ANOVA or F-Statistic

$R^2 = \frac{SSE}{SST} = 1 - \frac{\sum e_i^2}{\sum Y_i^2} = 1 - \frac{SSU}{SST} > 1 - \frac{SSU/(n-k)}{SST/(n-1)} = \overline{R}^2$, where in some literatures

i) SSU = SSE(Error Sum of Squares) or SSR(Residual Sum of Squares), and
ii) SSE = SSR(Regression Sum of Squares).

Now, $\frac{SSE/(k-1)}{SSU/(n-k)} \sim F_{k-1,n-k} = \left[\frac{n-k}{k-1}\right]\left[\frac{SSE/SST}{1-(SSE/SST)}\right] = \left[\frac{n-k}{k-1}\right]\left[\frac{R^2}{1-R^2}\right]$

Wald Test - Comparing Two Theories or Models

i) $Y_i = \beta_1 + \beta_2 X_{i2} + \cdots + \beta_k X_{ik} + \varepsilon_i$ vs.

ii) $Y_i = \beta_1 + \beta_2 X_{i2} + \cdots + \beta_k X_{ik} + \beta_{k+1} X_{ik+1} + \cdots + \beta_Q X_{iQ} + \varepsilon_i$, where Q>K.

To test $H_0 : \beta_{k+1} = \beta_{k+2} = \cdots = \beta_Q = 0$, then

i) SST = SSE_k+SSU_k
ii) SST = SSE_Q+SSU_Q
iii) $\frac{(SSE_Q - SSE_k)/(Q-k)}{SSU_Q/(n-Q)} \sim F_{Q-k,n-Q}$

If this F>Fc @ given α level, then theory ii) is valid indicating that

$SSE_Q \geq SSE_K \rightarrow \frac{SSE_Q}{SST} \geq \frac{SSE_k}{SST}, \quad i.e. R_Q^2 \geq R_k^2$

Conclusion: Adding new explanatory variables can never result in a reduction of R^2. However, this is not necessarily true of adjusted R^2.

Types of Distribution

i) χ^2 Distribution: If Z is a random variable ND(μ,σ^2), then $Z^2 \sim \chi^2$.

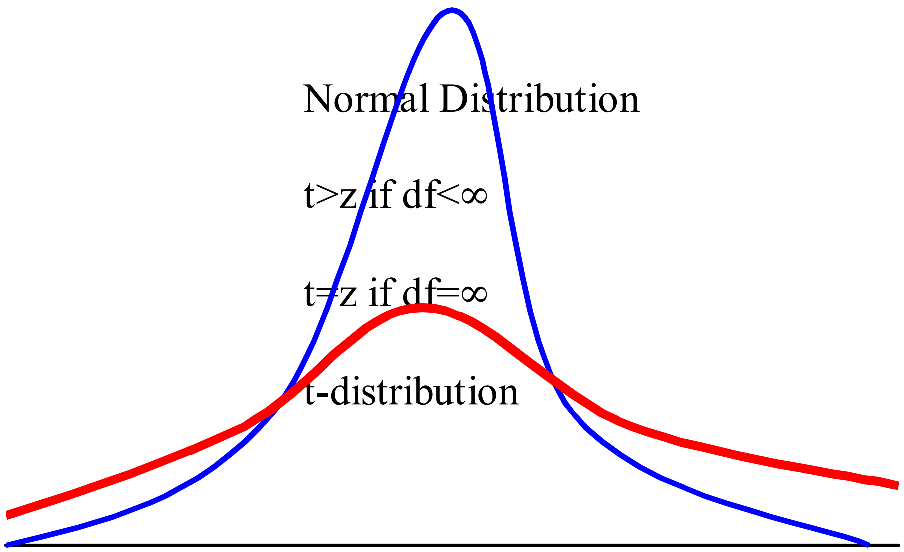
$$Z = \frac{X-\mu}{\sigma} \sim ND, Z^2 = \left(\frac{X-\mu}{\sigma}\right)^2 \sim \chi_n^2$$

ii) (Student) t Distribution: If Z~ND & U~χ^2_n, $t = \frac{Z}{\sqrt{U/n}} = \frac{\sqrt{n}}{\sqrt{U}} Z \sim t_n$.

iii) F Distribution: If $U \sim \chi_m^2$ and $V \sim \chi_n^2$, then $\frac{U/m}{V/n} \sim F_{m,n}$.

