Hyperbolic Utility Consumption-CAPM with Time Variation

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This study was motivated mainly by two issues. First, the fact that Consumption-CAPM, although a superior model to CAPM & APT in terms of theoretical development, has not been so successful in actual performance. Second, the asset pricing puzzle or the equity premium puzzle may be related to the poor performance of the Consumption CAPM.

Although it is generally believed that the poor performance of the CCAPM is due to the problem inherent in aggregate nature of data, I attribute it to the inadequate specification of the utility function and the resulting risk aversion mechanism. The risk aversion is simply one's tolerance about the uncertainty of the expected future return on investment. If an individual has high risk aversion, he/she has greater tendency to avoid risk, so his/her tolerance about the uncertainty will be low. Therefore, this person would rather use his/her income on consumption today rather than investing in uncertain return tomorrow. On the other hand, if his/her risk aversion is low, then, the person has high tolerance about this uncertainty and, therefore, willing to take more risk.

The existing literatures on CCAPM use a utility specification called power utility function that entails CRRA, which states that a representative individual's risk aversion is constant relative to income. Simply put, if a person's income is $1,000 and his risk aversion is high, so he/she would consume 90% of his/her income today and save/invest only 10% of it rather than any otherwise, this ratio will be maintained even when his/her income is $1,000,000 or $100. If we plot this consumption & saving/investing behavior against income, it will show a linear trend, because one's risk aversion is constant relative to income.

However, this conflicts with the actual data as well as theory, because the actual consumption behavior shows concave curvature as income increases. The theory has it that an individual doesn't have a perfect information about his/her lifetime income stream, so any transitory increase in income will not increase consumption proportionately, but will rather be saved to smooth out the consumption over one's life time. This is basically the idea of Milton Friedman's Permanent Income Hypothesis. Keynes also expressed a similar notion that the MPC out of transitory income or wealth declines with level of wealth. Then, it is only rational that uncertainty calls for concave consumption pattern, and it is essential to allow for decreasing absolute risk aversion

(DARA). Therefore, it becomes clear that the power utility function that entails CRRA is not an adequate utility specification.

This risk aversion argument also has an implication for the equity premium puzzle. If we plot the time series of U.S. consumption data against U.S. income data for the past 100+ years, we would find that the consumption shows smooth pattern relative to the income. This is due to the PIH as already explained above. Also, if we look at the time series of market risk premium in the U.S. over the past 100+ years, we will see that it has historically been in the 6% range, whereas the average real return on relatively risk free short-term securities has been only about 0.8%. If the consumption has been traditionally smooth, meaning that people generally have had low risk aversion, this risk premium certainly does seem too high to be justified. In other words, you wouldn't need to offer such a high incentive to lure people to invest, because they are already willing to take more risk than what the market thinks. This is the asset pricing puzzle.

To close this gap between the high risk premium and the actual consumption behavior, Mehra & Prescott experimented with varying values of risk aversion parameters from 2 to10, which are exorbitantly high for parameter values. However, playing with parameters while actual consumption data say otherwise still doesn't cure the fundamental cause of the problem. I think that Prescott & Mehra's approach may produce better results in replicating the high market risk premium without resorting to excessively high parameter values had they used Hyperbolic Decresasing Absolute Risk Aversion (HyDARA) rather than CRRA. HyDARA, of course, is subject to certain limitations, because it can better replicate the high risk premium only up to a marginally increasing range of the concave consumption. Yet, it is theoretically a better alternative to the CRRA with linear consumption curve, which certainly isn't suitable for nor even capable of replicating this marginally increasing range.

The model is initially derived from the Euler equation, which defines the intertemporal equilibrium relation between Consumption and Investment decisions.

I. Introduction

In the modern finance it seems as if there is a prevailing tendency to disregard the utility function when constructing an asset-pricing model. And whether the utility function is relevant or not in the model has almost become like a break-up point between finance and economics. Even in economics it has also been shown that the standard capital asset pricing model (CAPM hereafter) approximates asset pricing sufficiently when the marginal utility of consumption is highly correlated with the return on the stock market[1]. Some theories such as Lucasian tree model even contend that consumption is eventually replaced by dividends in equilibrium. Although attempts have been made in the past to bridge this gap between economics and finance, I believe the relevance of the

[1] Blanchard, O.J. & Fischer, S., *Lectures on Macroeconomics*, MIT Press, 1996, pp507~510

utility function must be addressed prior to pursuing further with any type of real asset pricing model.

This paper is motivated by the idea that hyperbolic absolute risk aversion (HARA) based time-varying asset pricing model might be an alternative solution to close this gap between consumption CAPM (CCAPM) and the actual financial market that several studies done in the past attempted unsuccessfully. First, Hall & Flavin contended that U.S. consumption is too sensitive to changes in income in their studies on consumption sensitivity puzzle, which was not the real puzzle at all since the actual consumption does not track the income process. Assuming that consumption is sensitive to income as in their studies, CCAPM would seem to work well. However, they failed to recognize the random walk possibility in modeling the process of permanent income.

Second, Mehra & Prescott found that U.S. consumption is too smooth to explain the observed actual risk premium and argued that higher risk aversion parameter must be used to replicate the volatility of the stock market. All these studies served adversely only to the detriment of the efficacy of CCAPM, but they did not explain why consumption tends to be so smooth. It is, of course, a widely accepted norm in economics that consumption is not sensitive to changes in transient income.

The objective of this paper, therefore, is to develop a new consumption-based real asset pricing model that is straightforward and rigorous in modeling technique, yet simple and easy to implement, and that can possibly replicate the actual asset returns more closely within the reasonable range of risk aversion. Modeling it as "consumption-based" and "real" inevitably involves utility function, and involving utility function in the model generally entails the assumption of constant relative risk aversion (CRRA). I, however, would propose to assume HARA, because i) it is a more comprehensive specification of the risk aversion mechanism.; ii) there has not been sufficient number of known studies using HARA utility function in the asset pricing model. Therefore, it would be a worthy effort to examine its possibility. The need for assuming HARA will be more evident as we proceed.

