A.P. CALCULUS "FIVES" SHEET

F=Fun at home **I**=Incunabula **V**=Variety **E**=Endeavors **S**=Specimens

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F #0 Find out about the rules of the class. Fill out data sheet (in class).
I What are some Brahma rules?
V Music Contest – Name 10, get a reward
E Find out about the rules of the class. Fill out data sheet (in class).
S Make sure you get the data sheet.

F #1 Worksheet #1 1–9 Always bring a covered book, pencil, pen, regular paper, graph paper, and your graphing Calculator to class.
I 1. Find radius of circle $x^2 + y^2 - 25 = 0$. 2. Name an integer.
V Use six 7's to write an expression that has a value of 110.
E 1. Go over rules of class. 2. Go over rules of contest. 3. Review coordinates and slopes: parallel and perpendicular lines; slope, midpoint, and distance formulae; forms of linear equations. 4. The contest begins–avoid being caught!
S 1. Which CD's would you choose if you won the contest?????
2. Find the slope: a) (1,3) (5,2) b) (1,3) (5,3) c) (1,3) (1,6)
3. Find the equation: (1,3) (5,2)
4. Find the slope, x-intercept, and y-intercept: $4x + 3y = 12$

F #2 Worksheet #2 1–11
I 1. Name two rational numbers that are not integers.
2. Name a rational number less than zero.
V If $2^n = 8$, what is the value of 3^{n+1} ?
E 1. Review writing an equation of a line.
S 1. Find the slope of the line through (2,3) and (–4,7).
2. Find the slope of the line $2x + 3y = 5$.
3. Write an equation for the line through (–2,–1) and (3,4).
4. Write the equation of the line through (2,1) and parallel to the line $2x + y = 3$.
5. Write the equation of the line through (2,1) and perpendicular to the line $2x + y = 3$.

F #3 p.7 (20,23,29), and p.25 (2,5,29,31,34,43c, 43d), and p.33 (3 a,b,c,d)
MAKE SURE THAT YOU READ EACH PROBLEM CAREFULLY AND SHOW ALL WORK FOR EVERY PROBLEM!!
I 1. 5 is what % of 75? 2. Change $\frac{3}{5}$ to a percent.
V Is tan x even, odd, or neither and explain why (a graph is not enough and a specific example is not enough — your solution must be a general one).
E 1. Find domain and range of a function. 2. Determine if a relation is even, odd, or neither.
3. Form a composition of two functions.
S 1. Give the domain and range of $y = \dfrac{1}{1-3x}$ 2. Give the domain and range of $y = |x|$.
3. Is $y = x^2 - 2$ even, odd, or neither?
4. If $f(x) = x - 2$ and $g(x) = \sin x$, then find $f[g(x)]$ and $g[f(x)]$.

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F #4 1. If points A and B are symmetric about the y–axis and point A has coordinate (6,2), then find the coordinates of point B. **ALSO DO THE FOLLOWING:**
p.7 (24,30), and p.15 (11,20,27,29,31 find x- and y-intercepts only), and p.25–26 (4,6 find domain only for problems 4 and 6, 32, 44c,44d,44g,44h)

I Solve for x: $\frac{1}{2x-1} = \frac{1}{9}$

V Use nine 8's to write an expression that has a value of 14.

E Review for upcoming test.

S Find the domain and range of $f(x) = \frac{1}{x-4}$

F #5 Worksheet #3 1–8

I SAT Sponges 1, 2

V p.26 (56a)

E Final review for tomorrow's first easy test.

S If g is an odd function, and g(4) = 9, then what is g(–4)?

F #6 For the following, find a) $f(x + h)$ b) $f(x + h) - f(x)$ c) $\frac{f(x+h)-f(x)}{h}$

1. $f(x) = x^2$ 2. $f(x) = \frac{1}{2x}$

Remember to show all work — don't just give an answer!!!

I On the SAT sponge worksheet 4

V What is the last digit in the number 7^{1000}?

E **Our first easy test—many more to come!**

S Are you following the rules?

F #7 Worksheet #4 (Values).

I None

V p.104 (2) Graph only. Do the graph on graph paper.

E To get you to think about what is important to you.

S What does it mean to you that someone is old? What do you value?

F #8 p.117 (4,6,13,17,51) and p.129 (4,6,8,12,41a)

I SAT Sponges 9, 10

V p.119 (50)

E 1. Continue with the definition of derivative and the power rule.

S Surveys have shown that the ability to get along with people is one of the top qualities that employers are looking for in an applicant. Do you have this quality?

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F #9 p.129 (17,20,22) and p.160 find the derivative, $\frac{dy}{dx}$: (10,12,17,20,27)

I SAT Sponges 11, 12

V p. 161 (54)

E To use the package, product, and the quotient rules in finding derivatives.

S 1. Find $\frac{dy}{dx}$ if $y =$ 3. Find $f'(x)$ if $f(x) = \frac{7x^2 + 1}{4x + 5}$

F #10 p.152 (2,4,8,11) and p.161 (15,17) and p.170 (19,21,28,37)

I SAT Sponges 13, 14

V p. 170 (44)

E To find derivatives of trigonometric functions and implicitly defined functions.

S 1. Find $\frac{dy}{dx}$ if $x^2 + xy + y^2 = 4$ 2. Find $\frac{dy}{dx}$ if $y^3 = \left(x^2+1\right)^2$ 3. Find $\frac{dy}{dx}$ if $y = \frac{\sin x}{1 - \cos x}$

4. Find $\frac{dy}{dx}$ if $y = \sec x \csc x$ 5. Find $\frac{d^2y}{dx^2}$ if $y^2 = x^2 + 2x$

F #11 p.170 (10,22,29,39,45) and p.465 (13,15,23) and p.472 (19,27,31)

I SAT Sponges 15, 16

V p.170 (58a) describe all points (find all x and y coordinates)

E 1. To find the derivatives of exponential functions and natural log functions.

S 1. Find $\frac{dy}{dx}$ if $y = e^{2x}$. 2. Find $\frac{dy}{dx}$ if $y = \ln(\sin x)$. 3. Find $\frac{d^2y}{dx^2}$ when $y^2 - 2x = 1 - 2y$.

4. Find $\frac{dy}{dx}$ if $y = \ln\left(\frac{x}{x-1}\right)$ 5. Find $\frac{dy}{dx}$ if $y = e^{\sin^2 x}$ 6. $x + \sin(xy) = 0$

7. Find $\frac{dy}{dx}$ if $y = \ln x^5$ 8. Find $\frac{dy}{dx}$ if $y = \tan\left(e^{x^2}\right)$

F #12 p.548 (1,4,5) and p.181 (5,11,13,42,45,54a,54b,63,71)

I SAT Sponges 17, 18

V p.183 (84) Find the equation of the normal only.

E 1. To review all of the rules for finding derivatives. 2. To prepare for upcoming test.

S 1. Find $\frac{dy}{dx}$ if $2y^3 = \cos\left(x^2\right)$. 2. $y = \ln\left(x^2 + x - 1\right)$ 3. Are you still in the contest? Be Careful!

F #13 Worksheet #5 1–12

I SAT Sponges 21,22.

V p.182 (72)

E 1. To practice AP–type differentiation problems. 2. To prepare for upcoming test.

S 1. Find $\frac{dy}{dx}$ if $y = \left[\sin^2\left(7x^2 - 2\right)\right]^2$ 2. Find the derivative of $\frac{1}{x}$ using the definition of derivative.

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F #14 Worksheet #6 1–15

I SAT Sponges 23, 24

V Find the equation of the tangent to the curve $y = x \sin x$ at the point $\left(\frac{p}{2}, \frac{p}{2}\right)$

E 1. Final review for tomorrow's easy test on derivatives.
2. Remember: $e^{\ln a} = a$, $\ln e^a = a$, $\ln e = 1$, $\ln 1 = 0$.

S Find $\frac{dy}{dx}$ for the following: 1. $\sin y + \cos x = 7$ 2. $y = \ln\left(\frac{e^x + 1}{e^x - 1}\right)$ 3. $x = \cos y$ 4. $y = \frac{5}{(3x^2 + 1)^3}$
5. Find f ' (x) using the definition: $y = 3x^2 - x + 1$ 6. Find the slope of the tangent line and the slope of the normal to the curve $y = \frac{3}{(x + 5)^2}$ at the point where $x = -4$.

F #15 p.472 (1–4 all)

I SAT Sponges 25, 26

V p.472 (10)

E **Another easy Calculus test – this one on derivatives**.
2. Simplify exponential and logarithmic equations. Remember: $e^{\ln a} = a$, $\ln e^a = a$, $\ln e = 1$, $\ln 1 = 0$.
Watch out!! Follow the rules!! Don't get a stamp!!!

S None

F #16 p.518 (3,14,21) and p.481 (11,43,44,45,46)
See your "Derivative Rules Sheet" and the textbook for formulae for the derivatives of inverse trig functions and also for the "special technique" for taking derivatives whose exponents contain variables.

