Genetic control applied to asset managements.

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Abstract. This paper addresses the problem of investment optimization using genetic control. Time series for stock values are obtained from data available on the www and asset prices are predicted using adaptive algorithms. A portfolio is optimized with the genetic algorithm based on a recursive model of portfolio composition obtained on-the-fly using genetic programming. These two steps are integrated into an automatic system - the final result is a real-time system for updating portfolio composition for each asset.

Introduction.

IBM and other companies are undertaking massive studies on the application of advanced computing technologies to stock brokerage and obtaining better results than the New York stock market's sharpest traders [1].

Every investor knows that there is a trade off between risk and reward: to obtain a greater than expected return on investment one must be willing to take on a greater risk [2]. Portfolio optimization theory assumes that for a given level of risk, investors prefer higher returns to lower returns. Similarly, for a given level of expected return, investors prefer less risk to more risk. It is standard to measure risk in terms of the variance, or standard deviation, of return.

The portfolio optimisation problem consists of obtaining the biggest return on investment with the least risk exposure necessary under the prevailing market dynamics. These dynamics are unpredictable due to both exogenous factors (such as government actions, market rumors, unexpected events, etc.) and endogenous factors (such as company and stock fundamentals).

Mahfoud and Mani [17] developed a rule-based system for managing each individual asset where the rules are of the form: IF (price < limit and EPS > value) THEN buy, where price is the asset price, limit is the buy threshold, and EPS is the earning per share.

Chang et al [18] used the genetic algorithm to find the portfolio of assets with different risk exposures (called the efficient frontier). Usually, this problem is solved with quadratic programming, but, for practical purposes it is desirable to limit the number of assets in a portfolio, as well as the proportion of the portfolio devoted to any particular asset.

Kivine and Warmouth [19] proposed that the portfolio vector itself encapsulate the necessary information from the previous price relatives. Thus, at the start of day t, the algorithm computes its new portfolio vector w^{t+1} as a function of w^t and the just observed price relatives x^t, using a linear regression. Helmbold et al [14] select a more complex function and Parkes and Huberman [16] generalize the idea for investment group model for the portfolio selection problem, adjusting their portfolio as they observe movements of the market over time and communicate to each other their current portfolio and its recent performance. Investors can choose to switch to any portfolio performing better than their own.

In this work the goal is to obtain a recursive mathematical law using genetic programming and the genetic algorithm that establishes a relation between the available information and the percentage of various assets to be held in the portfolio. The general framework, genetic control Werner [4], is represented in fig. 1. It uses data from experimental setup (the simulated market in this case) to feed genetic programming for the purpose of building a model of the market. Later, the genetic algorithm adapts real values to obtain the optimal percentages of the assets in the portfolio that will feed genetic programming, closing the loop.

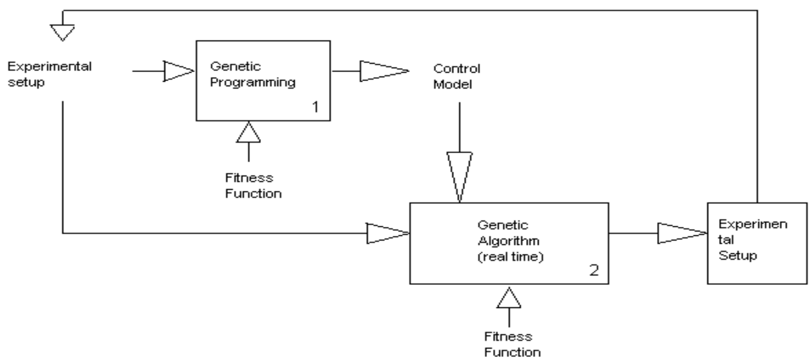


Fig. 1. Genetic control: obtaining the structure of the solution with genetic programming and adapting its parameters with genetic algorithm.

The genetic algorithm.

The genetic algorithm (GA) mimic the evolution and improvement of life through reproduction, where each individual contributes its own genetic information to the building of new ones adapted to the environment with higher chances of survival. This is the basis of genetic algorithms and genetic programming ([5], [6], and [7]).

