Week 5. <u>Output, Price & Profit - Marginal Analysis</u>

1. **Objective of a Firm: Maximize Total Profit**

a. **Total Profit** is the net earnings during some period of time.

Total Profit (TP) = Total Revenue (TR) — Total Cost (TC), where TP is also economic profit as TC includes opportunity cost.

cf.) A one-man business that hires no L and rents no K, but the owner's own K & L may incur no cost and thus yield accounting profit = TR from accounting point of view, whereas from economic point of view, the owner's own K & L consist an opportunity cost as they could have been rent out to and hired by other opportunities if they were not engaged in this business.

b. **Total, Average, & Marginal Revenue & Cost**

$TR = P\times Q = f(Q)Q = \alpha Q^2+\beta Q+\upsilon$ $\qquad$ $TC = (rK + wL)$

$AR = TR/Q = PQ/Q = P$ $\qquad$ $AC = TC/Q = (rK + wL)/Q = (rK + wL)$

$MR = \Delta TR/\Delta Q$ $\qquad$ $MC = \Delta(rK + wL)/\Delta Q = \Delta TC/\Delta Q$

$= \Delta(\alpha Q^2+\beta Q+\upsilon)/\Delta Q$ $\qquad$ $= \Delta[r\times f(Q) + w\times g(Q)]/\Delta Q$

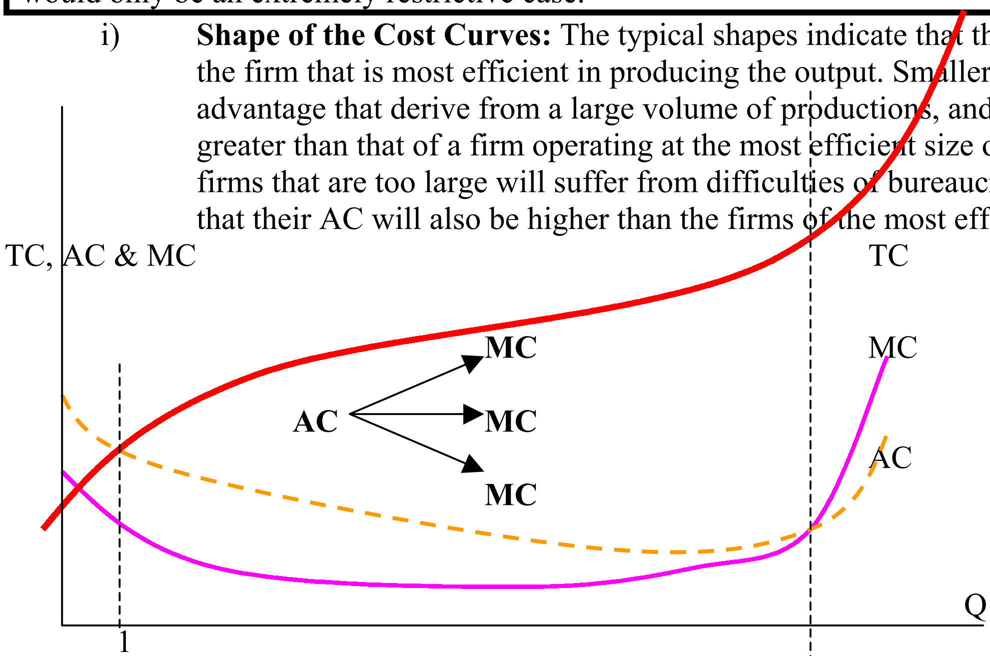
$= 2\alpha Q+\beta$

$$\therefore MP = MR - MC = \Delta(\alpha Q^2+\beta Q+\upsilon)/\Delta Q - \Delta(rK + wL)/\Delta Q$$