In this respect the HARA assumption seems to gain its ground as part of the crux of the model. If the model proves to be effective, it will solidify asset-pricing model's foundation on the utility function through the legitimate risk aversion mechanism.

Diagnosis of the risk aversion parameter prescribed by the assumption will test this, because the utility function in the model is justified if the model proves to add to the high explanatory power of the model.

The data used consist of time-series of 10~15 stocks selected according to the Dividend Yield Strategy.[2] The strategy is simple: once each year, adjust your portfolio so you own only the 10 highest yielding stocks in the Dow Jones Industrial Average. These 10 reportedly do better than the market during the down market and at least as well as the market during the up market. Therefore, these stocks can be said to rely heavily on the strategy that maximizes dividend-yield ratios. Using the dividend price ratio as a regressor thus gains a strong empirical rationale as well apart from the solid theoretical basis to be demonstrated later.

This will also enable us to construct a hypothetical portfolio consisting of one share each of these stocks and compare the portfolio's performance vis-à-vis the performance of the market portfolio/index to verify the validity of the strategy. These data are readily available for download from various internet sources such as Yahoo Finance.

The model's forecasting power is also checked with out-of-sample testing. If this model obtains significant results, it will support the validity of HARA-based asset pricing and increase efficiency and simplicity of the time-varying asset pricing model.[3] For instance, it can be compared with a pure data generating process such as autoregressive integrated moving average (ARIMA) technique. This paper will find out if this econometric model would perform better than a pure time series forecasting model in the short-term forecasting such as monthly or quarterly expected returns over SR forecasting horizon under one year. The schematics in Figure-1 would help understand the flow of this paper.

II. Consumption-Based Asset Pricing Models

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[2] This strategy is popularly dubbed "**Dogs of the Dow**" in the language of the financial market. There is a number of on-line resources for the Dogs of the Dow rationale. For more details refer to http://stocks.about.com/money/stocks/library/.

[3] This will be tested by RMSE of out-of-sample forecasts of the competing models.

A. CRRA-Based Time Varying Expected Returns

We begin with the "time-varying expected returns" technique developed by John Campbell and Robert Shiller, which they used heavily in many of their studies on expected dividends, dividend-price ratio, stock prices and earnings in the late 80's (Campbell & Shiller 1, 2, 3). The present-value relations among the variations of their model can be traced back to Gordon growth model, which is related to the utility-maximizing objective function in the following manner.

Objective function: According to Lucasian type models, individuals consume to

$$MaxE_t \sum_{\tau=0}^{\infty} \beta^{\tau} U(C_{t+\tau}) \quad s.t. \quad i)\ C_t + W_t = R_t W_{t-1} \ \&\ ii)\ R_t = \frac{P_t + D_t}{P_{t-1}} \tag{1}$$

where all the variables are expressed in aggregates and in their usual notation.
Then, the first order condition will be

$$\text{FOC: } U'(C_t) = \beta\ E_t U'(C_{t+1}) R_{t+1} \tag{2}$$

where $U'(C_t)$ or the marginal utility of consumption is equal to the opportunity cost of the foregone current consumption, which is $E_t \beta U'(C_{t+1}) R_{t+1}$ or the future market return adjusted by the marginal utility of future consumption discounted to the present.

Dividing both sides by $U'(C_t)$ we get $\frac{U'(C_t)}{U'(C_t)} = \beta\ E_t \frac{U'(C_{t+1})}{U'(C_t)} R_{t+1}$, assuming CRRA.

Define the net return $$r^*_{t+1} = \beta\ E_t \frac{U'(C_{t+1})}{U'(C_t)} R_{t+1} - 1 = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} - 1 \tag{3}$$

We may further define $$h_{t+1} = \log\left[\frac{U'(C_{t+1})}{U'(C_t)} R_{t+1} \right] = \log \frac{P_{t+1} + D_{t+1}}{P_t} - \gamma \log \frac{C_{t+1}}{C_t} \tag{4}$$

where h_{t+1} is a utility-adjusted log return of the asset that a utility optimizing investor will rationally choose, which is not directly observable. It can be observed only indirectly through $\log R_{t+1}$ or r_{t+1} which is an observable variable.

Through linearization of h_{t+1} in terms of *P* and *D*, first order Taylor series expansion of $f(p,d)$[4] around the steady-state points ***p**** and ***d****, and some algebraic manipulations we finally arrive at the structural equation between utility-adjusted log return and log dividend-price ratio, change in log dividend, and change in marginal utility of consumption[5]:

$$
\begin{aligned}
h_{t+1} &= (1-\rho)d_{t+1} + \rho p_{t+1} + k - \gamma \log\left(\frac{C_{t+1}}{C_t}\right) \\
&= \delta_t - \rho\delta_{t+1} + \Delta d_{t+1} - \gamma\Delta c_{t+1} + k \quad \approx \quad \xi_{t+1}
\end{aligned}
\tag{5}
$$

If $h = (1-\rho)d + \rho p + k - \rho - \gamma \log 1 = \log\frac{U'(C)}{U'(C)}R = \log R = \xi$ in steady state, since $-\gamma\,\Delta c_{t+1} = -\gamma \log 1 = 0$, a limiting case when $C_{t+1} = C_t$, then $h = \xi = \log R = (1-\rho)d + \rho p + k - \rho$. Therefore, $h = \xi$ in steady state. [6]

This may provide an insight as to why the utility function is generally overlooked in most financial asset pricing models. Intentionally or unintentionally, the financial model builders attest to an important point that utility function would drop out in the steady state under the assumption of CRRA.[7]

