Week 4. Production, Inputs and Cost: Supply Analysis

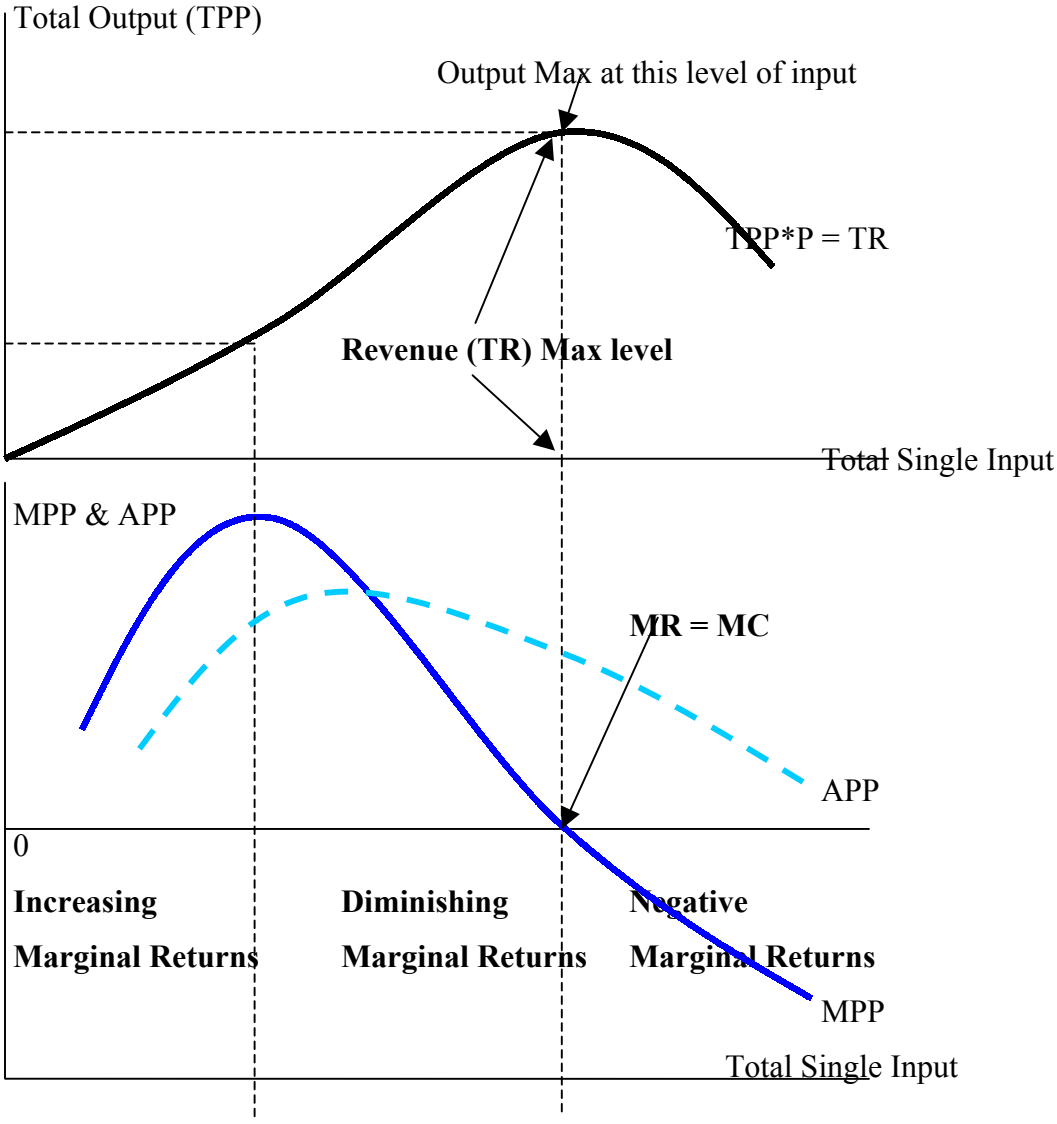
1. **Production, Input choice, and Profit Maximization (Cost Minimization)**

a. **Total, Average, and Marginal Physical Products**

i) Total Physical Product (TPP) or total output is the amount of output a firm obtains in total from a given quantity of input. Equivalent in money terms is Total Revenue, which is TR = TPP*P.

ii) Average Physical Product (APP): $APP = \frac{TPP}{K \text{ or } L}$, $AR = \frac{TR}{Q} = P$

iii) Marginal Physical Product (MPP) is Δ in TPP that results from a 1-unit Δ in the input , *Ceteris Paribus*: $MPP = \frac{\partial TPP}{\partial K \text{ or } L}$, $MR = \frac{\partial TR}{\partial Q}$



b. **Law of Diminishing Marginal Returns** states that an increase in the amount of any one input, *Ceteris Paribus*, leads to lower marginal returns to the expanding input.

