Calc. Worksheet #47

I. 1. $5p5 = \frac{5!}{0!} = 120$ 2. $5p3 = \frac{5!}{2!} = 60$ 3. $4! = 24$ 4. $5! = 120$

5. $5 \cdot 4 \cdot 3 = 60$ 6. $\frac{8!}{2! \cdot 2! \cdot 2!} = 5040$

---

1. $s(t) = \int_0^t (3w^2 - 6w)\,dw$

by Fundamental Theorem of Calculus: velocity = $s'(t) = 3t^2 - 6t$

acceleration = $v'(t) = 6t - 6$, so

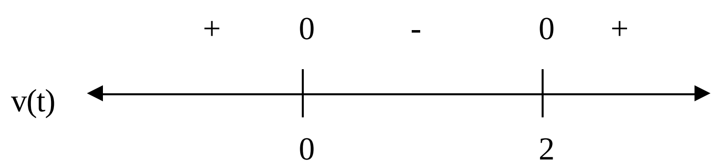
---

a) $a(t) = 6t - 6$

---

b) Particle moves in a positive direction when velocity is positive:

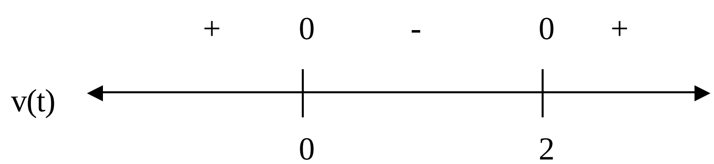
$v(t) = 3t^2 - 6t = 0 \Rightarrow 3t(t-2) = 0 \Rightarrow t = 0$ or $t = 2$

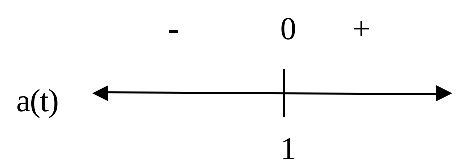


$\therefore$ when $t < 0$ or $t > 2$

---

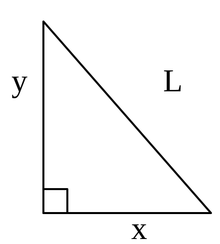
c) Particle is slowing down when v(t) and a(t) have opposite signs.





Thus, particle is "slowing down" when **$t < 0$ or $1 < t < 2$** as this is where v(t) and a(t) have different signs.
Note: Another way to do it … graph **SPEED** = | v(t) | and look at where **SPEED** is decreasing.

2.



x = "9"	$\frac{dx}{dt} = \frac{1}{2}$
y = "12"	$\frac{dy}{dt} = ?$
L = 15	$\frac{dL}{dt} = 0$

If x = 9 and L = 15, then $81 + y^2 = 225$ $y^2 = 144$ $y = 12$

$x^2 + y^2 = L^2$ so $2x\left(\frac{dx}{dt}\right) + 2y\left(\frac{dy}{dt}\right) = 2L\left(\frac{dL}{dt}\right) \Rightarrow x\left(\frac{dx}{dt}\right) + y\left(\frac{dy}{dt}\right) = L\left(\frac{dL}{dt}\right)$

$9\left(\frac{1}{2}\right) + (12)\frac{dy}{dt} = 15(0)$ $\frac{dL}{dt} = 0 \Rightarrow 12\left(\frac{dy}{dt}\right) = \frac{-9}{2}$ $\frac{dy}{dt} = \frac{-3}{8}\,\frac{\text{ft.}}{\text{sec.}}$

b) Area ? ABC $= \frac{1}{2}xy$ so $\frac{dA}{dt} = \left(\frac{1}{2}\right)\left(\frac{dx}{dt}\right)(y) + \left(\frac{1}{2}\right)(x)\left(\frac{dy}{dt}\right) = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)(12) + \left(\frac{1}{2}\right)(9)\left(\frac{-3}{8}\right) = \frac{21}{16}\ \frac{ft^2}{sec}$

3.
$f(x) = xe^{-x}$

$f\,'(x) = 1e^{-x} + x\left(e^{-x}\right)(-1)$

$f\,'(0) = 1 + 0 = 1$

$\therefore$ increasing A

4.
$f(x) = ax^4 + bx^2$ and $ab > 0$

$f\,'(x) = 4ax^3 + 2bx = 0$

$2x\left(2ax^2 + b\right) = 0$

at $x = 0$, $f\,'(0) = 0$ $\therefore$ has horizontal tangent

$f\,''(x) = 12ax^2 + 2b = 0$

$12ax^2 = -2b$

$x^2 = -\frac{b}{6a}$ recall that $ab > 0$, so $\frac{b}{a} > 0$ thus, $x^2 < 0$

$\therefore$ No inflection point. Answer = A

5. At point R, $f\,'(x) = 0$ (terrace) and $f\,''(x) = 0$ (inflection point)