Campbell also made similar point in "Intertemporal Asset Pricing without Consumption Data"(AER June '93). He replaces the covariance between the return on the i^{th} asset and consumption with the weighted average of "the covariance between the return on the i^{th} asset and the market return" and "the covariance between the return on the i^{th} asset and the upward revision of expected future returns":

i.e.) $E_t r_{i,t+1} - r_{f,t+1} = -\frac{V_{ii}}{2} + \gamma\ V_{im} + (\gamma-1)V_{ih}$. He, thereby, arrives at a real asset-pricing

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[4] p_{t+1}= logP_{t+1}, d_{t+1}= logD_{t+1}, and δ_t **= log(Div$_t$/P$_t$)**. A common name for δ_t is dividend yield. The ratio variables, as we see in the financial press, are used as indicators of fundamental value relative to price. If stocks are under-priced relative to fundamental value, returns tend to be high subsequently, and the converse holds if stocks are overpriced.

[5] For unabridged steps of derivation, refer to the Appendix.

[6] However, if in SS $\Delta c=g$ for example, $-\gamma\Delta c=-\gamma g$. Then, $h=\xi$ only approximately. See the Appendix for ξ.

[7] Another way to explain it is that consumption comes entirely from dividend in equilibrium as in Lucas Tree model or Cash-In-Advance model.

model without consumption variable that maps into the standard CAPM format.[8] Then, for an ***i^th*** asset, $h_{it+1} \approx \xi_{it+1} = \delta_{it} - \rho\delta_{it+1} + \Delta d_{it+1} + k$, where interpretation of h_{t+1} would be the utility-adjusted gross log return on the i^{th} asset at time t+1.

B. Hyperbolic Absolute Risk Aversion-Based Asset Pricing Model

1. About HARA

According to Carroll & Kimball (1996), hyperbolic risk aversion[9] is a more realistic alternative to the power utility used in CCAPM for the following reasons:

The power utility function is defined as $U(C) = \dfrac{C^{1-\gamma} - 1}{1-\gamma}$, with γ>0.

Then, $U' = C^{-\gamma}$ & $U'' = -\gamma\, C^{-\gamma-1}$, where $U'' < 0$. Since Arrow-Pratt risk aversion function is defined as $-\dfrac{U''}{U'} = \dfrac{\gamma}{C}$, as C increases, $\dfrac{\gamma}{C}$ decreases. This means decreasing risk aversion, which implies concavity of consumption.

This also conforms to what Keynes had argued about marginal propensity to consume - that MPC out of transitory income or wealth declines with level of wealth.[10] It is only rational that introduction of uncertainty requires concavity. Therefore, it is essential to allow for decreasing absolute risk aversion (DARA). However, since $-C\dfrac{U''}{U'} = \gamma$ is a constant, power utility belongs to a CRRA family. If the only form of uncertainty is in labor income Y_L, CRRA utility implies a linear consumption function. This means that MPC stays constant, even if wealth increases. Hence the power utility function needs to be generalized.

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[8] Campbell, J.Y., "Intertemporal Asset Pricing without Consumption", *American Economic Review*, 1993

[9] Blanchard and Fischer, Lectures on Macroeconomics, MIT Press, 1996 pp283~284.

[10] Vinod, H.D., "Concave Consumption, Euler Equation and Inference Using Estimating Functions", *Proceedings of Business and Economic Statistics section of American Statistical Association*, Alexandria, Virginia, 1997, pp118-123

HARA utility function is defined as

$$U(C) = H + \frac{1-\kappa}{(2-\kappa)A}[AC+B]^{(2-\kappa)/(1-\kappa)}, \quad where \quad \kappa = \frac{(1+\gamma)}{\gamma}. \tag{6}$$

The constant of integration H can be ignored by assuming ordinal utility function. Then, HARA $U'(C) = \left[AC+B\right]^{\frac{1}{1-\kappa}}$. Since $\frac{1}{1-\kappa} = -\gamma$, the power utility function with $U' = C^{-\gamma}$ is a special case of HARA with B=H=0, and A=1. Further, $\frac{U'''U'}{U''^2} = k > 0$ implies strictly concave consumption function required by the economic theory.

Therefore, κ is an important parameter characterizing 3 special cases of HARA functions:

i) κ = 0, then borderline case of quadratic utility function.

ii) κ = 1, constant absolute risk aversion (CARA) function *via* l'Hôpital's rule.

iii) κ >1, HyDARA.[11]

In the financial economics literature (Huang and Litzenberger 1988 or Ingersol 1987) the HARA class has been studied in the context of discrete time intertemporal portfolio selection problem. CCAPM are common in macroeconomics. The power utility remains a common assumption for Euler Equation estimates of CCAPM. Therefore, it would be a worthy attempt to propose new estimation of CCAPM under HARA as suggested by Vinod 1999.

2. HARA-Based CCAPM[12]

Once again, Euler equation is defined[13] as $E(g_t) = E(\beta R_t c_t^{-\gamma}) = 1$ (7)

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[11] If κ > 1, then 0 < γ < 1, which is the normal range for risk aversion parameter values. FYR if 0 < κ < 1, γ < −1, and if κ ≤ 0, then −1 ≤ γ < 0, which are both improbable and unrealistic values for risk aversion parameter.

