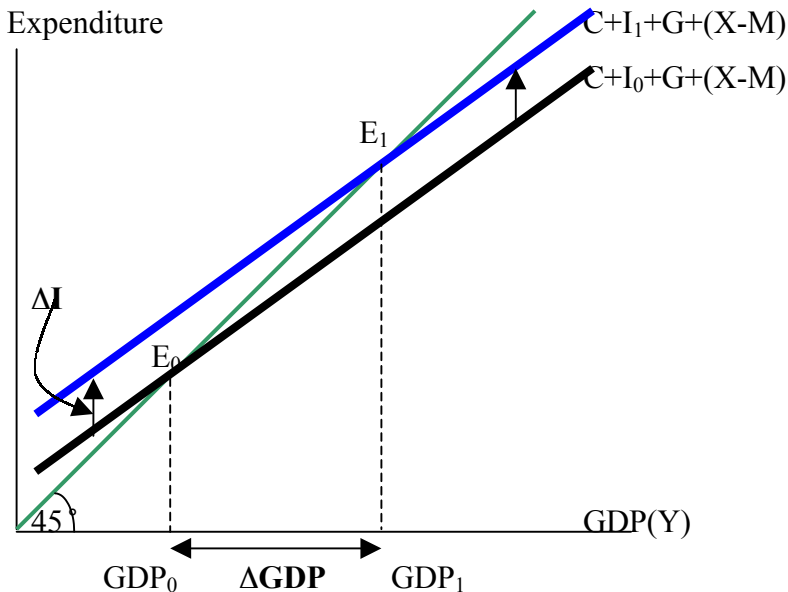


## Week 5. Changes on Demand-Side: Multiplier Analysis

### 1. Multiplier Effect of Investment

$$\text{Multiplier} = \frac{\Delta Y}{\Delta I} > 1$$



### 2. Multiplier Effect of Consumption

#### i) Two Types of Increase in Consumption

a. **Induced Increase in Consumption** is an increase in consumer spending that stems from an increase in consumer income ( $Y_d$ ) → movement along C-curve.

b. **Autonomous Increase in Consumption** is an increase in consumer spending w/o any increase in income. → **shift of the entire C-curve**.

∴ Only **Autonomous Increase in Consumption** will have multiplier effect.

#### ii) $\text{Multiplier} = \frac{1}{1 - MPC} > 1$

#### Derivation of Oversimplified Multiplier

Suppose expenditure or consumption is \$1 Million @  $MPC = .75$

$$\begin{aligned} C &= \$1\text{mil} + \$1\text{mil} \cdot .75 + \$1\text{mil} \cdot .75 + [(\$1\text{mil} \cdot .75) \cdot .75] \cdot .75 + \dots \\ &= \$1\text{mil} \cdot .75^0 + \$1\text{mil} \cdot .75 + \$1\text{mil} \cdot .75^2 + \$1\text{mil} \cdot .75^3 + \dots + \$1\text{mil} \cdot .75^K \\ &= \sum_{i=0}^K \$1\text{mil} \cdot .75^i = \lim_{K \rightarrow \infty} \sum_{i=0}^K \$1\text{mil} \cdot .75^i \text{ in the limit.} \end{aligned}$$

$$\text{Rewrite } C = \$1\text{mil}(1 + .75 + .75^2 + .75^3 + \dots + .75^K) \dots (1)$$

$$.75C = \$1\text{mil}(.75 + .75^2 + .75^3 + \dots + .75^K + .75^{K+1}) \dots (2)$$

$$\text{In the limit where } K \rightarrow \infty, C = \$1\text{mil}(1 + .75 + .75^2 + .75^3 + \dots + .75^\infty) \dots (1)$$

$$.75C = \$1\text{mil}(.75 + .75^2 + .75^3 + \dots + .75^\infty + .75^{\infty+1}) \dots (2)$$

$$(1 - .75)C = \$1\text{mil} \cdot 1 \dots (1) - (2)$$

$$\therefore C = \frac{1}{1 - .75} \times \$1\text{mil} = \frac{1}{1 - MPC} \times \$1\text{mil} = \$4\text{mil} \text{ through infinite geometric series}$$

*\*Oversimplified*, because it ignores other factors such as *international trade* ( $M \uparrow \rightarrow \text{Exp skdl becomes flatter} \rightarrow \text{multiplier} \downarrow$ ),  $\pi, \tau$  (income), *financial system* (*tight money policy*  $\rightarrow \downarrow \text{money multiplier}$ ) that *affects negatively* (*reduces/mitigates*) MPC on domestic consumption.

e.g.)  $\pi \uparrow \rightarrow \downarrow C, \tau \uparrow \rightarrow \downarrow C, \uparrow M \rightarrow \downarrow C. \downarrow C \rightarrow \downarrow MPC \rightarrow \frac{1}{(1 - MPC \downarrow)} \downarrow$