I SAT Sponges 27, 28

V p.518 (22)

E 1. To find the derivatives of functions whose exponents contain variables.
2. To find the derivatives of cot x, sec x, csc x, and inverse trig functions.

S 1. Find $\frac{dy}{dx}$ if $y = 2^x$. 2. Find $\frac{dy}{dx}$ if $y = x^{\cos x}$ 3. Find $\frac{dy}{dx}$ if $y = \sin^{-1}(2x^3)$

Example: (See "Special Technique" on Derivative Rules Sheet)

Find $\frac{dy}{dx}$ if $y = 2^x$:

	Take ln of both sides:	$\ln y = \ln 2^x$
	Move exponent down:	$\ln y = x \ln 2$
	Take derivative:	$\frac{1}{y}\frac{dy}{dx} = \ln 2$
	Multiply by y:	$\frac{dy}{dx} = (\ln 2)\, y$
	Replace y with its value in terms of x:	$\frac{dy}{dx} = (\ln 2)\, 2^x$

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F #17 p.481 (12,15,40), p.518 (13), p.160 Find $\frac{dy}{dx}$ in terms of x: (1,4,7), p.749 Find $\frac{dy}{dx}$ in terms of t: (1,5,9)

I 1. 25 percent of 16 is equivalent to $\frac{1}{2}$ of what number? A) 4 B) 8 C) 11 D) 13 E) 16

2. If $(y + 2)^2 = (y - 2)^2$, what is the value of y? A) 0 B) 1 C) 2 D) 4 E) 6

V p.749 (12) Find the equation of the **tangent line only**.

E 1. To find the derivative of parametric equations and to use the Chain Rule.

2. How many times can you win the lottery?

S 1. Find $\frac{dy}{dx}$ if $y = 3t^2 - 2$ and $x = 4t + 1$. 2. Find $\frac{dy}{dx}$ if $y = t^2 + 2t - 1$ and $t = x + 1$.

3. Find $\frac{dy}{dx}$ if $y = t - \frac{1}{t}$ and $x = 1 + \frac{1}{t}$.

F #18 Worksheet #7 1–16

I SAT sponges 29, 30

V The entire surface of a solid cube with edge of length 6 inches is painted. The cube is then cut into cubes each with edge of length 1 inch. How many of the smaller cubes have paint on exactly 1 face?

E 1. To find the derivative of ? with respect to Δ. 2. To get ready for the upcoming test on derivatives.

S 1. Find the derivative of y^2 with respect to ln x if $y = \sqrt{x+1}$. 2. Find $\frac{d\left(e^{3x}\right)}{d\left(\sin x\right)}$.

F #19 Worksheet #8 1–15

I 1. The solution set of $\frac{7}{x^2 + 8x + 23} = 1$ is A) {8,4} B) {8,–4} C) {–4,–4} D) {4,–4} E) {16,1}

2. Given the equation $\frac{7x}{3} = (a^4 + 1)^3$, and a = –1, solve for x. A) 16 B) $\frac{24}{7}$ C) 24 D) $\frac{20}{3}$ E) 0

V Which is the greater quantity: Choose A, B, C if the same, or D if there is not enough information.

A: The average (arithmetic mean) of –3, 1, and 3

B: The average (arithmetic mean) of –3, 2, and 3

E 1. **Soon, class, soon!** 2. To review old and new rules for test tomorrow on derivatives.

S Lots of "notes" problems.

F #20 p.65 (7,17,18,20,24)

I SAT sponges 31, 32

V The tangent to the curve $y^2 - xy + 9 = 0$ is vertical when

A) y = 0 B) $y = \pm\sqrt{3}$ C) $y = \frac{1}{2}$ D) y = ±3 E) none of these

E 1. **Easy test on derivatives**. 2. To review limits.

S 1. Find $\lim_{x\to 3} (6x-2)$ 2. Find $\lim_{x\to 3} \frac{x^2-9}{x-3}$ 3. Find $\lim_{x\to 3} \frac{x-3}{x^2-8x+15}$

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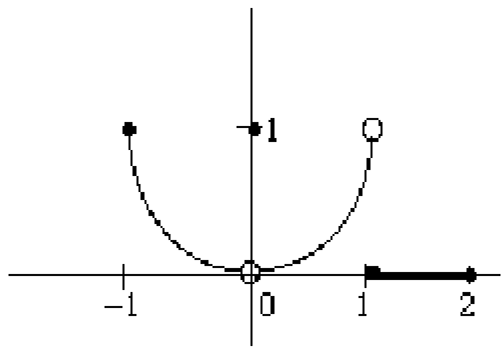
F #21 p.57–59 (4,5), p.65–67 (9,10,21,26, 34,49), p.83–86 (2 a through g only)
I SAT sponges 33, 34
V p.66 (50a)
E 1. To find the limit of a function. 2. To find one–sided limits graphically.
S 1. Find $\lim_{x\to 3}(4x-7)$. 2. Find $\lim_{x\to 2}\frac{x^2-4}{x-2}$. 3. Find $\lim_{x\to 3}\frac{x^2-9}{x-3}$

4. Find $\lim_{x\to 3}\frac{x-3}{x^2-8x+15}$ 5. Answer some limit questions about the graph of f(x) below:



F #22 p.83–86 (4,8,21,22,25), p.152–153 (35,37,42,45,47)
I SAT sponges 35, 36
V None
E 1. To find the limits of some trigonometric functions.
S 1. $\lim_{x\to 0}\frac{\sin 3x}{\sin 5x}$ 2. $\lim_{x\to 0}\frac{\sin^2 x\cos^3 x}{2x}$ 3. $\lim_{x\to 0}\frac{\sin^2 x\cos^3 x}{3x}$ 4. $\lim_{x\to 0}\frac{\tan x}{x}$

5. $\lim_{x\to 0^+}\frac{1}{5x}$ 6. $\lim_{x\to 2^-}\frac{3x}{x+2}$ 7. $\lim_{x\to 0}\frac{\tan 2x}{\sin 3x}$

F #23 p.230 (11a,14a,17a,19a) See p.223 "Summary for Rational Functions" for rules. **AND** p.152–153 (36,38,40,41,44,46,48), **AND** p.84 (2a,b,c,d,e,f,g)
I SAT sponges 37
V None
E To find the limit of functions where x goes to infinity (Principle of Dominance).
S 1. $\lim_{x\to\infty}\frac{3x^4-2x+1}{2x^4+3x^2}$ 2. $\lim_{x\to\infty}\frac{3x^5-2x+1}{2x^4+3x-2}$ 3. $\lim_{x\to\infty}\frac{3x^4-2x+1}{2x^5+3x-2}$

F #24 p.230 (11b,12ab,13ab,14b,15ab,16ab,17b,18ab,19b,20ab,21ab,22ab,23ab24ab)
I SAT sponges 39, 40
V p.182 (74)
E To find the limit of functions where x goes to positive infinity and negative infinity.
S See the rules for turning in your extra credit money.

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F #25 p.270 (51 – 56), p.163 (79), p.152–153 (33,39), p.83–85 (3,17,26) **and the following problems:**

1. $\lim\limits_{h\to 0} \dfrac{5(x+h)^4 - 5x^4}{h}$ 2. $\lim\limits_{h\to 0} \dfrac{\ln\{2(x+h)\} - \ln(2x)}{h}$

3. $\lim\limits_{h\to 0} \dfrac{(8+h)^{1/3} - 2}{h}$ 4. $\lim\limits_{h\to 0} \dfrac{e^{(x+h)^2} - e^{x^2}}{h}$ 5. $\lim\limits_{h\to 0} \dfrac{\cos h - 1}{h}$

I SAT sponges 41, 42

V p. 184 (104)

E 1. To find the limit of functions that are in the form of Newton's Quotient.

S 1. Find $\lim\limits_{h\to 0} \dfrac{(x+h)^3 - x^3}{h}$ 2. Find $\lim\limits_{h\to 0} \dfrac{(2+h)^3 - 8}{h}$

F #26 Worksheet #9 1–19

I SAT sponges 43, 44

V p.187 (13) Show all work.

E 1. To practice some limit problems for the upcoming test.
2. To find limits that involve bounded functions.

S 1. Find $\lim\limits_{x\to 0} \dfrac{\sin x - 1}{x}$ 2. Find $\lim\limits_{x\to 1} \dfrac{x^2 - 2x + 1}{x - 1}$ 3. $\lim\limits_{x\to\infty} \sin x$

4. $\lim\limits_{x\to\infty} \dfrac{\sin x}{x}$ 5. $\lim\limits_{x\to\infty} \dfrac{\cos x}{3x^2}$ 6. $\lim\limits_{x\to\infty} \dfrac{\sin \frac{x}{2}}{2x}$ 7. $\lim\limits_{x\to 1} \dfrac{3}{x-1}$

F #27 Worksheet #10 1–20

I SAT sponges 46

V p.230 (10)

E 1. Continue to work on finding limits. 2. Review of limits and some applications of derivatives for upcoming easy test.

