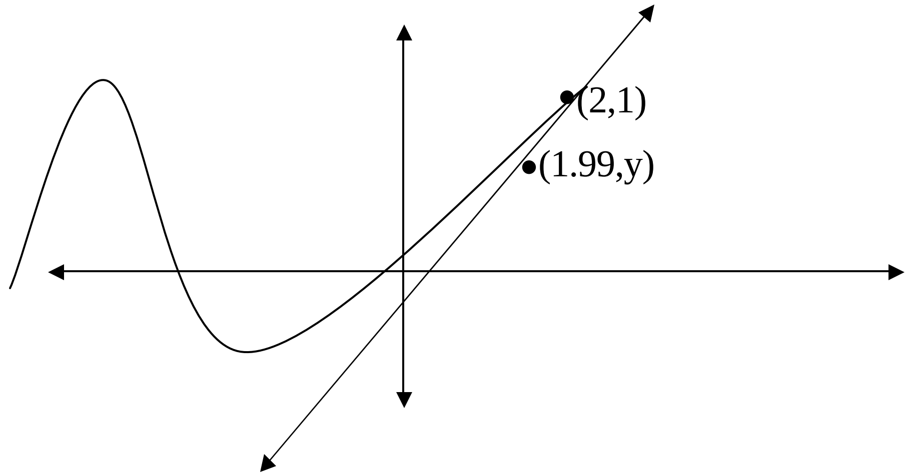
Linear Approximation



What it does: Estimates y-values by using tangent lines.

See figure above. f(x) is the given function. We want to find equation of tangent line. After we find equation of tangent, simply substitute x in order to get a linear approximation of y.

Example: Use Linearization to estimate f(1.99) of $f(x) = 2x^3 - 4x^2 + 1$

If $f(x) = 2x^3 - 4x^2 + 1$, then $f'(x) = 6x^2 - 8x$

Step #1 Use x = 2 … if x = 2, then y = 1 and f ' (2) = 8

Step #2 $y - y_1 = m(x - x_1) \Rightarrow y - 1 = f'(2)(x - 2) \Rightarrow y - 1 = 8(x - 2) \Rightarrow y = 8x - 15$

Step #3 Now, use x = 1.99 to approximate the y–value … $y = 8(1.99) - 15 \approx 0.92$

Step #4 Compare to actual value … $f(1.99) = 2x^3 - 4x^2 + 1 = 0.920798$ Not Bad !!

Another way to do the same problem,

Example: Use Linearization to estimate f(1.99) of $f(x) = 2x^3 - 4x^2 + 1$

If $f(x) = 2x^3 - 4x^2 + 1$, then $f'(x) = 6x^2 - 8x$

Step #1 Let a = 2 (nice number close to original)

Step #2 dx = real value – a = 1.99 – 2 = – 0.01

Step #3 $dy = f'(a) * dx = (24-16) * (-0.01) = -0.08$ (Note $f'(a) \approx dy/dx$)

Step #4 $f(1.99) = f(a) + dy = f(2) + dy = 1 + (-0.08) \approx 0.92$

Step #5 Compare to actual value … $f(1.99) = 2x^3 - 4x^2 + 1 = 0.920798$ Not Bad !!

My <u>favorite</u> way to do the same problem,

Let x = 2	$f(x) = 2x^3 - 4x^2 + 1$	$f'(x) = 6x^2 - 8x$
1. dx = real – a = 1.99 – 2 = – 0.01	2. f(2) = 16 – 16 + 1 = 1	3. f ' (2) = 24 – 16 = 8

4. $dy/dx = 8 \Rightarrow dy = 8 * dx \Rightarrow dy = 8 * -0.01 = -0.08$ Note: dy = 1 * 3
5. $f(x) = f(2) + dy = 1 + (-0.08) = 0.92$ Note: $f(x) \approx$ 2 + 4