[12] Carroll, C.D. and Kimball, M.S. On the Concavity of the Consumption Function, *Econometrica* 64 (4) , 1996, pp. 981~992

[13] Euler Equation equals 1 for the same reason as equation (3).

where $c_t = \frac{C_t}{C_{t-1}}$ assuming CRRA (Vinod 1999). However, the EuEqn is not amenable to testing as it is, because it is nonlinear in parameter. Therefore, it needs to be linearized. Using small sigma asymptotics (SSA)[14] and Jensen's error, we finally arrive at *Linearized Approximate SSA Regression Model for HARA*[15]:

$$\log R_t = -\log \beta + \gamma \log \frac{C_t}{C_{t-1}} + \gamma\phi\left(\frac{C_t}{C_{t-1}}\right) + u_t \tag{8}$$

If regression coefficients are $\beta_1, \beta_2, \beta_3$, the parameters of interest $\theta = (\beta, \kappa, \phi)$ can be found by using $k = \frac{1+\gamma}{\gamma}$ and $\beta = e - \beta_1$, $\kappa = \frac{1+\beta_2}{\beta_2}$ and $\phi = \frac{\beta_3}{\beta_2}$. (9)

C. Merging Time-Varying Technique with HARA-Based CCAPM

As already shown from the loglinearized HARA-Based CCAPM, if we let $g = RZ$, where $R = R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t}$ *and* $Z = \beta c_t^{-\gamma}$, we can express $\log g_t$ as

$$\begin{aligned}
\log g_t &= \log R_t + \log Z_t = \log\left[\frac{P_t + D_t}{P_{t-1}}\right] + \log \beta\, c_t^{-\gamma} \\
&= \log\left[\frac{P_t + D_t}{P_{t-1}}\right] + \log \beta - \gamma \log \frac{U'(C_t)}{U'(C_{t-1})} \\
&= f(p,d) - p_t + \log \beta - \gamma \Delta c_t \\
&= \lfloor (1-\rho)d_t + \rho p_{t+1} + k' - p_t \rfloor - \log \beta - \gamma \Delta c_t \\
&\approx \delta_{t-1} - \rho\delta_t + \Delta\delta_t - \gamma \Delta c_t + k, \quad \textit{where} \quad k = k' + \log \beta
\end{aligned}$$

As we have seen in the Time-Varying Expected Returns technique, we can approximate h_t as $h_t \approx \delta_{t-1} - \rho\delta_t + \Delta d_t - \gamma \Delta c_t + k$

$$\approx k + \delta_{t-1} - \rho\delta_t + \Delta d_t - \gamma \log\left[\frac{AC_t + B}{AC_{t-1} + B}\right] + \varepsilon_t . \tag{10}$$

Using linear approximation technique and SSA, we can rewrite (10) as:

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[14] *Journal of American Statistical Association*, 1970 p182 and *Journal of Econometrics*, 1976 p147

[15] For unabridged steps of derivation, refer to the Appendix.

$$h_t = k + \delta_{t-1} - \rho\delta_t + \Delta d_t - \gamma\left[\log\frac{C_t}{C_{t-1}} + \phi\left(\frac{1}{C_t} - \frac{1}{C_{t-1}}\right)\right] + \varepsilon_t$$

$$= k + \delta_{t-1} - \rho\delta_t + \Delta d_t - \gamma\,\Delta c_t - \gamma\phi\left(\frac{1}{C_t} - \frac{1}{C_{t-1}}\right) + \varepsilon_t$$

$$= k + \delta_{t-1} - \rho\delta_t + \Delta d_t - \gamma\,\log R_{mt} - \gamma\phi\left(\frac{1}{C_t} - \frac{1}{C_{t-1}}\right) + \varepsilon_t$$

$$= \beta_0 + \beta_1\delta_{t-1} + \beta_2\delta_t + \beta_3\Delta d_t + \beta_4\log R_{mt} + \beta_5\left(\frac{1}{C_t} - \frac{1}{C_{t-1}}\right) + \varepsilon_t. \quad (11)$$

Now, even a simple OLS can estimate $\beta_1 = \beta_3 = 1$, $\beta_2 = -\rho$, $\beta_4 = -\gamma$, $\beta_5 = \phi\beta_4$, and $\kappa = \frac{1+\gamma}{\gamma}$. Then, HARA assumption will be affirmed if $\kappa > 1$.

For testing the significance of this κ parameter, we need a κ statistic defined as $t_\kappa = \frac{\hat{\kappa} - 1}{\sigma(\hat{\kappa})}$, where Delta Method is used to compute the variance of κ, where

$\kappa = f(\gamma) = \frac{1+\gamma}{\gamma}$. Then, $\frac{df}{d\gamma} = \frac{\gamma - (1+\gamma)}{\gamma^2}$ and $\sigma^2(\kappa) = \left[-\frac{1}{\gamma^2}\right]^2 \sigma(\gamma)$.

Also, as discussed in the previous chapters, Δc_t can be proxied by $\log R_{mt}$. This framework would certainly imply what are the relevant variables in constructing any utility-maximizing CCAPM-based asset-pricing model.

This model has strengths in several points. First, HARA assumption is quite reasonable and conceivable in the light that the existing CRRA-based CCAPM models have not been able to replicate the volatility of the market to satisfaction as exhibited by asset pricing puzzles. As HARA assumes concave consumption with increasing income, it would naturally embrace the progressively smoothing consumption, not necessarily a sensitive one such as under CRRA with permanent income hypothesis (PIH). In this sense HARA might reasonably replicate the volatility of the market in response up to a certain point in transitory income.

Second, this is a very solid model in the sense that none of the variables are arbitrary, but all derived solidly from the fundamental present value relations. Therefore, it would be the strength of the model that it is theoretically complete. Another strength in a related issue would be the parsimony in the choice of variables. Adding extra variables cannot make stock returns unpredictable if they were already found to be predictable using fewer variables. Therefore, attempts to bring any other variables into the model would be totally unnecessary unless they are theoretically and mathematically derived.

