + 3 \sin x - 1}{3 + 2 \sin x - \sin^2 x} = \frac{1}{1 + \csc x}$

3. $\frac{\sec x - 1}{\sin^2 x} = \frac{\sec^2 x}{1 + \sec x}$

Trig derivatives and integrals, multivariable (non MC):

1. $\int_0^\pi \int_0^{\sec(y)} x \, dx dy$
2. $\int_{\pi/4}^{\pi/3} \int_1^{\cos(y)} x^2 \sin(y) \, dx dy$

3. Find all four second order partial derivatives for $f(x, y) = \sec(x) \cos(y)$
4. Find all four second order partial derivatives for $f(x, y) = \tan(e^{2y}) + x \sin(y)$