### 3. Multiplier Effect of Government Purchase

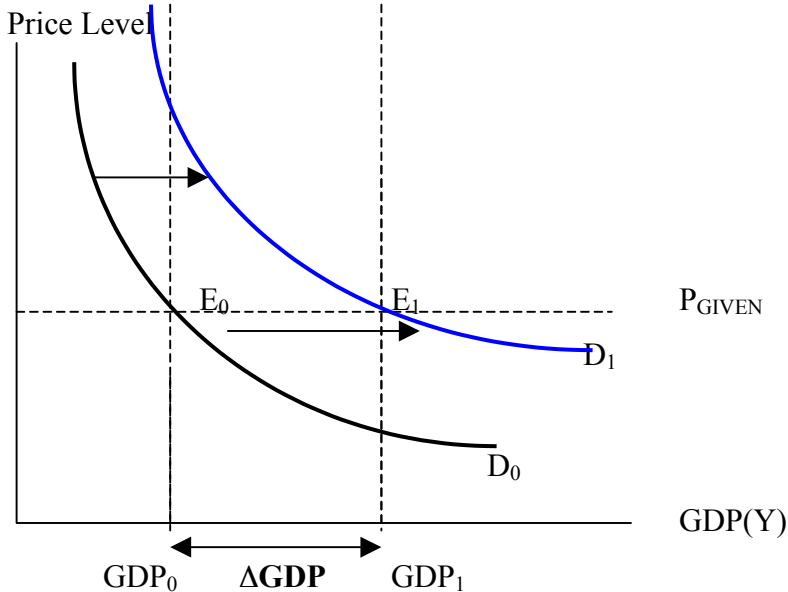
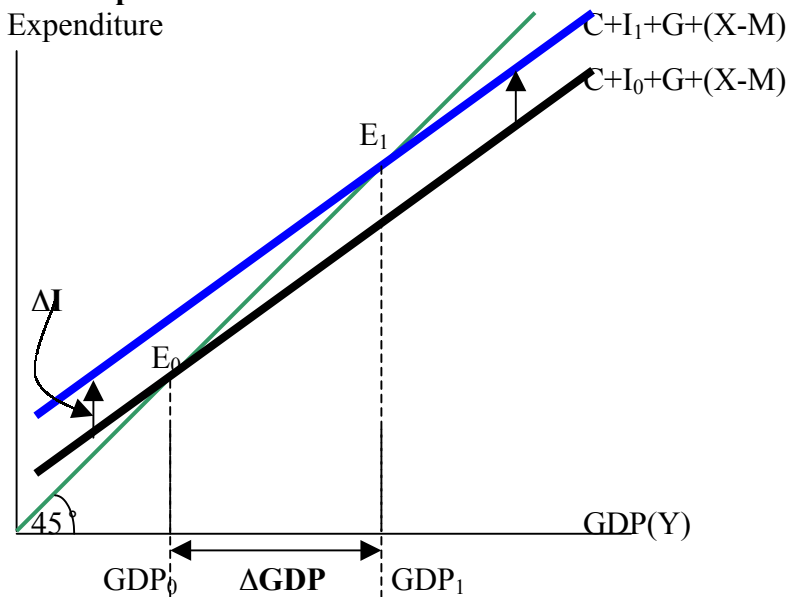
$$\text{Multiplier} = \frac{\Delta Y}{\Delta G} > 1$$

### 4. Multiplier Effect of Net Export

$$\text{Multiplier} = \frac{\Delta Y}{\Delta(X - M)} > 1 \quad \text{gets transmitted across national borders.}$$

### 5. Multiplier & AD Curve

Expenditure



- i) For a given level of price, the oversimplified multiplier measures the increase in real GDP demanded that would occur if the price level is fixed. It measures the horizontal shift of AD curve.
- ii) An autonomous increase in expenditure leads to horizontal shift of the AD curve by an amount given by the oversimplified multiplier.

**7. Simple Algebra of *Oversimplified Multiplier***

From the Algebra of Income Determination

$$Y = \frac{a - bT + I + G + (X - M)}{1 - b} \tag{1}$$

Now suppose any of the symbols in the numerator increases by 1 unit:

$$Y = \frac{a - bT + I + G + (X - M) + 1}{1 - b} \tag{2}$$

By comparing (2) w/ (1) we can compute  $\Delta Y$  as follows:

$$\Delta Y = \frac{a - bT + I + G + (X - M) + 1}{1 - b} - \frac{a - bT + I + G + (X - M)}{1 - b} = \frac{1}{1 - b}$$

**Proof of infinite geometric series**

Let  $S$  = Sum of the Series.

Let  $M$  = Any repeating Initial Value

Let  $0 < R < 1$

$$S = M + MR + MR^2 + MR^3 + \dots + MR^\infty$$

$$= M(1 + R + R^2 + R^3 + \dots + R^\infty) \tag{1}$$

$$RS = M(R + R^2 + R^3 + \dots + R^\infty) \tag{2}$$

$$S - RS = 1 + R + R^2 + R^3 + \dots + R^\infty$$

$$- \frac{R + R^2 + R^3 + \dots + R^\infty}{\hspace{10em}} \tag{1} - (2) = (3)$$

$$= \frac{1}{\hspace{10em}}$$

$$S(1 - R) = \frac{1}{\hspace{10em}}$$

$$\therefore S = \frac{1}{1 - R}$$