When the quantity of one input increases, *Ceteris Paribus,* the input whose quantity increases gradually becomes more and more redundant as well as inefficient. Therefore, adding more of abundant input will only do little good, and even harm.

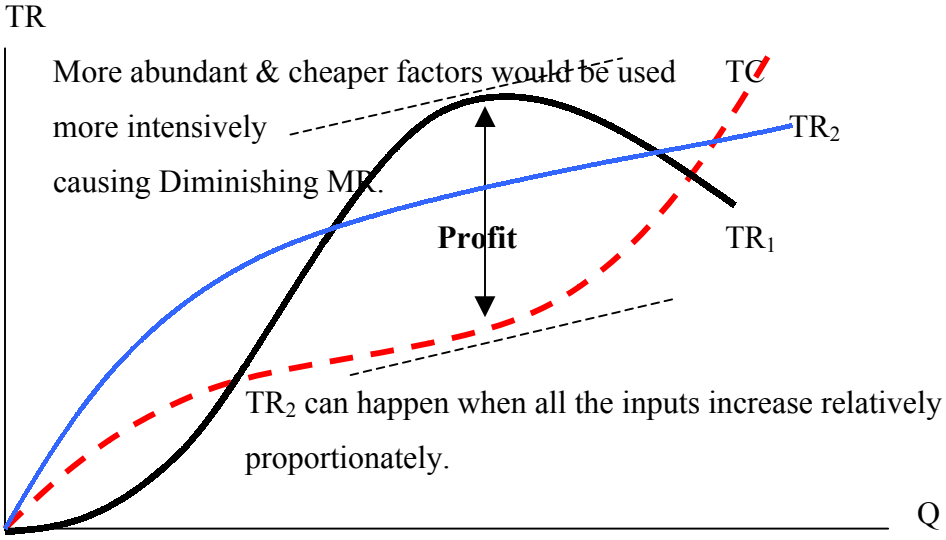
c. **Optimization (Max Output & min Input) and Diminishing Marginal Returns**

i) Marginal Revenue (Product) or MR(P) is the additional revenue to the producer form the increased sales when it uses an additional unit of the input.

$$MR = \frac{\partial TR}{\partial Q} = \frac{\partial (TPK, L \times P)}{\partial Q} = MPK, L \times P$$

ii) Total Output or TR is maximized if MR = 0. This condition is derived from $MR = \frac{\partial TR}{\partial Q} = \frac{\partial (TPP \times P)}{\partial Q} = MPP \times P$.

iii) Profit is maximized where cost is minimized if MR = MC. This condition is derived loosely from $Max(\pi = TR - TC) = \frac{\partial TR}{\partial Q} - \frac{\partial TC}{\partial Q} = MR - MC = 0$.



d. **Cost Curves and Input Quantities**

i) Total Cost increases as quantity of output Q_X increases, and must include the opportunity cost of any inputs. The opportunity cost would be the physical input or money price of the input + the possible return on this physical input or money that could have been used or invested in alternative productive activity or financial instrument. This return must be just as much as inputs in this production can yield. Otherwise, the inputs wouldn't be used in this production.