Specialized Markov Chains underline the theoretical bases of this algorithms change of states and searching procedures.

Each 'individual' of a generation represents a feasible solution to the problem, coding distinct algorithms/parameters to be evaluated by a fitness function.

GA operators are mutation (the change of a randomly chosen bit of the chromosome) and crossover (the exchange of randomly chosen slices of the chromosome).

The best individuals are continuously being selected, and crossover and mutation take place. Following a number of generations (Fig. 2), the population converges to the solution that performs better.

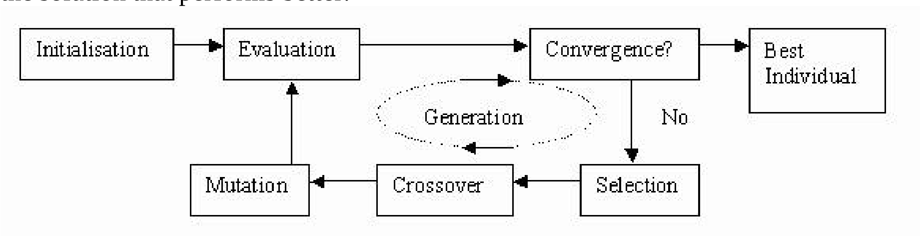


Fig. 2. Genetic algorithm: the sequence of operators and evaluation of each individual.

A generalization of the Genetic Algorithm is Genetic Programming (GP) (Holland [5] and Goldberg [6]) where each 'individual' in a generation represents, with its chromosome, a feasible solution to the problem; in our case, a mathematical function to be evaluated by a fitness function.

There are two kinds of information defined for the GP algorithm: terminals (variable values and random numbers) and functions (mathematical functions used in the generated model).

The virtual market.

The first problem when studying market investment is in what environment to test the concepts and how to obtain time series of assets, stocks and currency, and market index for a period of 2 years at least. To solve this problem we extracted time series values from the history graphics available in the Yahoo Finance site [3]. We built a database contains the following information [9] between July 1999 and July 2001: FTSE100 stock quotes, trade volume and quotes by sector; Fix interest; European and American indices; Stock exchanges indices around the world: Argentina, Brazil, Canada, Chile, Peru, Venezuela, Australia, China, Hong Kong, India, Indonesia, Malaysia, New Zealand, Pakistan, Philippines, Singapura, South Korea, Sri Lanka, Thailand, Taiwan, Austria, Tchec Republic, Finland, Greece, Nederland, Portugal, Russia, Slovakia, Spain, Swiss, Turkey, Egypt, Israel; Commodities: Gold, silver, paladio; Currency convergence to pound: USD, Australia, Canada, Argentina, Brazil, Euro, Franca, Germany, Hong Kong, Japan, Mexico, Russia and Swiss.

The benchmark for portfolio return.

The reference for return evaluation is the stock exchange index. In the case of London this is the FTSE 100 [10]. The investment operation would build a portfolio with reflects the same constitution as the FTSE 100 index, and its performance is the same as the market.

Asset forecast.

To develop a forecast of assets price (or any other time series value) there are two necessaries definitions: the mathematical function to be adjusted (termed filter) and the adaptation algorithm (responsible for calculate the parameter values of the filter following temporal changes of the series).

The FIR (*finite impulse response*) filter of N dimension is a filter with trivial poles (z=0) in its transference function:

$$W(z) = w_0 + w_1.z^{-1} + w_2.z^{-2} + ... + w_{n-1}.z^{-N+1} \qquad (1)$$

Let **W** the filter coefficient vector, and $\mathbf{X_k}$ last N inputs to the filter in k instant:

$$W = [\ w_0 \ \ w_1 \ \ w_2 \ \ w_3 \ \ .... \ \ w_{N-1}]^T$$
$$X_k = [x(k) \ \ x(k-1) \ \ x(k-2) \ \ x(k-2) \ \ .... \ \ x(k-N+1)]^T \qquad (2)$$

Filter output is defined as:

$$y(k) = \sum_{i=0}^{N-1} w_i \cdot x(k-i) = X_k^T \cdot W \qquad (3)$$

Any stock/asset contains in its price two components: one depending of its fundamentals and other completely random, modeled by the Brownian model. The intrinsic component would be adapted by Least Means Square LMS (see [11]), which cancels the random component.