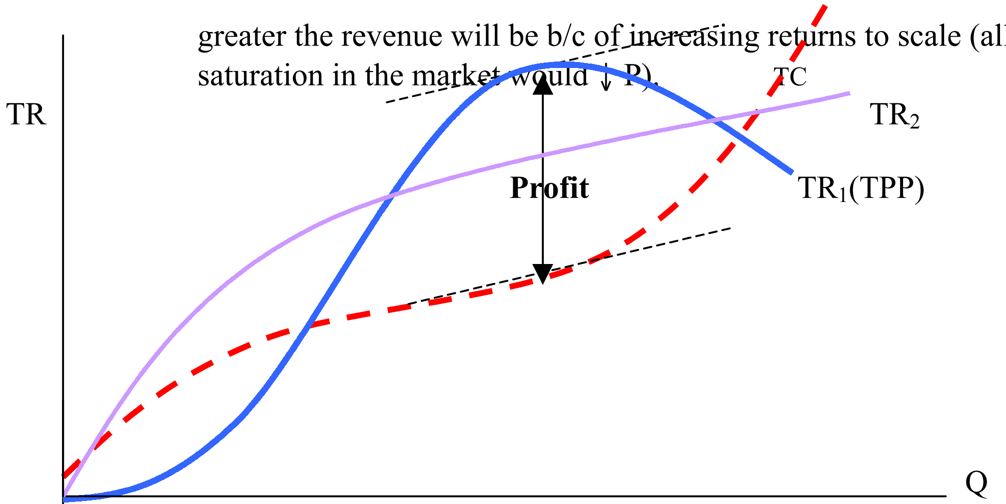
To **Maximize Total Profit** : MR – MC = 0 or MR = MC

This is only a simplified version where TR & TC are just linear functions of Q, which would have constant slopes. In such a case, AR = MR & AC = MC. However, normally TR and TC will be of some nonlinear functional form, and this loose simplification would only be an extremely restrictive case.

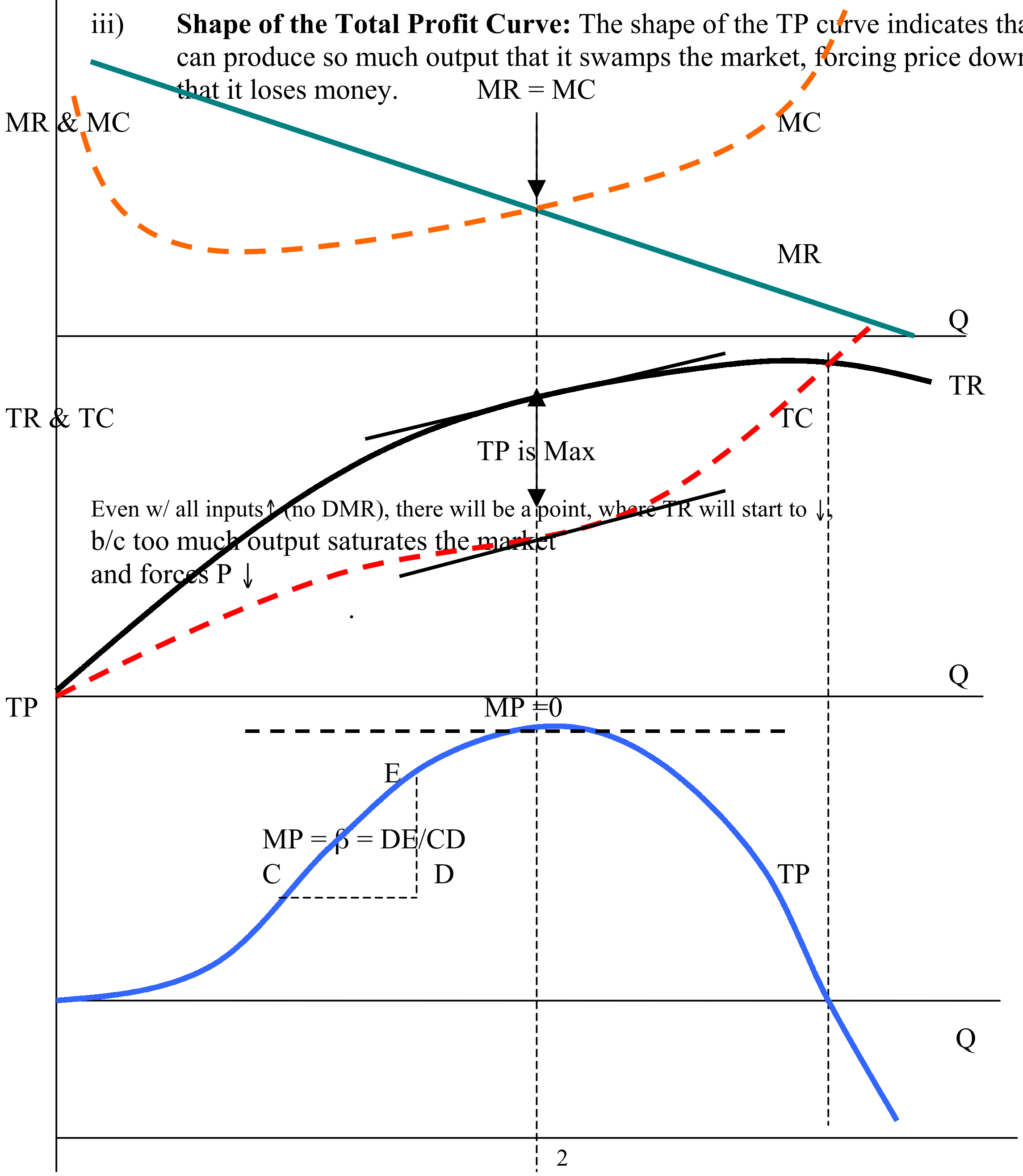
i) **Shape of the Cost Curves:** The typical shapes indicate that there is some size of the firm that is most efficient in producing the output. Smaller enterprises lose any advantage that derive from a large volume of productions, and so their AC will be greater than that of a firm operating at the most efficient size of output. Similarly, firms that are too large will suffer from difficulties of bureaucratic controls, so that their AC will also be higher than the firms of the most efficient size.