S 1. Find $\lim\limits_{h\to 0} \dfrac{\ln(e+h) - 1}{h}$ 2. Find slope of tangent line to curve $y = 3x^2 - 1$ at point (1,2).

F #28 Worksheet #11 1–15

I 1. If x and y are integers, for which of the following ordered pairs (x,y) is 2x + y an odd number?
A) (0,2) B) (1,2) C) (2,1) D) (2,4) E) 3,0)
2. If $25 * 16 * 9 = r^2 * 3^2$, then $r^2 =$ A) 4^2 B) 5^2 C) 10^2 D) 15^2 E) 20^2

V A triangle has a base of length 13 and the other two sides are equal in length. If the lengths of the sides of the triangle are integers, what is the shortest possible length of a side?

E 1. To practice some limit problems for upcoming test.

S Many "notes" problems.

F #29 Worksheet #12 1–19

I 1. If 3x = y and y = z + 1, what is the value of x when z = 29?
2. If $2^n = 8$, what is the value of 3^{n+1} ?

V p.66 (54a) Show graph.

E 1. Continue to work on finding limits.
2. Final review of limits and some applications of derivatives for tomorrow's easy test.

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S Be prepared for tomorrow's easy test!

F #30 Read p.275–279 Do p.280 (2c,6c,8c,11c,20,26) and p.296 (13,14)

I SAT sponges 47, 48

V p. 297 (42)

E **1. Take an easy test on limits and some derivatives.**

2. Begin finding the antiderivative (integral) of a function.

Studying for this test means knowing all of the different types of limit problems as well as those derivative problems.

F #31 p.296–297 Evaluate the antiderivatives. Ignore the instructions about substitutions. (5,6,7,8,19,22,36,38)

I SAT sponges 49, 50

V p.297 (44)

E 1. Learn the easy $\square$ –method for evaluating integrals (antiderivatives).

2. Learn a rule that makes doing some limit problems easier.

S 1. $\int(3x^2+4)dx$ 2. $\int\frac{x}{3x^2+4}\,dx$ 3. $\int\frac{x}{\sqrt{1-9x^2}}\,dx$ 4. $\int\frac{1}{\sqrt{1-9x^2}}\,dx$ 5. $\int xe^{x^2}\,dx$

6. $\int\frac{x}{1+x^2}\,dx$ 7. $\int\frac{1}{1+x^2}\,dx$ 8. $\lim_{x\to5}\frac{x^3-125}{x-5}$

F #32 p.548–549 (32,43,44,69), p.560 (1,7,25), p.518 (23,24,25)

I SAT sponges 53, 54

V p.560 (26)

E 1. Continue working on antidifferentiation. 2. Integrals that yield inverse trig functions.

S 1. $\int\frac{dx}{1+4x^2}$ 2. $\int e^{(\tan x)}\sec^2 x\,dx$ 3. $\int\frac{5\,dx}{\sqrt{1-9x^2}}$ 4. $\int\frac{dx}{\sqrt{16-x^2}}$ 5. $\int\frac{dx}{7+x^2}$

F #33 p.338 (1,4,9,10,11,15,30,33) and p.466 (51,58)

I SAT sponges 55, 56

V p.466 (66) Show all work without a calculator.

E 1. Work with definite integrals. 2. Learn the Fundamental Theorem of Calculus – Part 2

S 1. $\int_1^2(3x4)dx$ 2. $\int_{p/2}^{3p/2}\cos x\,dx$ 3. $\int_2^1\frac{dx}{x}$ 4. $\int_{-1}^{0}\frac{3}{3x-2}\,dx$

F #34 p.560–561 (5,7,9,18,47,49,52,55) and p.606–607 (23,68)

I SAT sponges 57,58

V $\int\cos^3 x\,dx$

E 1. Work with integrals where you have to divide before you integrate.

S 1. $\int\frac{3x-5}{x}\,dx$ 2. $\int\frac{x^2}{x-3}\,dx$ 3. $\int\frac{x}{x-3}\,dx$ 4. $\int_{-1}^{3}\frac{4x^2-7}{2x+3}\,dx$ 5. $\int\frac{3x^2-7x}{3x+2}\,dx$ 6. $\int\frac{3x+2}{\sqrt{1-x^2}}\,dx$

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F #35 p.288 (1,6,15), p.537–538 (10,12,14), p.550 (122), ALSO, FOR THE FOLLOWING, FIND THE FUNCTION, F(X), THAT SATISFIES THE GIVEN CONDITION:

1. $f'(x) = \sin x - \cos x$ and $f(0) = 3$ 2. $f'(x) = \left(\dfrac{x^2 - 3x + 2}{x - 2}\right)$ and $f(-2) = 0$

3. $f'(x) = \left(\dfrac{x^2 - 3x + 1}{x}\right)$ and $f(e) = 1$

I SAT sponges 59, 60

V Does any function satisfy the following. The second derivative is 0 for all x, $\dfrac{dy}{dx} = 1$ when x = 0, and x = 0 when y = 0? Explain your answer.

E 1. Finding the constant, C, when more information is given. 2. Solving simple differential equations using the method of <u>variables separable</u>. 3. Solving initial-value problems.

S 1. If $f'(x) = 3x^2 + 2$ and $f(0) = 5$, then find f(x). 2. Solve: $\dfrac{dy}{dx} = x^2y$

3. Find y: $\dfrac{dy}{dx} = 4y$ and y = 4 when x=0.

F #36 1. $\int \sin^2 2t\,dt$ 2. $\int \cos^2 3q\,dq$ 3. $\int_0^p \sin^2 x\,dx$ 4. $\int \dfrac{\sec^2 2x\,dx}{1 + \tan 2x}$

5. $\int \tan 3x\,dx$ 6. $\int \tan^2 4x\,dx$ 7. $\int \cos^3 2x\,dx$ 8. $\int \sin^3 x\,dx$ 9. $\int (\cos^3 x)(\sin^2 x)\,dx$

10. $\int \dfrac{x^2 + x - 1}{x^2 - x}\,dx$ 11. $\int_e^{e^2} \dfrac{1}{x \ln x}\,dx$ 12. If $f'(x) = 3x^2 - 2$ and if $f(2) = 14$, then $f(1)$=?

I 1. Find the inverse function $f^{-1}(x)$ if $f(x) = \sqrt[5]{x} + 1$ 2. Find $\lim_{x\to 0} \dfrac{(2+h)^3 - 8}{h}$

V $\int \cot^3 x\,dx =$

E 1. Integrate odd powers and 2nd powers of sines and cosines that don't have the correct children.

For odd powers: PEEL one of the sines or cosines, then substitute using: $\sin^2 x = 1 - \cos^2 x$ and $\cos^2 x = 1 - \sin^2 x$.

For even powers: Use the substitutions: $\sin^2\square = \frac{1}{2}\left(1 - \cos 2\square\right)$ and $\cos^2\square = \frac{1}{2}\left(1 + \cos 2\square\right)$

S 1. $\int \sin^3 x\,dx$ 2. $\int \sin^3 x \cos^2 x\,dx$ 3. $\int \cos^5 x\,dx$ 4. $\int \sin^2 x\,dx$

F #37 Worksheet #13 1–12

I 1. r + 3 > 5 Column A: r + 2 Column B: 4 What's bigger: A, B, equal, can't tell?

2. 6x – 2y < 0 Column A: x Column B: 0 What's bigger: A, B, equal, can't tell

V p.297 #46

E 1. Review integration for upcoming test.

S Notes problems.

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F #38 Worksheet #14 1–15

I 1. $\lim\limits_{x\to 2} \dfrac{x^2-2}{4-x^2}$ 2. $\lim\limits_{x\to 0} \dfrac{e^x-1}{x}$

V If $f(x)$ is a function whose derivative is constant, and if it contains the origin and the point (4,20), then find $\int_0^3 f\,'(x)\;dx$.

E 1. Review integration for upcoming easy test.

S 1. What do the symbols $\forall$, $\exists$, $\ni$, ε, α, β, δ, ϕ, θ, ρ, Σ mean? 2. Notes problems.

F #39 Worksheet #15 1–13

I SAT sponges 63, 64

V Let x > 0. Suppose $\dfrac{d}{dx}$ f(x) = g(x) and $\dfrac{d}{dx}$ g(x) = f($\sqrt{x}$); then $\dfrac{d^2}{dx^2}$ f(x^2) =

A) f(x^4) B) f(x^2) C) 2xg(x^2) D) $\dfrac{1}{2x}$ f(x) E) 2g(x^2) + 4x^2f(x)

E 1. Review integration, variables separable, l'hôpital's rule, and other stuff for tomorrow’s easy test.

S Several problems for review.

