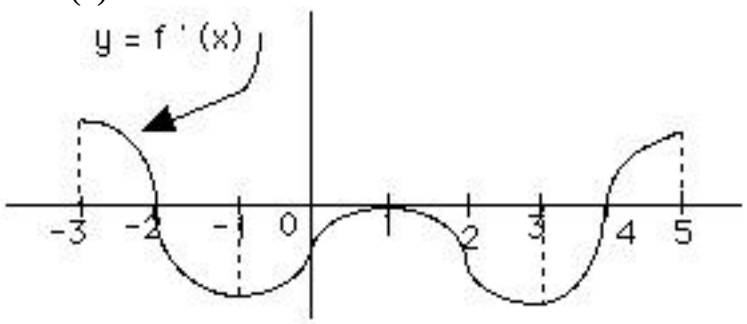
CALCULUS -- Worksheet #65

1996 Free Response (1):



Note: This is the graph of the derivative of f ' , not the graph of f.

The figure above shows the graph of f ', the derivative of a function f.
The domain of f is the set of all real numbers x such that $-3 < x < 5$.

(a) For what values of x does f have a relative maximum? Why?
(b) For what values of x does f have a relative minimum? Why?
(c) On what intervals is the graph of f concave upward? Use f ' to justify your answer.
(d) Suppose that f(1) = 0. Draw a sketch that shows the general shape of the graph of the function f on the open interval $0 < x < 2$.

Approximate each of the integrals using:
(a) 4 left-endpoint rectangles
(b) 4 right-endpoint rectangles
(c) 4 midpoint rectangles
(d) 4 trapezoids:

1. $\int_0^2 x\,dx$
2. $\int_0^2 x^2\,dx$
3. $\int_0^2 x^3\,dx$

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4. The average value of $f(x) = 3 + |x|$ on the interval $[-2,4]$ is

A) $2\frac{2}{3}$ B) $3\frac{1}{3}$ C) $4\frac{2}{3}$ D) $5\frac{1}{3}$ E) 6

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5. If $y = \ln\left(\frac{x}{\sqrt{x^2+1}}\right)$, then $\frac{dy}{dx}$ =

A) $\frac{1}{x^2+1}$ B) $\frac{1}{x(x^2+1)}$ C) $\frac{2x^2+1}{x(x^2+1)}$ D) $\frac{1}{x\sqrt{x^2+1}}$ E) $\frac{1-x^2}{x(x^2+1)}$