ii) **Shape of the Revenue Curves:** The typical TR curve would look either like a) TR_1 indicating that MR (MPP) will diminish past a certain point b/c of Law of DMR (only one input ↑); or b) TR_2 indicating that the more the firm sells, the greater the revenue will be b/c of increasing returns to scale (all inputs ↑ & no saturation in the market would ↓ P).



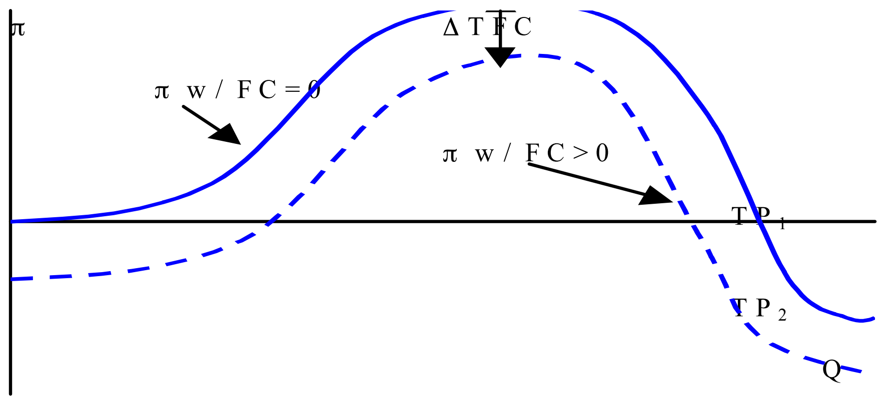
iii) **Shape of the Total Profit Curve:** The shape of the TP curve indicates that a firm can produce so much output that it swamps the market, forcing price down so low that it loses money.



2. **Generalization of Marginal Analysis & Maximization**
   Cost/Benefit Analysis: **Net Benefit = Total Benefit − Total Cost**
   Net Benefit is Maximized where **Marginal Benefit = Marginal Cost**
3. **Fixed Cost (Firm Size) & Profit-Maximizing Price**
   When a firm's fixed cost increases, its profit-maximizing price and output remains completely unchanged. This is because MC doesn't change just because the firm size changes, since

$$\frac{\Delta TFC}{\Delta Q} = 0 \quad \rightarrow \quad MC = \frac{\Delta TC}{\Delta Q} = \frac{\Delta TFC}{\Delta Q} + \frac{\Delta TVC}{\Delta Q} = \frac{\Delta TVC}{\Delta Q}.$$

Therefore, ΔTFC has no effect on MC. The FC increase will simply shift the TP curve straight downward by ΔTFC, but it doesn't shift the TP curve neither to the right nor to the left.



4. **Further Mathematical Insight to Production Optimization (Profit Maximization)**

a. **Output Maximization**: The firm's input decision has a dual nature. The optimum choice of K & L can be analyzed not only as the problem of choosing the lowest isocost line tangent to the production isoquant, but also as the problem of choosing the highest production isoquant tangent to a given isocost line.
   - i) **Objective Function:** $Max F(K,L) \quad s.t. \quad wL + rK = M$
   - ii) **Lagrangean:** $\Phi = F(K,L) - \mu(wL + rK - M)$ or $\Phi = F(K,L) + \mu(M - wL - rK)$, **where** μ is the Lagrange multiplier.
   - iii) **F.O.C.** for output (revenue) maximization are

$$\frac{\partial \Phi}{\partial K} = F_K - \mu r = 0, \quad \frac{\partial \Phi}{\partial L} = F_L - \mu w = 0, \quad \frac{\partial \Phi}{\partial \mu} = wL + rK - M = 0$$