F #40 Read p.474–477, especially Example 5. Do p.481 (11,47,49)

I SAT sponges 65, 66

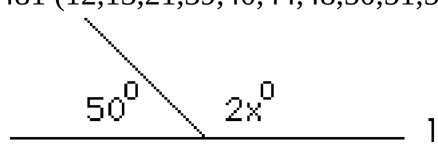
V p.550 (121)

E **1. Take an easy multiple–choice test on integrals.**

2. Find the integral where the exponent has a variable ("special technique").

S 1. SUCCESS comes from your intense, dedicated preparation!!

F #41 Read p.474–477, especially Example 5. Do p.481 (12,13,21,39,40,44,48,50,51,53)



I 1. In the figure to the right, what is the value of x ?

2. If $(x + 2)^2 = 25$ and x > 0, what is the value of x^2 ?

V p.481 (60)

E Find the derivative and integral where the exponent has a variable ("special technique").

S **Seniors:** Get those applications completed!!!

F #42 Read p.87–95. Do p.95 (1–10,13,14)

I SAT sponges 67, 68

V p.102 (26)

E Determine if a function is continuous at a point.

S 1. f(x) = $\dfrac{x^2-9}{x-3}$ for x $\neq$ 3 and f(x) = 6 when x = 3. Is f(x) continuous at x = 3?

2. If f(x) = 7 when x = 3, would f(x) be continuous at x = 3 ?

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F #43 p.95–96 (15,16,19,20,35–40,41 show graph)

I 1. $\lim_{x\to 4} \dfrac{x^3 - 4x^2 - x + 4}{x - 4} =$ 2. $\int \sin(4x + 2)\, dx =$

V $\int \sin^5 x\, dx =$

E 1. Another day determining if functions are continuous at a point.

S 1. If f(x)= $\dfrac{x^2 - 25}{x - 5}$ if x $\neq$ 5, and 10 if x=5, then is f(x) continuous at x=5?

2. If f(x) = $\dfrac{x^2 + 5x + 6}{x^2 - 3x - 10}$ if x $\neq$ –2 and 5, and f(–2) = k, then find k so that f is continuous at x = –2.

3. If f(x) = $\dfrac{1 - \cos x}{x}$ if x ? 0, and f(0) = k, then find k so that f is continuous at x = 0.

4. If f(x) = x^2 – 4 for x < 3, and 2kx for x = 3, then find k so that f is continuous at x = 3.

F #44 Read p.196–200. Do the Worksheet #16 1–10

I SAT sponges 71, 72

V p.96 (44)

E 1. To learn the Mean Value Theorem 2. To learn the special case of the MVT, Rolle's Theorem.

S Find c for the Mean Value Theorem if 1. f(x) = x^2 – 7x 2. f(x) = $3x^2$ – 5x + 1
Find c for Rolle's Theorem if 1. f(x) = x^2 – 2x on [0,2]. 2. f(x) = $2x^2$ – 3x + 1 on [1/2,1]

F #45 Worksheet #17 1–16

I SAT sponges 73, 74

V p.203 (10)

E 1. Review the Mean Value Theorem, Rolle's Theorem, and continuity of a function.

S 1. Find c for Rolle's Theorem if f(x) = x^2 – 2x – 3 on [–1,3].
2. Find c for the Mean Value Theorem if f(x) = $2x^2$ + 1 in [1,3].

F #46 Worksheet #18 1–17

I SAT Sponges 77, 78

V If f(x) = x cos $\dfrac{1}{x}$, then f ' $\left(\dfrac{2}{p}\right)$= A) $\dfrac{p}{2}$ B) – $\dfrac{2}{p}$ C) – 1 D) – $\dfrac{p}{2}$ E) 1

E 1. Review Mean Value Theorem, Rolle's Theorem, continuity, and higher level thinking on these topics for test Thursday.

S 1. Why does f(x) = $\dfrac{x^2 - 4x}{x - 2}$ on [0,4] **<u>not</u>** satisfy the hypotheses of Rolle's Theorem?

F #47 Worksheet #19 1–13

I 1. Which of the following integers is a divisor of both 36 and 90? A) 12 B) 10 C) 8 D) 6 E) 4
2. Point B is between points A and C on a line. If AB = 2 and BC = 7, then AC = A) 2 B) 3 C) 5 D) 7 E) 9

V The equation of the tangent of the curve y = e^x ln x, where x = 1, is
A) y = ex B) y = e^x + 1 C) y = e(x – 1) D) y = ex + 1 E) y = x – 1

E Review MVT, Rolle, continuity, variables separable, and anything else for upcoming test.

S 1. Solve: $\dfrac{dy}{dx} = \dfrac{x+1}{y-1}$ 2. f(x) = $\dfrac{x^2 - 16}{x - 4}$ when x = 4 and f(4) = k.

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Find k so that f is continuous at x = 4.

A.P. CALCULUS "FIVES" SHEET

F #48 Worksheet #20 1–13

I SAT Sponges 79, 80

V Suppose $\int_0^3 f(x + k)\,dx = 4$, where k is a constant. Then $\int_k^{3+k} f(x)\,dx$ equals

A) 3 B) 4 – k C) 4 D) 4 + k E) none of these

E Final review MVT, Rolle, continuity, variables separable, and anything else for test tomorrow.

S Lots of problems for your notes.

F #49 1. Find $\frac{dS}{dt}$ if $S = 4p\,r^2$. 2. Find $\frac{dV}{dt}$ if $V = \frac{1}{3} p\, r^2 h$. 3. Find $\frac{dz}{dt}$ if $x^2 + y^2 = z^2$.

4. If x = 4, y = 3, $\frac{dy}{dt} = -2$, and $\frac{dx}{dt} = 0$, then find $\frac{dz}{dt}$ for $x^2 + y^2 = z^2$.

5. If $V = \frac{4}{3} p\, r^3$ and $\frac{dV}{dt} = 3p$ and r=6, then find $\frac{dr}{dt}$

6. Using variables separable, solve the differential equation $\frac{dy}{dx} = \frac{3y}{x}$ if x = 1 when y = 1.

I SAT Sponges 81, 82

V If $f(x) = x^5 + 2$, then its inverse function $f^{-1}(x)$ is

A) $\frac{1}{x^5 + 2}$ B) $\sqrt[5]{x + 2}$ C) $\sqrt[5]{x} + 2$ D) $\frac{1}{\sqrt[5]{x - 2}}$ E) $\sqrt[5]{x - 2}$

E 1. **An easy test** on continuity, Rolle's Theorem, Mean Value Theorem, and variables separable.

2. Prepare for related rate problems.

Situation: Test. What to do: Study. Results: Success!! Success often depends on the right attitude! Think A+!

F #50 Read p.172–176. Do p.176–178 (10,13ab,18ab,21)

I SAT Sponges 85, 86

V p.177 (14)

E Learn the easy steps in solving related rates problems.

S 1. A ladder 15 ft long is leaning against a building. The bottom of the ladder is moved away from the building at the constant rate of $\frac{1}{2}\ \frac{\text{ft}}{\text{sec}}$. Find the rate at which the ladder is falling down the building when the ladder is 9 ft from the building.

2. Sand falls onto a conical pile at the rate of $10\ \frac{\text{ft}^3}{\text{min}}$. The radius of the base of the pile is always equal to one half its altitude. How fast is the altitude increasing when it is 5 ft. deep?

3. Water is withdrawn from a conical reservoir 8 ft in diameter and 10 ft deep (vertex down) at the constant rate of $5\ \frac{\text{ft}^3}{\text{min}}$. How fast is the water level falling when the depth of water in the reservoir is 6 ft?

F #51

1. A balloon is being filled with helium at the rate of 4 cubic $\frac{\text{ft}}{\text{min}}$ The rate, in square feet per minute, at which the surface area is increasing when the volume is $\frac{32p}{3}$ cubic feet is

   (A) $4p$ (B) 2 (C) 4 (D) 1 (E) $2p$

2. A circular conical reservoir has depth 20 feet and radius of the top 10 feet. Water is leaking out so that the surface is falling at the rate of $\frac{1}{2}\ \frac{\text{ft}}{\text{hr}}$ The rate, in cubic feet per hour, at which the water is leaving the reservoir when the water is 8 feet deep is

   (A) $4p$ (B) $8p$ (C) $16p$ (D) $\frac{1}{4p}$ (E) $\frac{1}{8p}$

3. The height of a rectangular box is 10 inches. Its length increases at the rate of $2\ \frac{\text{in}}{\text{sec}}$; its width decreases at the rate of $4\ \frac{\text{in}}{\text{sec}}$. When the length is 8 inches and the width is 6 inches, the volume of the box is changing, in cubic inches per second, at the rate of

   (A) 200 (B) 80 (C) –80 (D) –200 (E) –20

4. A 26 foot ladder leans against a building so that its foot moves away from the building at the rate of 3 $\frac{\text{ft}}{\text{sec}}$. When the foot of the ladder is 10 feet from the building, the top is moving down at the rate of r $\frac{ft}{\text{sec}}$, where r is (A) $\frac{46}{3}$ (B) $\frac{3}{4}$ (C) $\frac{5}{4}$ (D) $\frac{5}{2}$ (E) $\frac{4}{5}$

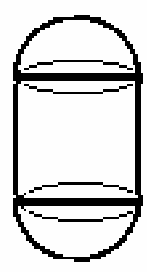
I SAT Sponges 87, 88

V p.179 (30 answer both parts)

E More practice with related rates.