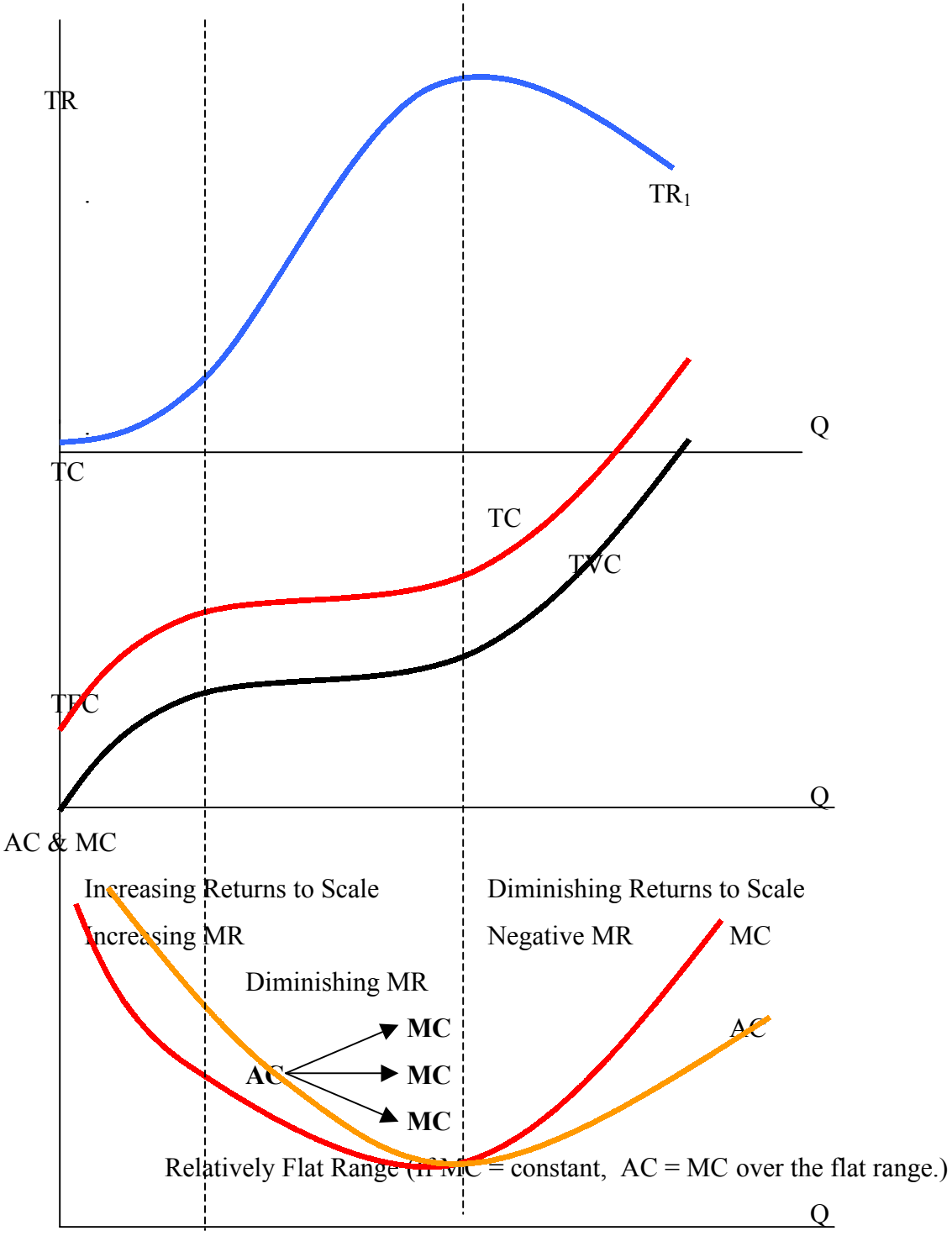
ii) The relationship of total cost to output is determined by the technological production relationships between inputs and outputs and by input prices.

e. **Production Function and the Firm's Cost Curves**

i) Production function summarizes the technical and engineering information about the relationship between a firm's output and all of its inputs.:

$Y = AK^{\alpha} L^{\beta} N^{\gamma}$ or $Y = AK^{\alpha} L^{1-\alpha}$

ii) It also indicates the maximum output that any particular combination of inputs is capable of producing. The total cost of producing any given quantity of output equals the sum of the cost-minimizing quantities of each of the inputs.