Third, another useful aspect of this model is that it casts light on the role of the dividend. Although standard finance literatures treat dividend policy as irrelevant in the firm valuation and give dividend no more credit than that of signaling effect, this model certainly attests to the realistic possibility that dividends do have a significantly material role in determining the asset return. Indeed, the proposed model is a sound loglinearized version of the Gordon return equation $R = \frac{D}{P} + G$ having the growth rate of dividend and the growth rate of market return for G term. [16]

Fourth, the market return which proxies the consumption growth rate instead of constant expected returns model has the advantage of self-adjustment with time. Intuitively it is only reasonable to assume that the stock market return converges to the macroeconomic output growth rate in the long run. Therefore, consumption is quite a relevant variable in the asset valuation model if we are using a long time series. This stipulates a fundamental component to be a legitimate part of any long-term asset valuation model.

Campbell & Shiller based their assumption on the premise that the return on reasonably long time series such as 10+ years must converge to the long mean by stationarity. However, the assumption of this long mean is not so infallible empirically in that until the economy has reached a steady state, the macro-variables would hardly manifest a constant long mean. Moreover, *ad hoc* exogenous shocks to the economic fundamentals make it hard to exactly determine when the market would attain the steady state. This sort of imperfection in the long-run constant return might be complemented by allowing it to change over time.

[16] Growth rate of market return in steady state can take two possible values as discussed previously. If in SS $C_{t+1} = C_t$, then presumably the growth rate would be 0. Or if in SS there still is a consumption growth at constant rate, say such as g, then, it would be $-\gamma g$.

Fifth, the model reflects more realistic and comprehensive risk aversion mechanism that decreases with concavity (DARA) as the transitory income/wealth increases. Combined with HyDARA the model is also easily amenable to testing as even a simple OLS can do the job.

III. Testing the Model

A. Sample and Data Sources

What I have tried to do is mainly twofold. First, I have estimated the parameters of the equation (11) for each of the selected stocks and diagnosed how well the HARA assumption applies by checking if the parameter $\kappa \geq 1$ as proposed by the model. Second, I have conducted an out-of-sample testing of the proposed model to measure its *ex post* forecasting power and compare its results with the forecast of a completely atheoretical DGP such as ARIMA.

1) **Choice of Time Horizon**: The decision on time horizon is fairly constrained by the availability of data, and the data available[17] to me mostly date back from 1970 and some from 1989 in quarterly form. Therefore, this study is basically looking into relatively medium to long-term data within 10~30 year range. Besides, i) excessively long-horizon forecasts will inevitably involve high degree of averaging. (*i.e.* law of large numbers); ii) there are those securities that have been in existence for relatively short period of time.

2) **Data Sources**: Stock price and dividend time series are available on-line from such off & on-line sources as Compustat, Yahoo Finance, Marketguide.com, Bigcharts.com, Bloomberg, Reuters, …etc. From these sources, returns series are obtained by the theoretically straightforward definition[18], $R_t = \frac{P_t + D_t}{P_{t-1}}$. Consumption

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[17] If the data are not readily available for every period, some techniques like "Winsorizing" that truncates values above and below the upper and lower bounds into the bound values may be used.

[18] I prefer to use theoretical returns over the reported returns data, because corporate earnings report practices are quite often dubious and fictitious and may contain fabricated data through "creative accounting procedures" all in an effort to present their performance favorably to the public.

Data are also available from on-line GDP data series provided by such websites as BEA-NPIA, NBER, FRED, BLS, and U.S. Census Bureau, Citibase…etc.

B. Stationarity Issues

Since all the variables used in the testing of the model are all in the first difference of the log, and particularly the log return defined as the log of the sum of the price and dividend minus log of the previous period price, which is roughly the first difference of the price, we can reasonably assume that most of these variables already have the stationarity taken care of.

Empirically dividend series are often found to be relatively stationary as discussed earlier in the theory part. Also empirically, return is generally considered a mean-reverting stationary process. So, we may proceed under the assumption that the log dividend-price ratio and growth rates of real dividends and prices are stationary, so that log dividends and prices are cointegrated processes.[19]

To verify this point I have also taken some measures to check stationarity. The following is the result of Durbin-Watson (DW hereafter) and Portmanteau *Q statistic* for the residuals of the 14 stock portfolio. This residual check for white noise and autocorrelation among the residuals supports stationarity as suggested by Campbell & Shiller.

ESS =		0.095076063
ρ =		0.000633528
DW		1.998732944
dL & dU @ 5% & 1%	1.59 & 1.76	1.46 & 1.63
Q stat		5.61901E-06
χ^2 w/ 4 df @1%		13.2767

Table 1. Stationarity check by DW and Q-statistic

DW is close to 2 and *Q statistic* is under critical χ^2 value indicating that this residual series is not autocorrelated and quite likely white noise.

Finally, the Dickey-Fuller (DF hereafter) test on residuals of the 14 stock portfolio over 10 year data range from 1989 through 1999 also produced the following results reaffirming that the residuals are stationary. The test was based on the model:

[19] Note that this is a conservative assumption in the sense that it leads to greater variability in the rational forecast of expected futures dividends, and less evidence of excess volatility in stock prices, than does the assumption that dividends and prices are stationary around a deterministic trend.

$$\Delta u_t = \gamma_1 + \gamma_2 t + \delta\, u_{t-1} + \zeta_i \sum_{i=1}^{\infty} \Delta u_{t-1} + \upsilon_t \; .$$

H_0: $\delta = \rho - 1 = 0$ indicates random walk (nonstationarity).

Dickey-Fuller for Residuals					
Multiple R	0.715671838				
R Square	0.51218618				
Adjusted R Square	0.51058679				
Standard Error	0.029119016				
Observations	613				
	df	*SS*	*MS*	*F*	*Significance F*
Regression	2	0.543071478	0.271535739	**320.23854**	8.26991E-96
Residual	610	0.517229443	0.000847917		
Total	612	1.06030092			
	Coefficients	*Standard Error*	*t Stat*	*P-value*	
Intercept	-0.000260925	0.001176153	-0.221846555	0.824507585	
u_{t-1}(H_0: $\delta = \rho - 1 = 0$)	-1.008799622	0.057264966	**-17.616349**	1.84396E-56	
Δu_{t-1}	-0.01504834	0.039632764	-0.379694423	0.704304411	

Table 2. Dickey-Fuller test for stationarity of the Portfolio residuals

Coefficient for u_{t-1} has a significant t-value rejecting H_0. This indicates that the residuals (u_t) are stationary, where DF critical τ at 1% with 2 df = −4.07. [20]

Therefore, judging from the above evidences we can conclude that the variables used in the testing of my model are cointegrated stationary processes.