   By solving the first two equations, we see that $\frac{F_K}{r} = \frac{F_L}{w}$.

b. **Cost Minimization:** The production function *F*(*K,L*) describes the maximum output that can be produced for every possible combination of inputs. We assume that each of the factors has positive, but decreasing marginal products. – *i.e.*) $\frac{\partial F}{\partial K} > 0, \frac{\partial^2 F}{\partial K^2} < 0, \frac{\partial F}{\partial L} > 0, \frac{\partial^2 F}{\partial L^2} < 0$.
   - i) **Objective function:** Minimize $M = wL + rK \quad s.t. \quad F(K,L) = \overline{Q}$.
   - ii) **Lagrangean:** $\Phi = wL + rK - \lambda[F(k,L) - Q]$ or $\Phi = wL + rK - \lambda(AK^{\alpha} L^{\beta} - Q)$, where λ is a Lagrange multiplier.
   - iii) **F.O.C.** for cost minimization are

$$\frac{\partial\Phi}{\partial K} = r - \lambda F_K = 0, \quad \frac{\partial\Phi}{\partial L} = w - \lambda F_L = 0, \quad \frac{\partial\Phi}{\partial\lambda} = F(K,L) = 0.$$

By combining the first two equations, we obtain $\frac{F_K}{r} = \frac{F_L}{w}$.

Finally, to evaluate the Lagrange multiplier, $\lambda = \frac{r}{F_K} = \frac{w}{F_L}$.

c. Profit Maximization

i) **Objective function**: $Max\pi = P \times F(L,K) - (wL + rK)$

ii) **FOC**:

$$\frac{\partial\pi}{\partial K} = PF_K = r \quad PF_K = P \times MPK = MRK = r$$

$$\frac{\partial\pi}{\partial L} = PF_L = w \quad PF_L = P \times MPL = MPL = w$$

- *E.g.*) **Objective function**: $MaxQ = 20K^{\frac{1}{3}}L^{\frac{2}{3}} \quad s.t. \quad 500 = 20K + 30L$

Lagrangean: $Z = 20K^{\frac{1}{3}}L^{\frac{2}{3}} + \lambda(500 - 20K - 30L)$

F.O.C:

$$\frac{\partial Z}{\partial K} = \frac{20}{3}K^{-\frac{2}{3}}L^{\frac{2}{3}} = 20\lambda \cdots\cdots\cdots (1)$$

$$\frac{\partial Z}{\partial L} = \frac{40}{3}K^{\frac{1}{3}}L^{-\frac{1}{3}} = 30\lambda \cdots\cdots\cdots (2)$$

$$\frac{\partial Z}{\partial \lambda} = 500 = 20K + 30L \cdots\cdots\cdots (3)$$

$$\frac{1}{2}\frac{L}{K} = \frac{2}{3} \cdots\cdots\cdots \frac{(1)}{(2)}$$

$$L = \frac{4}{3}K \rightarrow (3)$$

$$500 = 20K + 30\frac{4}{3}K = 60K$$

$$\therefore K = \frac{25}{3}, L = \frac{100}{9} \rightarrow Q^* = 20\left(\frac{25}{3}\right)^{\frac{1}{3}}\left(\frac{100}{3}\right)^{\frac{2}{3}} = 1{,}5022$$

d. Estimable Cobb-Douglas Production Function

$F(K,L) = AK^{\alpha}L^{\beta}$ or $\log F(K,L) = \log A + \alpha\log K + \beta\log L$ by taking the log transform, where $\alpha < 1$, $\beta < 1$, and $\alpha + \beta = 1$ in case of constant returns to scale, $\alpha + \beta > 1$ in case of increasing returns t scale, $\alpha + \beta = 1$ incase of diminishing returns to scale. Also, α & β are elasticities.

$$\alpha = \frac{\partial Q}{\partial K}\frac{K}{Q} = \frac{\alpha\, Q}{K}\frac{K}{Q}, \text{ and } \beta = \frac{\partial Q}{\partial L}\frac{L}{Q} = \frac{\beta\, Q}{L}\frac{L}{Q}$$

e. Functional Form of SR Cost Function

$$TVC = aQ + bQ^2 + cQ^3$$

$$AVC = \frac{TVC}{Q} = a + bQ + cQ^2$$

$$MC = \frac{\partial TC}{\partial Q} = \frac{\partial TFC}{\partial Q} + \frac{\partial TVC}{\partial Q} = 0 + \frac{\partial TVC}{\partial Q} = a + 2bQ + 3cQ^2$$