Let us define the performance function

$$\xi = \xi(W) \qquad (4)$$

a quadratic function with one minimum point. For any initial condition W, evaluate new values of W into the contrary direction of the performance hyper surface gradient (with indicate the maximum direction). Following the contrary direction, certainly will hit the minimum.

To evaluate the gradient of ξ is necessary do some approximations. Let

$$\xi = E[e^2(k)] \cong e^2(k) \qquad (5)$$

Where E is the average of stochastic variable e, the error between the real value d and the adapted by fitting y. Then:

$$\nabla x \cong \begin{bmatrix} \frac{\partial e^2(k)}{\partial w_0} \\ \frac{\partial e^2(k)}{\partial w_1} \\ \vdots \\ \frac{\partial e^2(k)}{\partial w_{N-1}} \end{bmatrix} = 2.e(k).\begin{bmatrix} \frac{\partial e(k)}{\partial w_0} \\ \frac{\partial e(k)}{\partial w_1} \\ \vdots \\ \frac{\partial e(k)}{\partial w_{N-1}} \end{bmatrix} \qquad (6)$$

and because

$$e(k) = d(k) - X_k^T \cdot W \Rightarrow \frac{\partial e(k)}{\partial w_i} = -x(k-i) \qquad (7)$$

Then

$$\nabla \boldsymbol{x} \cong -2.e(k).X_k \qquad (8)$$

LMS algorithms consists in iteratively calculating the vector filter coefficient W, with a little vector in the gradient contrary direction:

$$W_{k+1} = W_k - m\nabla \boldsymbol{x} \qquad (9)$$

where μ is an arbitrary constant to control the iteration step. Replacing $\nabla\xi$ into W iterative equation:

$$\mathrm{W}_{k+1} = \mathrm{W}_k + 2.\mu.\mathrm{e}(k).\mathrm{X}_k \qquad (10)$$

This is the LMS algorithm equation, a very simple mathematical model easy to be implemented.

LMS stability depends of the μ value. If (see [12])

$$0 < \mu < \frac{2}{3} \cdot \frac{1}{\mathrm{N} \cdot \sigma_x^2} \qquad (11)$$

where σ_x^2 – average power of x(k) and N is the filter order, the algorithm will be convergent and stable. If the value is too big it will be unstable and if it is too little convergence will take too much time.

Assets data pre processing.

Let S(t) be the quote of a stock. The market works with the function

$$Ln\frac{S(t)}{S(t-1)} \qquad (12)$$

for return and with the recursive variation:

$$\sigma_{ij,t} = \lambda \sigma_{ij,t-1} + (1+ \lambda) r_{i,t-1} r_{j,t-1} \qquad \textbf{(13)}$$

where $\sigma_{ij,t}$ is the conditional variation of the period t, λ= 0.94 is the decay parameter of exponential smoothing , $r_{x,t}$ is the return of x in period t. The initial condition is:

$$\sigma_{ij,0} = \frac{1}{T-2} \sum \left(r_{i,t} - \overline{r_i}\right)\left(r_{j,t} - \overline{r_j}\right) \qquad \textbf{(14)}$$

$$\overline{r_x} = \frac{1}{T} \sum_{t=1}^{T} r_{ij,t}$$

The variation matrix Σ of risk factors and correlation matrix are:

$$\Sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1k} \\ \sigma_{21} & \sigma_{22} & \cdots & \sigma_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{k1} & \sigma_{k2} & \cdots & \sigma_{kk} \end{bmatrix} \qquad \textbf{(15)}$$

$$C = \begin{bmatrix} 1 & \rho_{12} & \cdots & \rho_{1k} \\ \rho_{21} & 1 & \cdots & \rho_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{k1} & \rho_{k2} & \cdots & 1 \end{bmatrix} \qquad \textbf{(16)}$$

$$\rho_{ij} = \frac{\sigma_{ij}}{\sigma_i \sigma_j}$$

The diagonal terms of matrix Σ are $\sigma_{ii} = \sigma_i^2$

Portfolio optimization.