S A balloon is in the shape of a cylinder with hemispherical ends of the same radius as that of the cylinder. The balloon is being inflated at the rate of $261p\ \frac{\text{cm}^3}{\text{min}}$. At the instant the radius of the cylinder is 3 cm, the volume of the balloon is $144p$ cm^3 and the radius is increasing at the rate of $2\ \frac{\text{cm}}{\text{min}}$. (The volume of a cylinder is $p\ r^2h$ and the volume of a sphere is $\frac{4}{3}p\ r^3$)

1. At this instant, what is the height of the cylinder?
2. At this instant, how fast is the height of the cylinder changing?



A.P. CALCULUS "FIVES" SHEET

F #52 An exciting Thanksgiving Adage
I None
V p.354 (28) Hint: Use Simpson's Rule
E Review related rates, continuity, Rolle's and Mean Value Theorems for upcoming easy test.
S Preparation for getting stuffed!

F #53 p.184–185 (117), p.203 (2,5), p.96 (40) **and also do the following:**

1. A 20 ft. ladder leans against a building. If the bottom slides away horizontally at $2\,\frac{\text{ft}}{\text{sec}}$ how fast is the ladder sliding down the building when the ladder is 12 ft. above the ground?
2. A water tank is in the shape of an inverted cone with altitude 12 ft. and base radius of 6 ft. If water is pumped in at $10\,\frac{\text{ft}^3}{\text{min}}$ how fast is the water level rising when the water is 3 ft deep?
3. Find c for Rolle's Theorem for $f(x) = 3x^2 - 12x$ on [0,4].
4. $f(x) = \frac{x}{x^2 + 1}$ when $x > -1$, $f(x) = \frac{x + 1}{x^2 - 1}$ when $x < -1$, and $f(-1) = \frac{1}{2}$ Find all points of discontinuity of f(x).

I SAT Sponges 89, 90
V p.331 (34c)
E Review related rates, continuity, Rolle's and Mean Value Theorems for upcoming test.
S More related rates problems.

F #54 Worksheet #22 1–10
I SAT Sponges 91, 92
V None
E Review related rates, continuity, Rolle's and Mean Value Theorems for upcoming easy test.
S HEY SENIORS: Are your college applications in? Don't procrastinate any further–get them in! After this is done, get your scholarship applications finished!

F #55 Worksheet #23 1–14
I 1. $\lim\limits_{x \to 2} [x]$ (where [x] is the greatest integer in x) is A) 1 B) 2 C) 3 D) 8 E) none of these

2. $\lim\limits_{h \to 0} \dfrac{\sin(\frac{p}{2} + h) - 1}{h}$ is A) 1 B) –1 C) 0 D) 8 E) none of these

V $\displaystyle\int \frac{\cos x}{4 + 2\sin x}\,dx$ equals A) $\sqrt{4 + 2\sin x} + C$ B) $-\dfrac{1}{2(4 + \sin x)} + C$

C) $\ln\sqrt{4 + 2\sin x} + C$ D) $2 \ln|4 + 2\sin x| + C$ E) $\frac{1}{4}\sin x - \frac{1}{2}\csc^2 x + C$

E Review related rates, continuity, Rolle's and Mean Value Theorems for tomorrow's test.
Study right! Do problems! That's how success is gained on a Calculus test!

A.P. CALCULUS "FIVES" SHEET

F #56 Read p.309–320. Do p.320–321 (29a,b,c) **Graph and compute the area for each part.** Approximate the **INTEGRAL** using rectangles using the left-hand endpoint, the right-hand endpoint, and the midpoint. This means that the heights of the rectangles are the y values at the right endpoint, the left endpoint, and the midpoint, even if they are negative. Include the graphs for each example. REMEMBER – This is an easy assignment after a test. Make sure that you read the pages assigned in the book and really try problems.

I If $\frac{r^2}{r-1} \geq r$, then A) $r \geq 0$ B) $r \leq 0$ C) $r \leq 0$ or $r > 1$ D) $r \leq 0$ or $r \geq 1$ E) $0 \leq r \leq 1$

V p.320–321 (32) Follow the same directions as in the homework problems.

E 1. **Take an easy test.** 2. Approximate the area under a curve (an integral) using rectangles.

S Do your best on this test!

F #57 p.321 (30abc,32abc) Approximate the **AREA** using rectangles using the left-hand endpoint, the right-hand endpoint, and the midpoint. This means that the heights of the rectangles are the **POSITIVE** y values at the right endpoint, the left endpoint, and the midpoint. Include the graphs for each example. **AND ALSO THE FOLLOWING:** p.353 (1,2,3,11a – For problems 1,2,3: (a) Estimate the integral using the trapezoidal rule with n = 4, and (b) Evaluate the integral directly. For problem 11a: Do parts (a) and (c) but do not find the error, E_T.)

I If f ' (c) = 0 for $f(x) = 3x^2 - 12x + 9$, where 0 = x = 4, then c = A) 2 B) 3 C) 0 D) 1 E) $\frac{1}{3}$

V p.179 (28)

E 1. Approximate the area under a curve using rectangles and trapezoids.

S Approximate using rectangles and the trapezoidal rule and compare with exact value:

1. $\int_0^4 x^2\,dx$ (n = 4) 2. $\int_1^2 x^2\,dx$ (n = 4) 3. $\int_0^2 x^3\,dx$ (n = 4) 4. $\int_0^2 x^3\,dx$ (n = 6) 5. $\int_0^1 5x^4\,dx$ (n = 4)

F #58 p.331 (21 – 22, and 23 – 26 part b only) and also the following:

Approximate to 3 decimal places the integral $\int_0^4 \sqrt{x}$ with 4 equal intervals using:

a) rectangles whose height is the right-hand endpoint b) rectangles whose height is the left-hand endpoint
c) rectangles whose height is the midpoint of the interval d) trapezoids (trapezoidal rule)
e) Evaluate the integral directly.

I 1. The solution set of $\frac{7}{x^2+8x+23} = 1$ is A. {8,4} B. {8, – 4} C. { – 4, – 4} D. {4, – 4} E. {16,1}

2. Given the equation $\frac{7x}{3} = (a^4+1)^3$, and a = – 1, solve for x. A) 16 B) $\frac{24}{7}$ C) 24 D) $\frac{20}{3}$ E)0

V p.373 (50)

E Finding the exact area under a curve.

S Find the area under the following curves: 1. $y = 1 - x^2$ on [0,1] 2. $y = 1 - x^2$ on [0,3]

3. y = sinx on $\left[0, \frac{3p}{2}\right]$ 4. $y = 4 - x^2$ on [0,2] 5. $y = 4 - x^2$ on [0,3]. 6. $y = x^2 - 7x + 10$ on [3,7]

7. $y = 4x - x^2$ on [– 1,6]

F #59 p.372 (13 - 16) also for the following problem:
Approximate the area under $y=(x + 1)^2$ on [0,4] using
(a) 4 rectangles whose height is given using the left endpoint,
(b) 4 rectangles whose height is given using the right endpoint,
(c) 4 rectangles whose height is given using the midpoint, and
(d) 4 trapezoids.

I If $\frac{f(x_1)}{f(x_2)} = f\left[\frac{x_1}{x_2}\right]$ for all real numbers x_1 and x_2, where $x_2 \neq 0$ and $f(x_2) \neq 0$, which of the following could define f? A) $f(x) = \frac{1}{x}$ B) $f(x) = x^2 + 3$ C) $f(x) = x + 1$ D) $f(x) = \ln x$ E) $f(x) = e^x$

V p.373 (46)

E Find the area between curves.

S Find the area between the curves: 1. y = 3x, y = 4, and the y = axis. 2. $y = x^2$, y = 3, and x = 0 3. $y = x^2$ and $y = x^3$

F #60 p.371-372 (1,5,8,11,19) and the following problem:
Approximate the area under $y = (x-1)^2$ on [0,4] using
(a) 4 rectangles whose height is given using the left endpoint
(b) 4 rectangles whose height is given using the right endpoint
(c) 4 rectangles whose height is given using the midpoint
(d) 4 trapezoids.

I $\int_{-2}^{-1} x^{-2}dx$ A) $-\frac{1}{2}$ B) $\frac{7}{16}$ C) $\frac{7}{4}$ D) $-\frac{7}{4}$ E) $\frac{1}{2}$

V p.373 (52)

E Find the area between two curves.

S Find the area between the curves: 1. $y = 4x^2$ and $y = 6x - 2$ 2. $y = \sqrt{x}$ and $y = x^3$ 3. $y = x^2$ and $y = -x^2 + 6x$ 4. $y = x^2$, $y = 6 - x$, y = 0, in 1st quadrant.