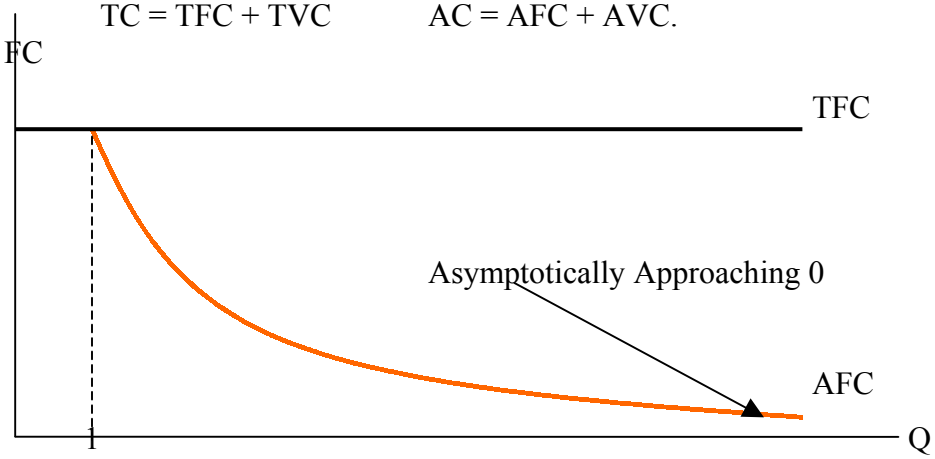
f. **Fixed Costs & Variable Costs**

i) Fixed cost is the cost of the smallest (least expensive) batch of inputs that the firm can buy in order to produce any output at all. The total amount of money

spent on these inputs does not increase when output rises in the SR. However, if the capacity (firm size) is to be increased in the LR, fixed cost will increase as well.

a) Total fixed cost (TFC or sunk cost) is horizontal.
b) Average fixed cost (AFC) decreases steadily as TFC is spread among more and more units of output. AFC is asymptotic meaning that it gets lower and lower as output increases.

ii) Variable cost is the cost that is not fixed. Therefore, it rises when output rises.
TC = TFC + TVC AC = AFC + AVC.



g. **Shape of Average Cost Curve**

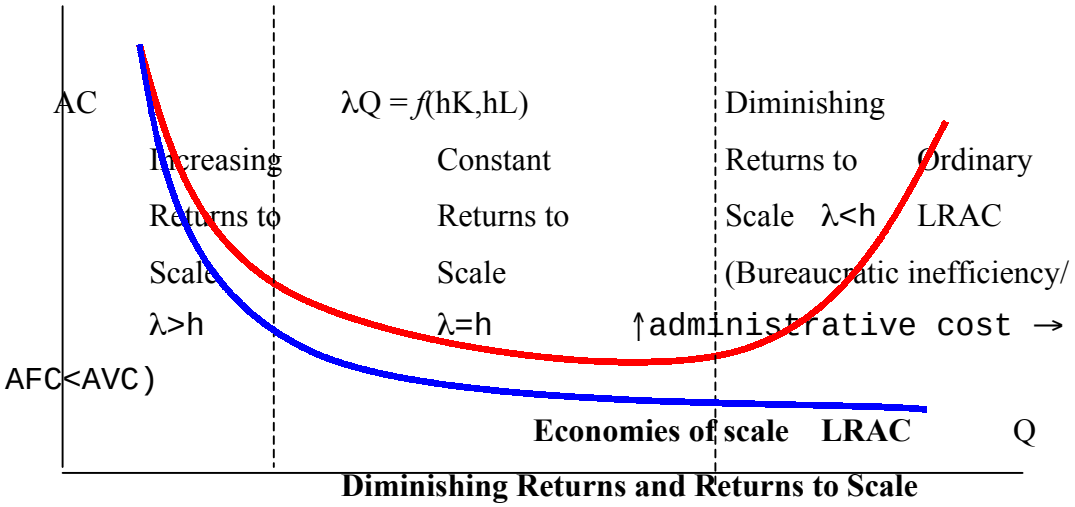
i) AC slopes downward in the left-hand portion of the curve for
a) increasing marginal returns or increasing MPP (output side) in the segment of increasing returns to scale (cost side);
b) the firm spreads its fixed cost over greater level of outputs. Since AC = AFC + AVC, AC falls as AFC falls steeply at first when output increases.

ii) AC slopes upward in the right-hand portion of the curve for
a) the **diminishing returns to scale** or decreasing MPP (output side);
b) the administrative (bureaucratic) problems of large organizations - *i.e.* costly to manage. Bureaucratic cost rises disproportionately. (problem of sheer size).

h. Economies of Scale (pertains to TR(AR)>TC(AC))

i) Economies of scale or increasing returns to scale refers to the situation where the output is increased by more than inputs are increased. It is generally represented by the decreasing AC curve. The same LRAC curve can feature all three aspects of returns to scale.

ii) Economies of scale give larger firms cost advantages over smaller ones, thereby foster larger firm sizes. Technology generally determines whether or not a specific economic activity is characterized by economies of scale. *e.g.*) industries w/ large sunk cost (FC) would tend to feature economies of scale, because the firms spread their fixed cost over larger and larger outputs, and since AC = AFC + AVC, AC falls as AFC falls steeply as output increases.