The methodology for searching for the constitution of a portfolio was established by Markowitz [13] 50 years ago and has been central to research activities in extending, improving and revising this approach in unforeseeable and dynamic markets. The aim is to obtain a mix of assets to maximize the relation between mean return and risk, a model optimization problem:

$$\max\left[(1.0-\lambda)*\left[\sum_{i=1}^{N} w_i \mu_i\right]-\lambda\left[\sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{i,j}\right]\right] \quad \textbf{(17)}$$

$$\sum_{i=1}^{N} w_i = 1$$

$$\delta \le w_i \le \xi, \quad i = 1,\ldots,N$$

where each composition is limited between the limits δ and ξ. The case λ=0 represents maximize expected return (irrespective of the risk involved) and the optimal solution will involve just the single asset with the highest return. The case λ=1 represents minimize risk (irrespective of the return involved) and the optimal solution will typically involve a number of assets. Values 0<λ<1 represents explicit trade off between risk and return, generating solutions between the extremes. We assume λ=0.3 in this paper.

A recursive mathematical model for portfolio selection.

The fulcrum point of the problem of portfolio optimisation consists in obtaining a model:

$$w_i^{t+1} = F(w_i^t, return, risk, past\ price, prediction\ price, market\ index) \quad \textbf{(18)}$$

of the future percentage of each asset available, with a function dependent only of the forecast and past condition. A general theory for this approach is available in [14], [15] and [16].

Software implementation and results with virtual market.

The software consists in three procedures, running in sequence before trade work:

1. Given time series values up to last period, obtain the parameters of an adaptive filter for each asset, which model its fundamental behaviour, with a 20th order approximation, meaning that all effects with a period less than 10 days are modelled. With these parameters forecast the return for next period using a FIR model adapted by the LMS algorithm.
2. Genetic algorithms optimize the percentages w for each asset, with the available information of the last period, giving, as a result, the optimum portfolio. For each asset we suppose that its composition in the portfolio is limited between 10% and 30%, eliminating the effects of small fluctuations in the selection.
3. With the information of the evolution of w, two approaches were applied:
   - Use GP to obtain equation (18) that predicts the function with and without constraints.

- Use GP to obtain equation (18) that gives the best return for the period, on-the-fly. Two possibilities are explored: with and without optimal composition of last period obtained by Genetic algorithm.

Bearing in mind the assumptions described in the introduction about market behavior and forecast, the first result consists in evaluating the error distribution of forecast assets using the adaptive method, with an accuracy better than 5% for FTSE100 assets forecast over 2 years.

The next step consists in obtaining the optimal portfolio with all assets available using past information. The problem consists in solving equation 17 using genetic algorithms, with a chromosome with binary coded float variables for each assets representing the amount into the portfolio. This evaluation uses return, standard deviation and variance matrix of the last 20 time series values available.

The return sum for all period intervals gives the efficiency of the strategy. Figure 3 and 4 show the results for free and with constraints ($0.1<\omega<0.3$), which fixes the percentage of any asset between 10% and 30%.

Fig. 3. Daily return for unconstrained portfolio against FTSE100 return (-0.089).

Σreturn=15.33

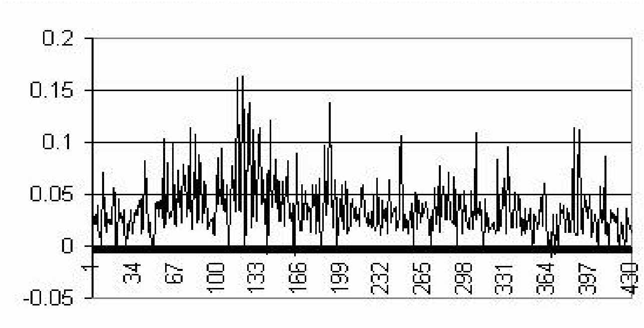
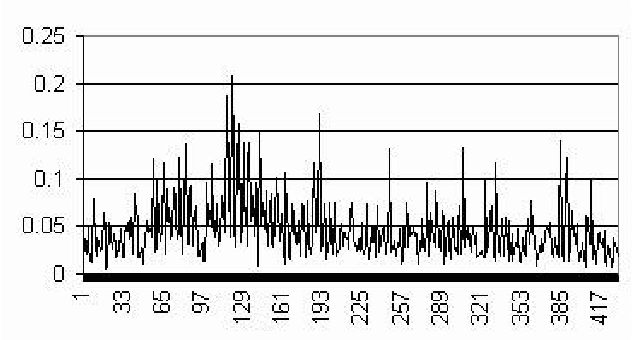


Fig. 4. Daily return for constrained portfolio against FTSE100 return (-0.089).