C. Regression Results

1. Estimation of Parameters

The following individual regression results of 14 stocks used S&P 500 index to calculate the market return. They all present significantly high ***t*** and ***F*** ratios, R^2s and very small standard errors.

Table 3. Regression Results [21]

Bold italic indicates t significant @ 1% level. Boldface indicates t significant @ 5% level.

[20] However, estimated u is based on the estimated cointegrating parameter β. Therefore, DF and ADF τ_c and $\boldsymbol{F_c}$ are not quite appropriate. One needs to find critical values in the following references:

1. Engel & Granger, Econometrica vol. 55 1987, pp. 251-276
2. Engel & Yoo, B.S., Journal of Econometrics vol. 35, pp. 143-159
3. Long-run Economic Relationship: Readings in cointegration, Oxford Univ. Press, 1991, Chapter 12.

[21] In the earlier version of the paper, I ran the regression with 3 different softwares, *i.e*., Excel, SAS, & Gauss, and each of them all came up with practically the same estimates.

		AT&T			**Caterpillar**	
Variables	Estimates	se	t	Estimates	se	t
Intercept	0.0403504	0.000870401	**46.35840368**	0.1153128	0.00911073	**12.6568**
δ_{it-1}	**0.99985908**	0.000382716	-0.368210277	**0.992920628**	0.007677268	-0.922121273
δ_{it}	**-0.99316579**	0.000378098	**-2626.738674**	**-0.97011**	0.00731731	**-132.578**
Δd_{it}	**0.999206739**	0.003371166	-0.235307496	**0.990476779**	0.008956358	-1.063291606
r_{mt}	0.000660775	0.000752846	0.877702987	0.009528	0.01513176	0.6296714
$(1/C_t - 1/C_{t-1})$	-12.28583862	40.48811735	-0.30344307	-19.65477	435.085036	-0.0451745
F	**1879766.061**			**4214.72**		
R Square	**0.999995957**			**0.99688**		
Adjusted R	**0.999995425**			**0.99664**		
df (k, n-k)	5, 38			5, 66		

		Chevron			**Du Pont**	
Variables	Estimates	se	t	Estimates	se	t
Intercept	0.041226759	0.005996151	**6.8755366**	0.306364488	0.051296728	**5.9723983**
δ_{it-1}	**0.9246365**	0.035258702	-2.137443971	**0.9472273**	0.130611466	-0.404043394
δ_{it}	**-0.9169311**	0.035297468	**-25.977249**	**-0.8840685**	0.130836721	**-6.7570364**
Δd_{it}	**0.9268854**	0.035338184	-2.068997094	**1.0518353**	0.136812447	0.378878539
r_{mt}	0.12346752	0.045002076	**2.7435961**	-0.18682533	0.186913601	-0.999527744
$(1/C_t - 1/C_{t-1})$	-5111.457304	290.331276	**-17.605603**	-5599.729307	1834.330507	-3.052737381
F	**336.99304**			**47.74824181**		
R Square	**0.9371512**			0.676816893		
Adjusted R	**0.9343703**			0.662642195		
df (k, n-k)	5, 113			5, 114		

		Exxon			**GM**	
Variables	Estimates	se	t	Estimates	se	t
Intercept	0.049073766	0.000580863	**84.484299**	0.183732312	0.015169703	**12.111794**
δ_{it-1}	**0.9982327**	0.000637797	**-2.77094436**	**1.0040958**	0.026587084	0.154052246
δ_{it}	**-0.989768**	0.000632436	**-1565.0082**	**-0.964449**	0.026605491	**-36.250014**
Δd_{it}	**1.0009805**	0.00154995	0.632601052	**1.0265366**	0.026726874	0.992880799
r_{mt}	-1.65844E-05	0.000627697	-0.026420962	-0.06546763	0.043482762	**-1.5055996**
$(1/C_t - 1/C_{t-1})$	-79.8182461	34.40803055	**-2.3197563**	-1463.58924	394.5504635	**-3.7095109**
F	**640706.03**			**473.3979**		
R Square	**0.9999881**			**0.9544353**		
Adjusted R	**0.9999866**			**0.9524192**		
df (k, n-k)	5, 38			5, 113		

		Goodyear			**Int'l Paper**	
Variables	Estimates	se	t	Estimates	se	t
Intercept	0.127870131	0.006337964	**20.175268**	0.154546024	0.006565884	**23.537733**
δ_{it-1}	**1.0025474**	0.008233144	0.309407925	**0.9714867**	0.01181025	-2.414284202
δ_{it}	**-0.9779884**	0.008266476	**-118.30778**	**-0.9393902**	0.011936918	**-78.696213**
Δd_{it}	**1.0028502**	0.010696469	0.266461764	**0.9790171**	0.015414652	-1.36123086
r_{mt}	-0.020614447	0.015410053	**-1.3377272**	0.026371506	0.017768714	1.48415389
$(1/C_t - 1/C_{t-1})$	-273.3185545	141.9836115	-1.92500072	-644.1647136	155.5426339	**-4.1414029**
F	**3880.1303**			**2467.4407**		
R Square	**0.9941582**			**0.9908443**		
Adjusted R	**0.993902**			**0.9904427**		
df (k, n-k)	5, 114			5, 113		

