Newton's Method

What it does: Estimates zeroes by using tangents.
How the formula is derived:

See figure above. f(x) is the given function. We want to find slope of tangent line.

Slope = $\dfrac{f(x_n) - 0}{x_n - x_{n+1}}$ or $f'(x_n) = \dfrac{f(x_n)}{x_n - x_{n+1}}$

Now comes the algebra in three easy steps. We need to get x_{n+1} by itself!

Step #1 $f'(x_n)\,[x_n - x_{n+1}] = f(x_n)$ by cross multiplication

Step #2 $\dfrac{f(x_n)}{f'(x_n)} = x_n - x_{n+1}$ divided both sides by $f'(x)$

Step #3 $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$ moved x_{n+1} to the right and fraction to the right

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This will give us a new x–value for our "new and improved" tangent line. We continue the process until we have "zeroed in" on our root.

Example: Use Newton's method to estimate one of the zeroes of $f(x) = 2x^3 - 4x^2 + 1$

If $f(x) = 2x^3 - 4x^2 + 1$, then $f'(x) = 6x^2 - 8x$

Step #1 Let $x_0 = 1$ (I just guessed, you can use a different number)

Step #2 $x_1 = x_0 - [f(x_0) \div f'(x_0)] = 1 - [f(1) \div f'(1)] = 1 - [(-1) \div (-2)] = 1 - (1/2) = 0.5$

Step #3 Repeat step 2 with new x each time.

$x_2 = x_1 - [f(x_1) \div f'(x_1)] = .5 - [f(.5) \div f'(.5)] = .5 - [(.25) \div (-2.5)] = .5 - [-.1] = .6$

$x_3 = x_2 - [f(x_2) \div f'(x_2)] = .6 - [f(.6) \div f'(.6)] = .6 - [(-.008) \div (-2.64)] = .6 - [.003] = 0.597$

Stop when you get the same answer twice, the answer to this one is 0.597

Now you try it!
Use $x_0 = 2$. You will be able to locate another root. To find the last root, use $x_0 = -1$

(You can check your answers by calculating the roots on your graphing calculator)