F #61 p.371 – 372 (3,10,23,24,26) and the following problem:
Approximate the area under $y= x^2 - 1$ on [0,4] using
(a) 4 rectangles whose height is given using the left endpoint
(b) 4 rectangles whose height is given using the right endpoint
(c) 4 rectangles whose height is given using the midpoint
(d) 4 trapezoids

I If $f(x)=\pi^2$, then $f'(1)=$ A) 2π B)0 C) π D)1 E) π^2

V p.372 (22)

E Sometimes you have to turn your head sideways to find the area between two curves!!

S Find the area between the curves: 1. $x = y^2$ & x = 3 2. $x = -y^2$ & y = x + 6 3. $x = 12y^2 - 12y^3$ & $x = 2y^2 - 2y$

F #62 Worksheet #24

I If $3x^2 + 2xy + y^2 = 2$, then the value of $\frac{dy}{dx}$ at x = 1 is
A) –2 B) 0 C) 2 D) 4 E) Not defined

V p.331 (35) You must show all work for credit.

E 1. Review all types of area problems. 2. Find the average value of a function on a given interval.

S Find the area: 1. under $y = x^2 - 6x + 8$ on [1,5] 2. under $y = x^2 - 6x + 8$ 3. under $y = 3 - 2x^2$ on [–1,2]

A.P. CALCULUS "FIVES" SHEET

4. between $y = x^2 – 3$ & $y = 1$ 5. between $x + y = 11$ & $x = y^2 – 1$

6. Find the avg. value of $f(x) = 3x^2 – 3$ on [0,1]

F #63 Worksheet #25 (1-10)

I A telephone number consists of 7 digits. How many different telephone numbers exist if each digit appears only one time in the number? A) 7! B) 10^7 C) 70 D) 7^7 E) $\frac{10!}{3!}$

V Let n be an integer greater than or equal to 1. Which of the following is greater than 1?
A) $–(n)^3$ B) $(n)^{–1/3}$ C) $–(–n)^3$ D) $–(n + 1)^3$ E) $(–n + 1)^5$

E Lots of review will help on Thursday's test.

S Many exciting problems.

F #64 Worksheet #26 (1 – 12)

I $\lim_{x\to 0} \frac{3x}{\tan x} =$ A) – 8 B) 0 C) 1 D) 3 E) 8

V If $f(x) = \begin{cases} 2x & \text{for } x \le 1 \\ 3x^2 - 1 & \text{for } x > 1 \end{cases}$ then $\int_0^2 f(x)\,dx =$ A) 7 B) 6 C) 5 D) 2 E) 1

E Lots of review will help on upcoming test.

S Many exciting problems.

F #65 Worksheet #27 (1 – 10)

I If $V = \frac{4}{3} p\, r^3$, what is $\frac{dV}{dr}$ when r = 3? A) $4p$ B) $12p$ C) $24p$ D) $36p$ E) $42p$

V $\lim_{h\to 0} \frac{\sin(1+h) - \sin 1}{h}$ is approximately A) 0 B) 0.54 C) 0.63 D) 0.89 E) none of these

E Review for tomorrow's easy test on average value of a function and area under and between curves (and, of course, anything else that might come up).

S 1. For tomorrow's test, study average value problems and problems that involve finding area:
(1) approximating using rectangles and trapezoids ;
(2) under a curve
(3) between two curves; and other applications of derivatives and integrals that we have covered.

2. Find the area between $y^2 = x$ and $x + y = 6$.

3. Find the area between $y = e^x$ and $x = 3$ in the 1st quadrant.

4. If $f(x) = \frac{\sqrt{3x+6} - \sqrt{x+8}}{x}$ if x ? 1 and f(1) = k, find k so f(x) is continuous at x = 1.

A.P. CALCULUS "FIVES" SHEET

F #66 Read p.309–311 about summation notation. Do p.320 (1,2,3)

I $\lim_{h\to 0} \dfrac{\sin(p+h) - \sin p}{h} =$ A) 1 B) 0 C) –1 D) + 8 E) – 8

V p.320 (16)

E 1. **Take an easy test on average value and area under and between curves.**
2. Using summation notation.

S None

F #67 Worksheet #28 (1–6) (Know the Rules for Definite Integrals – Table 4.5 – on p.324.)

I A function f that is continuous for all real numbers x has $f(3) = -1$ and $f(7) = 1$.
If $f(x) = 0$ for exactly one value of x, then which of the following could be x?
A) – 1 B) 0 C) 1 D) 4 E) 9

V $\displaystyle\sum_{k=0}^{5} \frac{1}{2^k} =$

E 1. Review properties of integrals and summation notation.

S If $\int_1^4 f(x)\,dx = 7$, then find $\int_4^1 f(x)\,dx =$

F #68 Worksheet #29 1-5

I If $f(x) = \dfrac{x}{1-x}$, then $f'(x) =$ A) – 1 B) $\dfrac{-1}{1-x}$ C) $\dfrac{-1}{(1-x)^2}$ D) $\dfrac{1}{1-x}$ E) $\dfrac{1}{(1-x)^2}$

V $\int x^3 \ln x \; dx =$

E 1. Some functions cannot be integrated using normal methods. We will learn how to use the method called **"Integration by Parts" (p.562–567 in your book)**. 2. Review properties of integrals and summation notation. 3. What is a surprise test? 4. Review finding the integral of inverse trig functions.

S 1. If $\int_1^4 f(x)\,dx = 7$, then find $\int_4^1 f(x)\,dx =$ 2. Find $\int \dfrac{2}{1+x^2}\,dx$ 3. $\int x(\ln x)\,dx$ 4. $\int x(\cos x)\,dx$
5. $\int 2^x\,dx$

F #69 Worksheet #30 1 – 14

I Find the area enclosed between the graphs of $x + 2 = y^2$ and $y = x$.
A) $\dfrac{9}{2}$ B) $\dfrac{7}{2}$ C) $\dfrac{19}{2}$ D) $\dfrac{26}{6}$ E) none of these

V $\lim_{x\to 0} \dfrac{\sin x}{|x|} =$ (Explain your answer for credit.) A) –1 B) 0 C) 1 D) $\dfrac{1}{2}$ E) none of these

E 1. Review finding the integral of inverse trig functions. 2. Continue work on Integration by Parts.
3. Work on functions that repeat in parts.

S 1. $\int 2^{\cos x} \sin x\,dx$ 2. $\int e^x \cos x\,dx$

F #70 That's right! Another wonderfully designed **HOLIDAY ADAGE !! IT'S THE LAST ADAGE OF THE YEAR!!** Be sure that you show all work on your own paper for full credit. The message on this adage could prove to be very valuable to **you!!** Remember that the adage is designed to be done at the end of winter break so that you come back renewed and refreshed and ready for more exciting CALCULUS!

I None

V p.358 (56)

E **** HAPPY HOLIDAYS ****
**** WE WILL SEE YOU IN JANUARY ****

S 1. Be good, have fun, and **be careful** this holiday.

F #71 Worksheet #31 1–10

I If $f(x) = ax^2 + 3x + 1$ and $f(1) = 6$, then $f(2) =$ A) –1 B) 7 C) 11 D) 12 E) 15

V $\int x\cos(ax)\,dx$

E 1. Review finding the integral of inverse trig functions. 2. Continue work on Integration by Parts.
3. Work on functions that repeat in parts.

S 1. Find $\int \frac{2}{4 + x^2}\,dx$ 2. $\int x^2e^x\,dx$ 3. $\int 4^x\,dx$ 4. $\int 5^{(x+3)}\,dx$ 5. $\int x^2e^{-5x}\,dx$

6. $\int \ln\sqrt{x}\,dx$ 7. $\int 7^{4x}\,dx$ 8. $\int e^x\cos x\,dx$ (preview of tomorrow's homework)

F #72 Worksheet #32 1 – 11

I If $f(x) = \sqrt[5]{x^3 - 2x}$, then $f'(\sqrt{3}) =$
A) 0.129 B) 0.902 C) 0.906 D) 1.116 E) 2.173

V $\int e^{ax}\sin(bx)\,dx$

E 1. Review integrating functions where the exponents include variables and integration by parts.

S 1. $\int 6^{2x-1}\,dx$ 2. $\int \sin^{-1}x\,dx$

A.P. CALCULUS “FIVES” SHEET

F #73 Find the volume of the region formed by revolving the following regions about the given axis:

1. $y = x^2$, $x = -1$, $x = 4$, $y = 0$ (x–axis) 2. $y = x^2$, $y = 2$, $x = 0$ (y–axis)

3. $y = \sqrt{x}$, $y = 2$, $x = 0$ (y–axis) 4. $y = x^4$, $x = 1$, $y = 0$ (x–axis)

ALSO DO THE FOLLOWING:

5. $\displaystyle\int_0^1 \frac{e^x}{(3 - e^x)^2}\,dx$ equals A) $3 \ln (e - 3)$ B) 1 C) $\dfrac{1}{3 - e}$ D) $\dfrac{e - 2}{3 - e}$ E) none

6. If $\dfrac{dy}{dx} = -y$ and $y = 1$ when $x = 2$, then y equals

A) $2e^{-x}$ B) $-2e^{-x}$ C) e^{2-x} D) e^{x-2} E) $\dfrac{1}{2}e^{-x}$

I Which of the following is an equation of an asymptote for the graph of $y = \dfrac{2x^2 - x - 1}{x^2 - x}$?