		JP Morgan			Kodak	
Variables	Estimates	se	t	Estimates	se	t
Intercept	0.065094262	0.001377322	**47.261481**	0.158195352	0.007702217	**20.538938**
δ_{it-1}	**0.9998868**	0.000748677	-0.15120005	**0.9920884**	0.01330993	-0.594413344
δ_{it}	**-0.9879338**	0.000775753	**-1273.5153**	**-0.9602367**	0.013391195	**-71.706572**
Δd_{it}	**0.9989132**	0.001785735	-0.60860094	**1.009159**	0.013732946	0.666936286
r_{mt}	-0.000701589	0.001291558	-0.543210959	-0.004624746	0.021160858	-0.21855193
$(1/C_t - 1/C_{t-1})$	-42.71987096	45.69379179	-0.934916305	407.7764428	144.9531625	2.813160029
F	**727009.75**			**1624.1684**		
R Square	**0.9999873**			**0.9908443**		
Adjusted R	**0.999986**			**0.9904427**		
df (k, n-k)	5, 46			5, 113		

		3M			Philip Morris	
Variables	Estimates	se	t	Estimates	se	t
Intercept	0.157801589	0.005176658	**30.483295**	0.432595599	0.026440296	**16.361224**
δ_{it-1}	**0.994659**	0.012415152	-0.43020013	**0.914615**	0.06563431	-1.300920205
δ_{it}	**-0.9619646**	0.012457401	**-77.220334**	-0.790331256	0.066741719	**-11.841638**
Δd_{it}	**1.0011354**	0.015526984	0.073124311	**0.9386661**	0.076546639	-0.801261829
r_{mt}	0.013817702	0.016312316	0.847071727	0.243161418	0.115281191	**2.1092896**
$(1/C_t - 1/C_{t-1})$	134.2197	107.7232327	1.245967993	-6732.27522	1111.628563	**-6.0562273**
F	**2570.8715**			**266.79974**		
R Square	**0.9913623**			**0.9219073**		
Adjusted R	**0.9909766**			**0.9184519**		
df (k, n-k)	5, 112			5, 113		

		SBC			Texaco	
Variables	Estimates	se	t	Estimates	se	t
Intercept	0.157801589	0.005176658	**30.483295**	0.076039167	0.008474924	**8.9722535**
δ_{it-1}	**0.994659**	0.012415152	-0.43020013	**0.9733498**	0.042060391	-0.633617505
δ_{it}	**-0.9619646**	0.012457401	**-77.220334**	**-0.9647292**	0.041899541	**-23.024816**
Δd_{it}	**1.0011354**	0.015526984	0.073124311	**0.9726338**	0.042093901	-0.650122686
r_{mt}	0.013817702	0.016312316	0.847071727	-0.00295811	0.047307923	-0.062528848
$(1/C_t - 1/C_{t-1})$	134.2197	107.7232327	1.245967993	-3511.542915	354.7763682	**-9.8979054**
F	**2570.8715**			**146.716**		
R Square	**0.9913623**			**0.8665218**		
Adjusted R	**0.9909766**			**0.8606157**		
df (k, n-k)	5, 53			5, 113		

The above table presents *t* values of estimates based on the H_0: $\beta_i = 0$. However, the H_0 for β_1 and β_3 are not 0, but 1 as specified by the model. Therefore, we need additional test statistic for β_1 and β_3, because H_0: $\beta_1 = \beta_3 = 1$. The adjusted *t*-statistic is " $t = \frac{\beta_i - 1}{se_i}$. The table reports results of the adjusted *t*-statistic for β_1 and β_3. BLUE characteristics still undefeated. This time, the low *t* values for β_1 and β_3 confirm our adjusted H_0: $\beta_1 = \beta_3 = 0$ (that adjusted β_1 and β_3 are not significantly different from zero.) which confirms our original H_0 except for Exxon and International Paper. [22]

2. Diagnosis HARA

Next, we proceed to the main objective of this paper – *i.e.*) to diagnose if HARA is the correct assumption for risk aversion mechanism. As long as $\kappa > 0$, we have strictly concave consumption required by the economic theory and will exhibit 3 cases of HARA: i) quadratic utility function if $\kappa = 0$, ii) CARA if $\kappa = 1$, iii) HyDARA if $\kappa > 1$. The raw κ estimates can be found in the Appendix. As their statistical significance can only be determined by the appropriate test statistic, we need a test statistic to check how significant these κ estimates are, and as κ is a nonlinear parameter, delta method is the best choice. The ***t*** statistic for kappa was calculated according to the following steps at the top of the table and the results are shown below.

Table 4. κ Statistic

Company	$\sigma\gamma$	$\sigma^2\kappa = (-1/\gamma^2)^2\sigma\gamma$	$\sigma\kappa$	$t\kappa = (\kappa-1)/\sigma\kappa$
AT&T	0.000660775	3466080579	58873.42847	**-0.0257055**
Chevron	0.045002076	193.6518495	13.91588479	**-0.582018**
Du Pont	0.186913601	153.4256148	12.38650939	**0.4321309**
Exxon	0.000627697	8.29757E+15	91091006.03	**0.0006619**
GM	0.043482762	2367.067515	48.65251808	**0.313955473**
Int'l Paper	0.017768714	36738.0643	191.6717619	**-0.197836734**
JP Morgan	0.001291558	5330682653	73011.52411	**0.019522068**
Kodak	0.021160858	46257454.59	6801.283304	**0.031792249**
3M	0.016312316	447478.6684	668.9384639	**-0.10818773**
Philip Morris	0.115281191	32.97461893	5.742353083	**-0.716168871**
Texaco	0.047307923	617840826.6	24856.40414	**0.013600265**
Caterpillar	0.015131763	1836011.838	1354.995143	**-0.077456657**
SBC Comm	0.016312316	447478.6684	668.9384639	**-0.10818773**
Goodyear	0.015410053	85333.0712	292.1182487	**0.166061755**