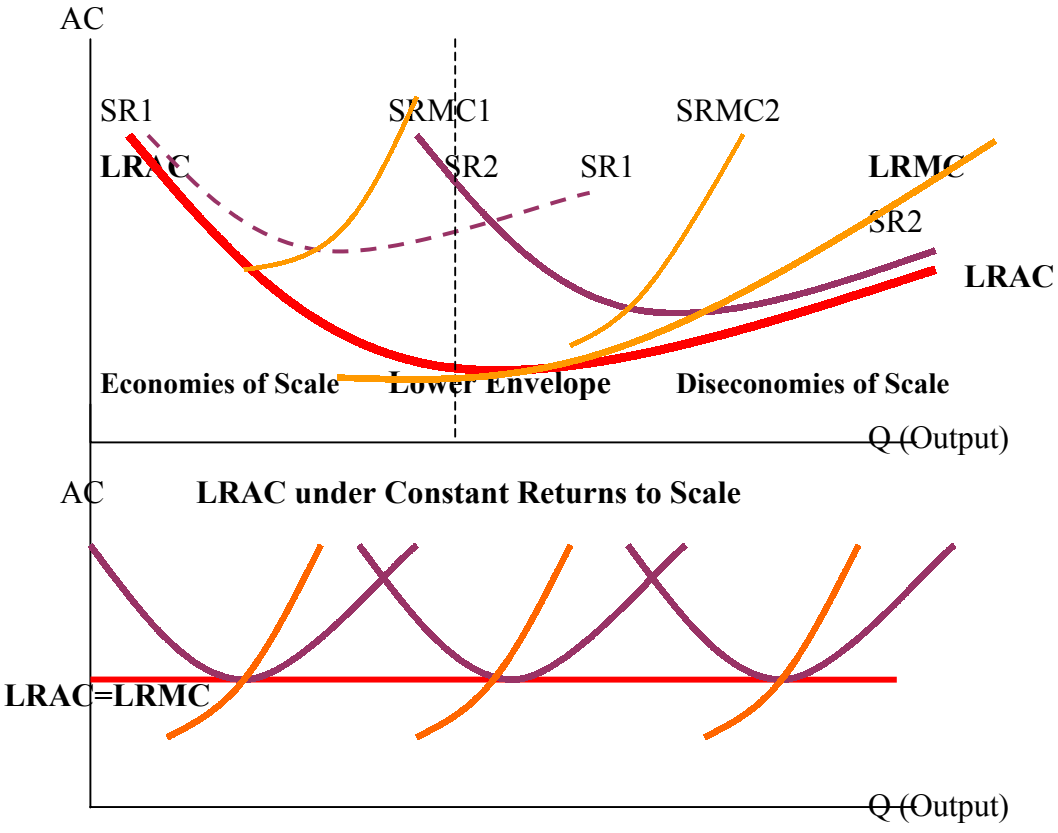
Diminishing Returns and Returns to Scale

i) Returns to a Single Input: How much does output expand if a firm increases just one input? Law of diminishing (marginal) returns pertains to this question and is compatible with any sort of returns to scale..

ii) Returns to Scale: How much does output expand if all inputs are increased simultaneously by the same proportion? This question pertains to proportionate increases in all inputs.

iii) Returns to scale and returns to a single input refer to two distinct aspects of a firm's technology. A production function that displays diminishing returns to a single input may show diminishing, constant, or increasing returns when all inputs are increased proportionately.

i. Long-Run vs. Short-Run Average Cost



i) Short-Run is the time frame constrained by the firm size or FC, which is the pre-committed sunk cost during which input price (w) rigidity prevails.

ii) Long-Run is the time period long enough for all of the firm's sunk commitment to come to an end, where all inputs, including the firm size, adjust perfectly.

iii) The LRAC curve consists of all the lowest possible segments of the SRAC curves.