Also, If $t \sim t_n$, then $t_n^2 \sim F_{1,n}$

t-Statistic

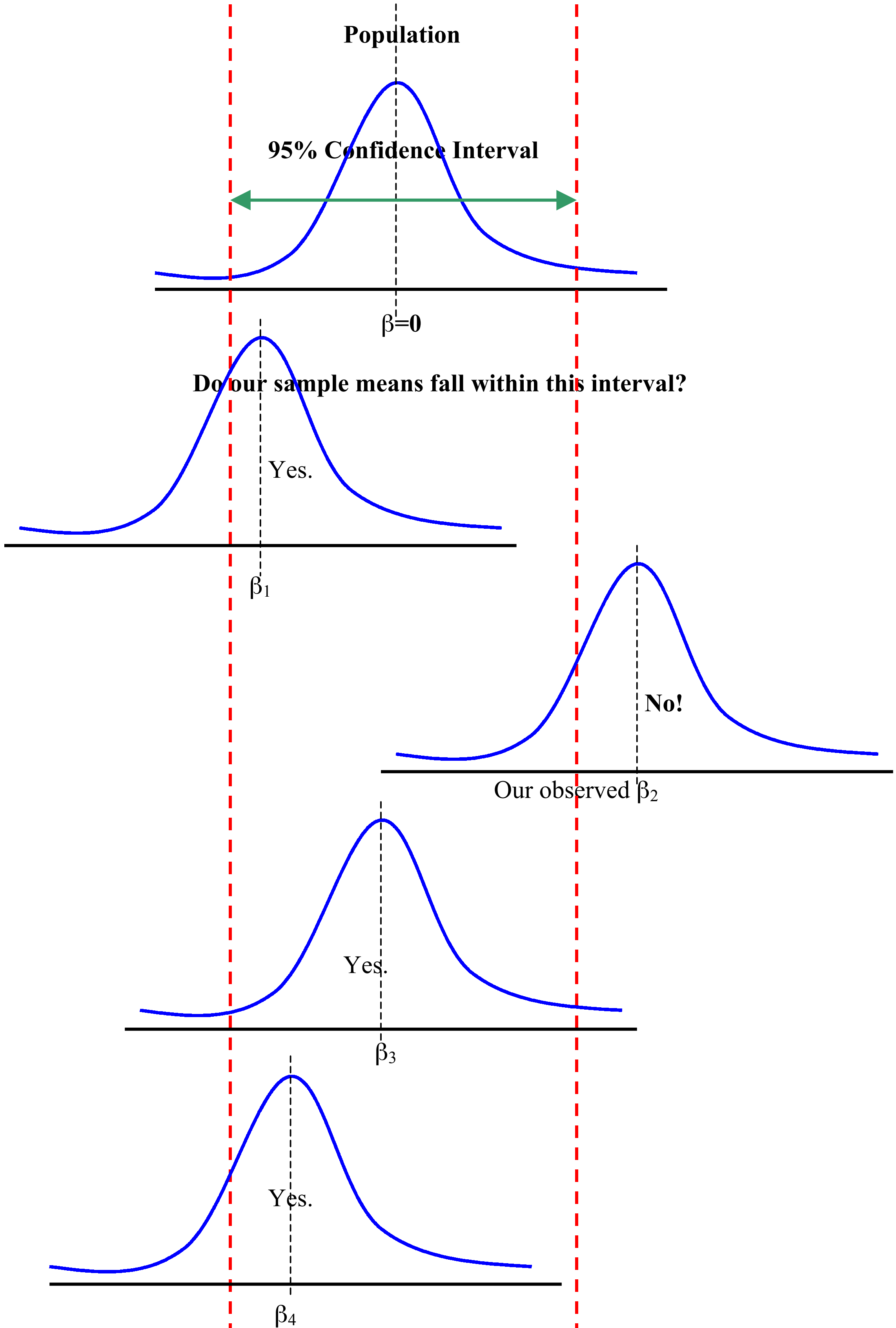


Even if we got β≠0, β may not be truly different from 0 (no effect on the dependent variable). How can we be sure that we got the β≠0 purely not by fluke?

If we had 100 samples that come from same distribution, we would get 100 different sample means. If we found that 95 out of 100 fall within 95% interval from the true (population) mean (standardized β = t = 0), then the 5 sample means that fall outside this 95% interval would be extremely rare cases. And these rare occurrences would not be by chance, because if it were to happen by chance, evidently the chances are higher for these sample means to fall within this 95% interval (95% vs. 5%).

Then, these 5 means may come from samples drawn from totally different populations, and can be said significantly different from the most likely β=0 at 5% significance level. - *i.e*. these 5 β's are significant from this group of 100 sample means of β's and therefore, can be relied on to be different from the true mean of β=0.The t-value @ α % level is, therefore, the criterion for judging whether the observed β is significantly different from the hypothesized β value, which is usually set at 0. This makes it easy to tell if our explanatory variable is effective in explaining the dependent variable, because β is the very measure of the effect of the explanatory variable on the dependent variable.

Therefore, t-value is the indicator, and, if the calculated t from our observed β is > critical t_C @ given significance level of α %, our observed β is concluded significant. If we use 5% significance level as is routinely conducted, the most common rule-of-thumb critical t value is 2.


Population
95% Confidence Interval
β=0
Do our sample means fall within this interval?
Yes.
β1
No!
Our observed β2
Yes.
β3
Yes.
β4