[22] *t critical @ 1% =2.617 for 120 df & 2.576 for infinite df*
t critical @ 5% =1.980 for 120 df & 1.960 for infinite df
F critical @1%=3.17 for n-k=120, k=5 & 3.02 for n-k = infinite, k=5
t critical @ 1% =2.704 for 40 df & 2.576 for infinite df
t critical @ 5% =2.021 for 40 df & 1.960 for infinite df
F critical @1%=3.51 for n-k=40, k=5 & 3.02 for n-k = infinite, k=5

All of 14 stocks produced *t* statistic not significantly different from 0 at 1% level, which strongly support HARA. So, it seems that CARA in 11 and HyDARA in 3 cases are rather quite relevant risk aversion mechanisms in the investment decision in these stocks.[23] This may have an important implication to the financial asset-pricing models that do not give proper credit to risk aversion and utility function – an implication that the utility function with proper risk aversion assumption may be essential in building a better model.

D. Out-of-Sample Testing

The full 5-quarter out-of-sample forecasts by the Model and by ARMA can be found in the Appendix. For the economy of space, only the RMSE comparison results are reported here. The last column shows the difference between ARMA RMSE and the Model RMSE. Any negative value would indicate that ARMA RMSE > Model RMSE meaning that the Model has a better out-of-sample forecasting power.

Table 5. Model vs. ARMA

Company	Model RMSE	ARMA RMSE	Spikes at lags	Model RMSE – ARMA RMSE
AT&T	0.000805217	0.081074343	AR(1,2)	**-0.080269126**
Caterpillar	0.008411008	0.20588372	MA(1,4,8)	**-0.197472713**
Chevron	0.018878357	0.108144714	AR(1,2,19)	**-0.089266357**
Du Pont	0.128729533	0.148504033	MA(2,5)	**-0.019774499**
Exxon	0.000425439	N/A		N/A
GM	0.069199542	0.098727606	MA(3,4)	**-0.029528064**
Goodyear	0.005079097	0.279652886	MA(1)	**-0.274573789**
Int'l Paper	0.02325495	N/A		N/A
JP Morgan	0.001103416	0.181492892	MA(1)	**-0.180389476**
Kodak	0.011757591	0.143889223	AR(1,2,12)	**-0.132131631**
3M	0.017417936	0.130691033	MA(1)	**-0.113273097**
Philip Morris	0.088943441	0.243820011	MA(1)	**-0.15487657**
SBC	0.098330159	0.37191014	AR(1,3,26)	**-0.273579981**
Texaco	0.044173777	0.111672488	ARMA(1,2,22:1)	**-0.067498711**

In all of the cases where ARMA process could be identified, the model produced smaller RMSE than ARMA indicating that this model has better out-of-sample forecasting power than ARMA.

[23] The same thing can be said for both HyDARA and CARA as long as the test statistic is designed to check if the parameter is significantly different from 1 prior to adjusting. However, it is also curious from CRRA perspective that only 8 have the correct sign for γ.

IV. Conclusion

Despite insignificant multicollinearity[24] in some data series, the overall test results proved to be quite robust in the light of the different other variants of tests performed such as auxiliary regressions, data pooling, and detrending.[25] All of the regressions estimated β_1 and β_3 to be not significantly different from 1 exactly as prescribed by the model and β_2 to be within the reasonably consistent range with Campbell & Shiller's estimates.

HARA possibility has also been diagnosed and validated theoretically and empirically. With HARA, the relevance of utility function in the asset-pricing model has also been reinstated. Transient income does not affect consumption. Therefore, change in transient income is not paralleled by a matching change in consumption, and consumption tends to be smooth. The built-in progressively decreasing curvature of the hyperbolic consumption ensures this smooth response of consumption to income. Therefore, it is only natural that the variance of consumption becomes smaller than the stock market volatility past a certain point on the curve. This means that HARA is a more realistic explanation for why consumption tends to be smooth vis-à-vis the market.

CRRA cannot explain progressively smoothing consumption, because CRRA assumes proportionate changes in consumption in response to changes in relative income. This would imply that there is a point where hyperbolic MPC starts to be less than power or quadratic MPC somewhere along the curvature of the consumption. Even if HARA-based consumption may not completely track income in reality, the model still closely tracked and forecasted the returns path. This may be another strength of hyperbolic utility model.

Certainly, the puzzle is that the risk premium is too high to be explained by the smooth consumption. This paper may answer at least partially this puzzle without resorting to exorbitant risk aversion parameters. Under the non-stochastic permanent income hypothesis, CRRA may be a relevant assumption, but with the stochastic permanent income, HARA may be the most relevant one. Intuitively, it indicates that no

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[24] See the Appendix for multicollinearity check using condition number and the remedy thereof through ridge regression.

[25] These were done in the longer version of the paper.

matter whether the windfall gain in income is great or small, as long as it is transient, people's general attitude toward risk should be absolutely averse, not relatively averse.

The model also outperformed a simple DGP such as ARMA in an out-of-sample testing, which is a rare feat for a structural model. Empirically, the hypothetical portfolio using dividend yield strategy also fared well vis-à-vis the market suggesting the practical value of this model.[26] Therefore, the main contribution of this paper may be summed up in 2 propositions: i) The risk-aversion mechanism in the utility function was diagnosed and HARA was found to be quite a relevant factor. This also conforms to Vinod's (1999) test result that HARA models are statistically significantly different from the traditional CRRA.; ii) It was verified that the utility function is an integral part of the long run asset-pricing. Even aside from the theoretical exercise, it is also logically quite probable that the more the true consumption path is revealed in the long run, and thus the permanent income path as well, the more the asset return would track the consumption.

[26] This was done in the longer version of the paper.

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