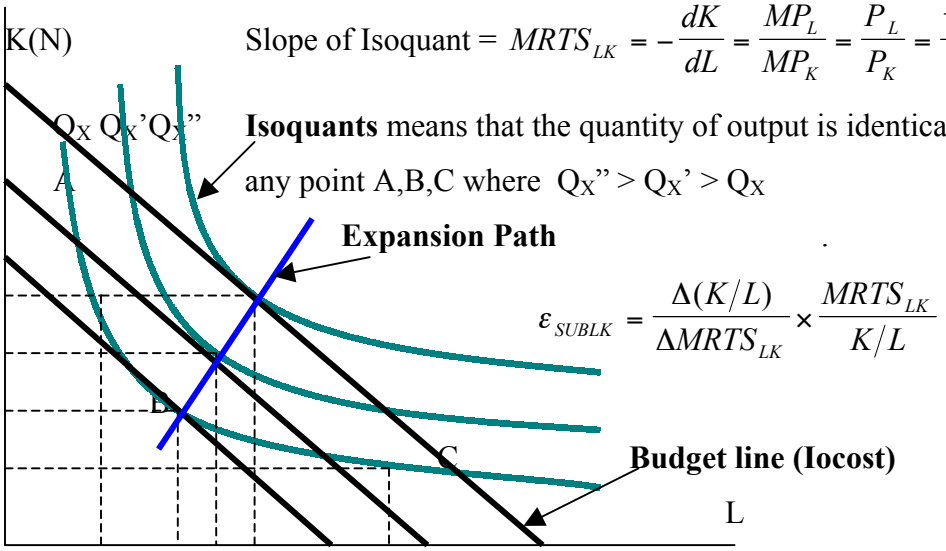
2. **Choice of Input Combinations with Multiple Inputs**

a. **Production Indifference Curve (Isoquant)** shows all the possible combinations of inputs (K & L) that yields the same output. All the points on the same curve are technologically indifferent in the sense that they represent each a bundle of inputs capable of yielding the same quantity of outputs.

i) Higher curves correspond to larger outputs.

ii) An indifference curve will generally have a negative slope. If a firm reduces the quantity of one input, it must use more of another input.

iii) An indifference curve bends inward toward the origin around its middle reflecting the law of diminishing returns to a single input.



b. **Budget Line (Isocost)** shows all the combinations of inputs the firm can purchase with a fixed amount of budget. Therefore, all the points on a budget line represent the same cost to the firm. Hence, isocost. *i.e.*) $P_K K + P_L L = M$, where M is constant.

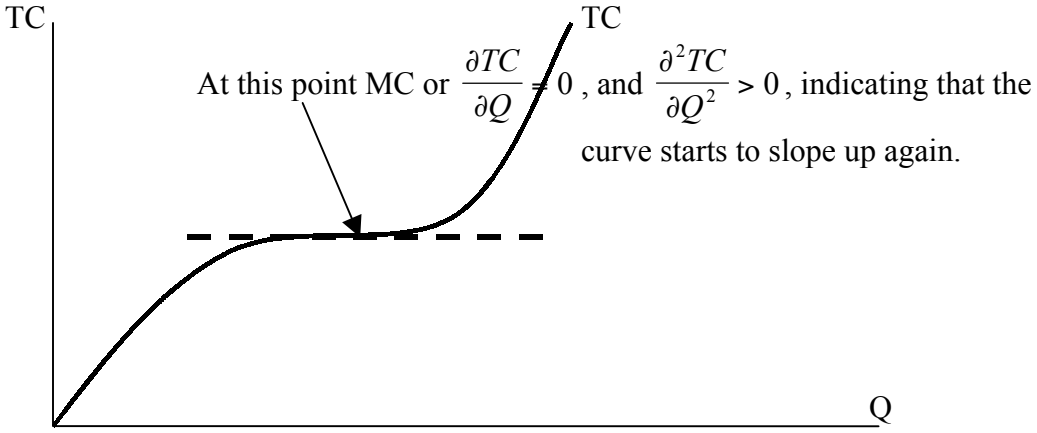
c. **Cost Minimization by Input Combination (K, L & N)** is attained at the point where isoquant is tangent on isocost.