Σreturn=19.49



The constrained portfolio is necessary due operational problems, otherwise the portfolio is formed of too many assets.

The next step consists in evolving the solution to equation 18 using genetic programming to forecast the percentage of each asset in the portfolio. A tree built with the functions: multiply (*), sum(+), subtraction (-) and division (Div), and terminals: ERC(0..1), price forecast using adaptive algorithm, return in [t-1] and [t-2], mean return for 20 last days period, standard deviation of last 20 days, trade volume is evaluated for the period of 2 years, and optionally the last optimal percentage of the assets obtained by genetic algorithm.

The best model gives the return shown in figs 5 and 6, using, with or without, the last assets optimal percentage with constraints of percentage between 10% and 30% for any assets in the portfolio.

Fig. 5. Portfolio without last optimal percentage with constraint against FTSE100 return (-0.089).

Σreturn=20.66

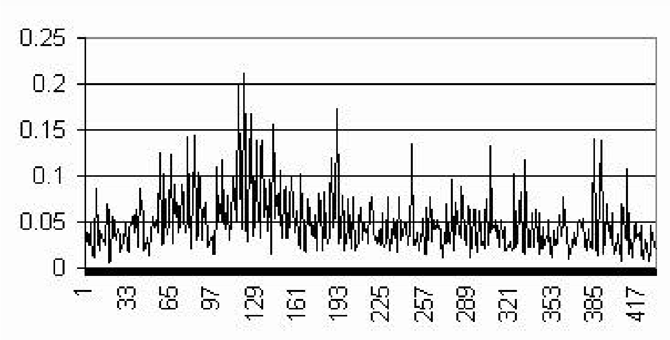
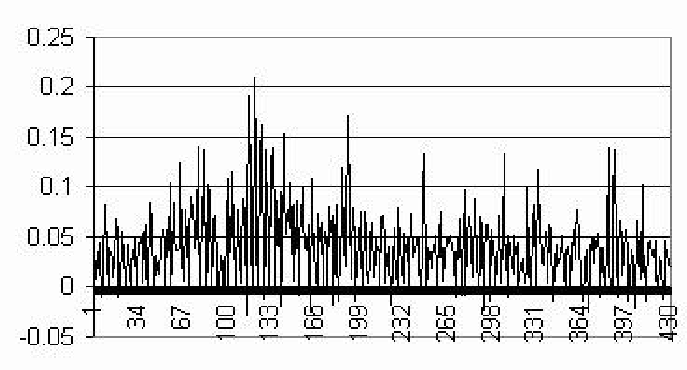


Fig. 6. Portfolio with last optimal percentage with constraint against FTSE100 return (-0.089).

Σreturn=17.53



The returns are evaluated against the next time period's return, to establish the efficacy of mathematical model. The asset selection model using equation 18 obtained by GP forecast as good results as the optimization using equation 17, with λ=0.3.

The difference between figures 3 and 4, and 5 and 6 is that in the first case, genetic algorithm uses only available data to optimize the problem, and the second case uses GP to forecast a future solution based in assets forecast using adaptive algorithm.

Conclusions.

Genetic programming for producing a predictive model for portfolio assets percentage associated with genetic algorithm to obtain portfolio optimal values obtains good results when comparing with FTSE100 index, and to the same level as obtained by genetic algorithms calculating with available data. This framework could be applied in assets management, taking care with exogenous influence in the market.

The next step of the project consists in apply the algorithm to different scenarios, to verify the adaptability of the predictive model.

The software concept is adequate to real time application in assets management, with adequate tests and adaptation of man machine interface and broadcast data acquisition.

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