A) $x = 2$ B) $y = 0$ C) $x = 1$ D) $y = 2$ E) $y = 1$

V Let f be the function given by $f(x) = 3e^{2x}$ and let g be the function given by $g(x) = 6x^3$. At what value of x do the graphs of f and g have parallel tangent lines?

A) –0.701 B) –0.567 C) –0.391 D) –0.302 E) –0.258

E 1. Find volumes when the area is revolved around the x– and the y–axis. Look for discs today!

S Find the volume: 1. $y = x^2$, $y = 4$, $x = 0$, (y–axis) 2. $y = x^2$, $y = 2$, $x = 0$, (x–axis)

3. $y = x^3$, $x = 2$, $y = 0$ (y–axis)

F #74 Find the volume of the region formed by revolving the following regions about the given axis (axis is in parentheses):

1. $y = x$, $y = 3x - x^2$ (x–axis) 2. $y = x^3$, $x = 2$, $y = 0$ (x)

3. $y = 2x^2$, $y = 0$, $x = 5$ (x) 4. $y = 4x^2$, $x = 0$, $y = 16$ (y)

5. $y = \dfrac{x}{2}$, $x = 0$, $y = 2$ (y) 6. $y = \dfrac{x}{2}$, $x = 0$, $y = 2$ (x)

7. $y = \sqrt{x}$, $x = 0$, $y = 2$ (y) 8. $y = \sqrt{x}$, $x = 0$, $y = 2$ (x)

I 1. $\displaystyle\int_0^{\frac{p}{4}} \tan x\,dx$

V What are all values of k for which $\displaystyle\int_{-3}^{k} x^2\,dx = 0$? A) –3 B) 0 C) 3 D) –3 and 3 E) –3, 0, and 3

E 1. Continue to investigate volumes of revolution. 2. Look for washers today.

S Find the volumes: 1. $y = x$, $y = 2x - x^2$ (x–axis) 2. $y = x^2$, $y = 4$, $x = 0$, (x–axis)

3. $y = x^2$, $x = 2$, $y = 0$ (y–axis)

A.P. CALCULUS "FIVES" SHEET

F #75 Worksheet # 33 1 – 12

I $\int \sin^2 3x \, dx$ =

V A smooth curve with equation y = f (x) is such that its slope at each x equals x^2. If the curve goes through the point (–1,2), then its equation is

A) $y = \frac{x^3}{3} + 7$ B) $x^3 - 3y + 7 = 0$ C) $y = x^3 + 3$ D) $y - 3x^3 - 5 = 0$ E) none

E 1. Find volume of solid of revolution when the region is revolved around a line other than the x- or y-axis.
2. Review properties of definite integrals, integration by parts, area between curves, and volume by disc/washer methods for test next week.

S Find volume: 1. $y = \sqrt{x}$, x = 4, y = 0 (line x = 4) 2. $y = \sqrt{x}$, $y = x^2$ (line x = 1)

F #76 Worksheet #34 (1 – 14)

I If c represents the number defined by Rolle's Theorem, then, for the function
$f(x) = x^3 - 3x^2$ on the interval [0,3], c = (A) 2 (B) 1 (C) 0 (D) $\sqrt{2}$ (E) none

V $\lim_{x\to 0} x \sin \frac{1}{x}$ =

E 1. Review properties of definite integrals, integration by parts, area between curves, and volume by disc/washer methods for upcoming test.

S Notes problems for upcoming test.

F #77 Worksheet #35 (1 – 13)

I The average (or mean) value of $\frac{1}{2}t^2 - \frac{1}{3}t^3$ over the interval $-2 \le t \le 1$ is

A) $\frac{1}{36}$ B) $\frac{1}{12}$ C) $\frac{11}{12}$ D) 2 E) $\frac{33}{12}$

V Find the volume generated by revolving the region in the first quadrant bounded by $y = x^3$ and y = 4x about the x – axis.

E 1. Review properties of definite integrals, integration by parts, area between curves, and volume by disc/washer methods for test.

S Lots of "notes" problems for review.

F #78 Worksheet #36 (1 – 5) and Read p.387–391 in your book.

I 1. If $f(x) = x^{3/2}$, then $f'(4)$ = A) – 6 B) – 3 C) 3 D) 6 E) 8
2. The radius of a circle is increasing at a nonzero rate, and at a certain instant, the rate of increase in the area of the circle is numerically equal to the rate of increase in its circumference. At this instant, the radius of the circle is A) $\frac{1}{\pi}$ B) $\frac{1}{2}$ C) $\frac{2}{\pi}$ D) 1 E) 2

V Let $f(x) = \int_1^{2x} f(x)\, dx$ Assume that f(x) is continuous over the real numbers.
Use the Fundamental Theorem of Calculus to find f ' (10), given f(20) = 5.

E 1. **Take an easy test on volumes of solids of revolution** using discs or washers, properties of integrals, and integration by parts. 2. Finding volumes of solids of revolution by shells or pipes.

S None

A.P. CALCULUS "FIVES" SHEET

F #79 Find the volume if the following areas are revolved around the given axis.

1. $y = x$, $y = x^2$ (y–axis) SHELL
2. $x = 2y - y^2$, $x = 0$ (x–axis) SHELL
3. $y = 4x - x^2$, $y = 0$ (x–axis) DISC
4. $y = 4x - x^2$, $y = 0$ (y–axis) SHELL
5. $y = \sqrt{x}$, $y = 0$, $x = 4$ (y–axis) YOUR CHOICE
6. $y = \sin x$, $y = 0$, $0 = x = p$, (y–axis) YOUR CHOICE

I 1. If $x^3 + 3xy + 2y^3 = 17$, then in terms of x and y, $\dfrac{dy}{dx} =$

A) $-\dfrac{x^2 + y}{x + 2y^2}$ B) $-\dfrac{x^2 + y}{x + y^2}$ C) $-\dfrac{x^2 + y}{x + 2y}$ D) $-\dfrac{x^2 + y}{2y^2}$ E) $\dfrac{-x^2}{1 + 2y^2}$

2. If the second derivative of f is given by $f''(x) = 2x - \cos x$, which of the following could be $f(x)$?

A) $\dfrac{x^3}{3} + \cos x - x + 1$ B) $\dfrac{x^3}{3} - \cos x - x + 1$

C) $x^3 + \cos x - x + 1$ D) $x^2 - \sin x + 1$ E) $x^2 + \sin x + 1$

V None

E 1. Finding the volumes of solids of revolution by shells or pipes.

S Find the volume: 1. $y = x^2$, $y = 0$, $x = 2$ (y) 2. $y = 2x$, $y = x^2$ (y) 3. $x = y - y^2$, $x = 0$ (x)

F #80 p.392 (9,17,18,26a,26b) **AND**

1. If $f(x) = (2x + 1)^4$, then the 4th derivative of $f(x)$ at $x = 0$ is

A) 0 B) 24 C) 48 D) 240 E) 384

2. $\displaystyle\int_0^1 xe^{-x}\,dx$ = A) $1 - 2e$ B) -1 C) $1 - 2e^{-1}$ D) 1 E) $2e - 1$

I 1. If $f(x) = \cos^2 x$, then $f''(p) =$ A) –2 B) 0 C) 1 D) 2 E) $2p$

2. If $f(x) = \dfrac{5}{x^2+1}$ and $g(x) = 3x$, then $g(f(2)) =$ A) –3 B) $\dfrac{5}{37}$ C) 3 D) 5 E) $\dfrac{37}{5}$

V $\displaystyle\int \frac{5}{\sqrt{9-4x^2}}\,dx$ = A) $-\dfrac{5}{2}\ln\left|9-4x^2\right| + C$ B) $-\dfrac{5}{8}\ln\left|9-4x^2\right| + C$ C) $-\dfrac{5}{8}\sqrt{9-4x^2} + C$

D) $-\dfrac{5}{2}\sqrt{9-4x^2} + C$ E) $\dfrac{5}{2}\arcsin\dfrac{2x}{3} + C$

E 1. Learning more about finding volumes using the shell method. 2. Revolving an area around a line other than the x- or y-axis. 3. A look ahead at another type of volume problem.