i) From the input perspective, the ratio of the marginal return to the cost must be equal from one input to another by the law of diminishing marginal returns (MPP) to excessive use of one input or another. - *i.e.*

$$MRTS_{LK} = -\frac{\Delta K}{\Delta L} = \frac{MPP_L}{MPP_K} = \frac{MP_L}{MP_K} = \frac{P_L}{P_K} = \frac{w}{r} \rightarrow \frac{MPP_K}{P_K} = \frac{MPP_L}{P_L}$$

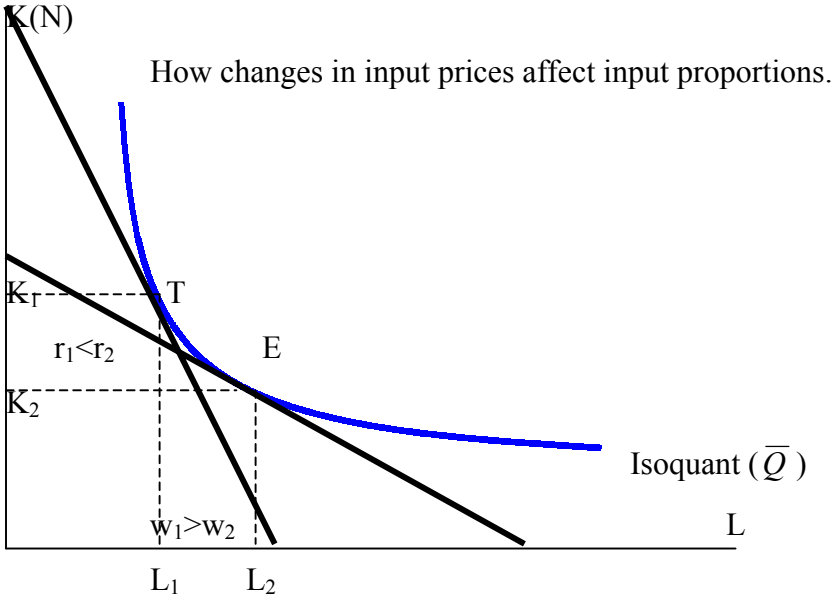
Intuitively, by switching away from the input with lower MPP/$ to more of the input w/ higher MPP/$, the firm can reduce cost w/o reducing its output. *e.g.*) Pizzeria w/ K = oven & L = workers: Too many workers/oven will get in the way of each other and result in higher cost & lower output. Too many ovens/worker will be redundant & inefficient and also result in higher cost & lower output.

ii) From the output perspective, a) FOC: $\frac{\partial TC}{\partial Q} = 0$ & b) SOC: $\frac{\partial^2 TC}{\partial Q^2} > 0$ by the cost minimization rule.



d. **Expansion Path** is the locus of the firm's cost minimizing input combinations for all relevant output levels.

e. **Effects of Changes in Input Prices**



3. **More on Marginal Rate of Technical Substitution**

Since MRTS represents various possible combination of K & L keeping output Q constant (Hence, isoquant), moving along on a given isoquant the total change in output

must be 0. – *i.e.*

$MP_L \times \Delta L + MP_K \times \Delta K = 0$ *or* $MP_L dL + MP_K dK = 0$ *or* $\frac{\partial TR}{\partial L} dL + \frac{\partial TR}{\partial K} dK = 0$, where

$MP_L = \frac{\Delta TR(TPP)}{\Delta L} = \frac{\Delta PQ}{\Delta L} = \frac{\partial TR}{\partial L}$, $MP_K = \frac{\Delta TR(TPP)}{\Delta K} = \frac{\Delta PQ}{\Delta K} = \frac{\partial TR}{\partial K}$ & $TR = TPP \times P = F(K, L) = PQ$

Then, $\frac{MP_L}{MP_K} = -\frac{\Delta K}{\Delta L} = -\frac{dK}{dL} = MRTS_{LK} = \frac{P_L}{P_K} = \frac{w}{r}$. It can also be verified as follows.

$\frac{MPP_L}{MPP_K} = \frac{\partial TPP / \partial L}{\partial TPP / \partial K} = \frac{\partial TPP}{\partial L} \times \frac{\partial K}{\partial TPP} = \frac{\partial K}{\partial L}$