S 1. Find the volume when the area bounded by $y = x^{1/2}$, $x = 4$, and $y = 0$ is revolved about the: (a) x–axis (b) y–axis. (c) the line $x = 4$

2. $y = \sqrt{x}$, $y = 2$, $x = 0$ (a) x-axis (b) y-axis (c) line $x = 4$ (d) line $y = 2$

3. $y = x^2$, $y = \sqrt{x}$ (y–axis) Shell

4. The cross sections of a solid cut by planes perpendicular to the y-axis are equilateral triangles whose base is in the x-y plane and is bounded by the parabola $x^2 = 8y$ and the line $y = 4$. Find the volume of the solid. (Upcoming problem)

A.P. CALCULUS "FIVES" SHEET

F #81

1. Find the volume of the solid whose base is the region bounded between the curves $y = x$ and $y = x^2$, and whose cross sections perpendicular to the x-axis are squares.
2. The base of a certain solid is the region enclosed by $y = \sqrt{x}$, $y = 0$, and $x = 4$. Every cross section perpendicular to the x-axis is a semicircle with its diameter across the base. Find the volume of the solid.
3. Find the volume of the solid generated when the region enclosed between $y = \sqrt{x}$, $x = 1$, $x = 4$, and the x-axis is revolved about the y – axis.
4. Find the volume of the solid generated when the region enclosed between $x = 2y - 2y^2$ and the y-axis is revolved about the x – axis.
5. Find the volume of the solid that is generated when the region that is enclosed by $y = x^3$, $y = 1$, and $x = 0$ is revolved about the line $y = 1$.

I $\int_1^2 \frac{1}{x^2}\,dx =$ A) $-\frac{1}{2}$ B) $\frac{7}{24}$ C) $\frac{1}{2}$ D) 1 E) $2 \ln 2$

V Find $\frac{dy}{dx}$ and simplify If $y = \sec^4 x - \tan^4 x$

E Finding the volume when a cross section is given.

S 1. The base of a solid in the x – y plane is a right triangle bounded by the axes and $x + y = 2$. Cross sections of the solid perpendicular to the x – axis are squares. Find the volume.
2. The base of a solid is the circle $x^2 + y^2 = 9$. Cross sections of the solid perpendicular to the x-axis are semicircles. Find the volume of the solid.

F #82 p.392 – 393 (28a,28b,28c,28d,29a,29b) **AND**

1. Find the volume of the solid whose base is the circle $x^2 + y^2 = 9$ and whose cross sections perpendicular to the x – axis are squares.

I 1. $\int \ln x\,dx$ 2. $\int \tan^2 x\,dx$

V p. 297 #52

E 1. Learning more about finding volumes using the shell method. 2. Finding the volume when a cross section is given.

S The semester ends soon! Are you all caught up?

A.P. CALCULUS "FIVES" SHEET

F #83

1. Find the volume of the solid generated by revolving the region bounded by $y = 3x^4$, $x = 1$, and $y = 0$ about the (a) x – axis (b) y – axis (c) line $x = 1$ (d) line $y = 3$.
2. The base of a solid is a circle $x^2 + y^2 = 25$, and every plane cross section perpendicular to the x-axis is a square. The solid has volume: A) 72 B) 54π C) 108p D) 2000/3 E) 216p
3. The base of a solid is the region bounded by the parabola $x^2 = 8y$ and the line $y = 4$, and each plane section perpendicular to the y-axis is an equilateral triangle. The volume of the solid is:

   A) $\frac{64}{\sqrt{3}}$ B) $64\sqrt{3}$ C) $32\sqrt{3}$ D) 32 E) None of these

I 1. If the function f is continuous for all real numbers and if $f(x) = \frac{x^2 - 4}{x + 2}$ when $x \neq -2$, then $f(-2) =$

A) -4 B) -2 C) -1 D) 0 E) 2

2. If the definite integral $\int_0^2 e^{x^2}\,dx$ is first approximated by using two <u>inscribed or left–endpoint</u> rectangles of equal width and then approximated by using the trapezoidal rule with $n = 2$, then the difference between the two approximations is A) 53.60 B) 30.51 C) 27.80 D) 26.80 E) 12.78

V p.392 (24c)

E 1. Learning more about finding volumes using the shell method. 2. Finding the volume when a cross section is given. 3. Could a surprise be in your future?

S 1. Find the volume of the solid that lies between planes perpendicular to the x – axis at $x = 0$ and $x = 1$. The cross sections perpendicular to the x – axis between these planes are circular disks whose diameters run from the parabola $y = x^2$ to the parabola $y = \sqrt{x}$.
2. Find the volume of the solid whose base is the region in the first quadrant between the line $y = x$ and the parabola $y = 2\sqrt{x}$. The cross sections of the solid perpendicular to the x – axis are equilateral triangles whose bases stretch from the line to the curve.

F #84 Read p.205-208 Do p.208-209 (2abc,3abc,4abc,9ab,12ab,13ab,16ab,17ab)
Do not graph problems 2,3,4. **DO** graph all other problems. Show all work and show graphs on graph paper. Use your graphing calculator to **check** your answer, not to do the work. Show your World Famous First Derivative chart to show where the 1st derivative is zero, undefined, positive, and negative.

I 1. An equation of the line tangent to the graph of $y = \frac{2x + 3}{3x - 2}$ at the point (1, 5) is

A) $13x - y = 8$ B) $13x + y = 18$ C) $x - 13y = 64$ D) $x + 13y = 66$ E) $-2x + 3y = 13$

2. If $\frac{dy}{dx} = 2y^2$ and if $y = -1$ when $x = 1$, when $x = 2$, $y =$

A) $-\frac{2}{3}$ B) $-\frac{1}{3}$ C) 0 D) $\frac{1}{3}$ E) $\frac{2}{3}$

V None

E 1. Review curve analysis using the first derivative.

S Determine for what values of x are the following increasing and decreasing, and find all critical points, then graph f(x): 1. $f(x) = -3x^2 + 9x + 5$ 2. $f(x) = 6x - x^3$
Do the same as above given f '(x): 3. $f'(x) = (x - 1)(x + 2)$ 4. $f'(x) = x(x - 2)^2(x + 1)^3$

A.P. CALCULUS "FIVES" SHEET

F #85 Worksheet #37 1–8

I 1. If h is the function given by $h(x) = f(g(x))$, where $f(x) = 3x^2 - 1$ and $g(x) = |x|$, then $h(x) =$ A) $3x^3 - |x|$ B) $\left|3x^2 - 1\right|$ C) $3x^2|x| - 1$ D) $3|x| - 1$ E) $3x^2 - 1$

2. $\lim_{q \to 0} \dfrac{1 - \cos q}{2\sin^2 q}$ A) 0 B) $\dfrac{1}{8}$ C) $\dfrac{1}{4}$ D) 1 E) nonexistent

V None

E Continue curve analysis and sketching using the first derivative test.

S Give the total number of maximum and minimum points of the function whose **derivative** is given by $\dfrac{dy}{dx} = x(x + 2)^2(x - 5)^3$

F #86 Read p.209 – 216 DO p.217 (1,10,11,15,18,21,69) **Find and <u>label</u> intervals where curve is increasing, decreasing, concave up, concave down, critical points, points of inflection, and graph the curve on graph paper**

I 1. The fundamental period of $2\cos(3x)$ is A) $\dfrac{2\pi}{3}$ B) 2π C) 6π D) 2 E) 3

2. The function f given by $f(x) = x^3 + 12x - 24$ is
A) decreasing for all x
B) decreasing for $x < -2$, increasing for $-2 < x < 2$, decreasing for $x > 2$
C) increasing for all x
D) increasing for $x < -2$, decreasing for $-2 < x < 2$, increasing for $x > 2$
E) decreasing for $x < 0$, increasing for $x > 0$

V None

E Use the second derivative to determine concavity of a curve and to find inflection points.

S 1. $y = 4 + 3x - x^3$ 2. $x^4 + 4x^3 + 5$ Find and label the critical points and points of inflection points, and find the intervals where the curve is increasing, decreasing, concave up, concave down, and sketch the curve.

F #87 Read p.233 – 242 Do Worksheet #38 1–7

I 1. $\dfrac{d}{dx}\left(2^x\right) =$

2. $\displaystyle\int \frac{3x^2}{\sqrt{x^3 + 1}}\,dx =$ A) $2\sqrt{x^3 + 1} + C$ B) $\dfrac{3}{2}\sqrt{x^3 + 1} + C$ C) $\sqrt{x^3 + 1} + C$

D) $\ln\sqrt{x^3 + 1} + C$ E) $\ln\left(x^3 + 1\right) + C$

V None

E 1. Solving word problems that involve maximizing or minimizing a function.

S 1. Find two positive numbers whose sum is 20 and whose product is as large as possible. 2. A square piece of cardboard has 12 inches in a side. An open box is formed by cutting out equal square pieces at the corners and bending upward the projecting portions that remain. Find the maximum volume that can be obtained. 3. A farmer has 200 feet of fencing and wants to enclose a rectangular plot of land. The land borders a river so no fence is required on that side. What should the dimensions of the rectangle be in order that it include the largest possible area?
