

Optimization of batch fermentation through genetic algorithms and optimal techniques.

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Abstract: This paper presents a comparison between genetic algorithms and optimal techniques used to optimize a batch fermentation. The results are the convergence to the same value of substrate adiction, showing that the genetic algorithms is the optimal solution to the problem. The methodology used is discussed.

Keywords: industrial optimization, genetic algorithms, fermentation, optimal control.

1. Introduction.

The mathematical modelling of an industrial plant has the objective to represent as accurately as possible its real behavior in time through differential equations, including the relevants variables of the process, the plant dynamics, and the control.

The solutions obtained with the integration in time of the differential equations represents the values that maximizes the performance index related to the operational costs and parameters/variables constraints.

The paper compares two optimization techniques, one using genetic algorithms and the other optimal control for the fermentation process, that is very important in the brasilian energetic matrix, where the PROALCOOL project has millions of dollars of investments and thousands of employees.

2. Batch fermentation.

When a population of *Saccharomyces cerevisiae* (microorganisms) is placed in a fermentation tank together with cane sugar, a biochemistry reaction occurs (Krebs cicle and glicolitic way) to obtain energy (ATPs) and essential compounds to the yeast.

Alcohol is a process residue expeled to the extra-cellular environment. A centrifugal process separates yeast from the wine, that will be distilled to obtain the alcohol.

The process studied here is modelled through experiments in a real plant and successful models of the literature.

Secondary compound where not considered in the differential equations. The limiting factors were considered only in the

performance index. The temperature was controlled between 30-32 degrees Celsius, where it doesn't interfere in the process.

The steps followed were:

-field experiments (to get values of the main variables).

-studies of the fermentation process^{[4][5][6][7][8]}.

-development of the mathematical models (mainly mass and energy balances of the differents reaction channels).

-development of the performance index of the process.

-fitness of some parameters.

-time integration of differential equations to maximize performance index.

-adherence analysis of the model and actual process.

3. Mathematical modelling of batch fermentation.

The differential equations of the batch fermentation are:

$$\frac{dP}{dt} = \mu X_v \frac{SK_s}{(K_s + S^{3/4})} \frac{K_p}{(K_p + P)} + \frac{(P_0 - P)E_s}{V} \quad (1)$$

$$\begin{aligned} \frac{dS}{dt} = & -(\mu X_v \frac{SK_s}{(K_s + S^{3/4})} \frac{K_p}{(K_p + P)} + \\ & + \mu_m X_v \frac{K_{sp}}{(K_{sp} + P + P^2)} \frac{S}{(K_{ss} + S)} (1 - K_i)) + \\ & + \frac{(S_0 - S)E_s}{V} \end{aligned} \quad (2)$$

$$\frac{dX_v}{dt} = +\mu_m X_v \frac{K_{sp}}{(K_{sp} + P + P^2)} \frac{S}{(K_{ss} + S)} + \frac{(X_{v0} - X_v) F_s}{V} \quad (3)$$

$$\frac{dV}{dt} = F_s \quad (4)$$

where

S= sugar concentration in the tank.

P= alcohol concentration in the tank.

X_v= yeast concentration in the tank.

μ, μ_m, K_s, K_p, K_{sp}, K_{ss}, K_i= process parameters.

F_s= sugar input.

V= tank volume.

P₀= initial concentration of alcohol.

S₀= initial concentration of sugar.

X_{v0}= initial concentration of yeast.

The performance index is:

$$J = J_F + \int [J_P + J_M + C] dt \quad (5)$$

$$J_F = \sum a_i P(t_{final}) * V(t_{final}) - \sum b_i S(t_{final}) * V(t_{final})$$

$$J_P = c_1 \frac{P^2}{(1 + \exp((P - \lim_P) / \text{dif}_P))}$$

$$J_M = d_1 \frac{S^2}{(1 + \exp((S - \lim_S) / \text{dif}_S))}$$

$$C = e_1 \frac{F_s^2}{(1 + \exp((F_s - \lim_F) / \text{dif}_F))}$$

where

a, b, c, d and **e** are weights of each compound.

J_F is the final state dependent.

J_P is the product time dependent.

J_M is sugar time dependent.

C is the control component.

lim is the concentration limit.

dif is the diffusivity of concentration limit.

Placing the concentration limits in the performance index helps the solution of the differential equations.

4. Optimal control techniques.

Taking the process differential equations and the performance index below:

$$\frac{dx}{dt} = a(x(t), u(t), t) \quad (6)$$

$$J = h(x(t_f), t_f) + \int g(x(\eta), u(\eta), \eta) d\eta \quad (7)$$

the Hamiltonian can be defined as:

$$H(x(t), u(t), p(t), t) = g(x(t), u(t), t) + p^T(t) a(x(t), u(t), t) \quad (8)$$

where p^T are the Lagrange's multiplier. By using variational calculus it follows:

$$\frac{dx^*}{dt} = \frac{\partial H}{\partial p}(x^*(t), u^*(t), p^*(t), t) \quad (9)$$

$$\frac{dp^*}{dt} = - \frac{\partial H}{\partial x}(x^*(t), u^*(t), p^*(t), t) \quad (10)$$

$$0 = \frac{\partial H}{\partial u}(x^*(t), u^*(t), p^*(t), t) \quad (11)$$

The initial values to costate equation are:

$$p^1(t_f) = dh(x(t_f), t_f) / dP$$

$$p^2(t_f) = dh(x(t_f), t_f) / dS$$

$$p^3(t_f) = dh(x(t_f), t_f) / dX_v$$

$$p^4(t_f) = dh(x(t_f), t_f) / dV$$

where h(x(t_f), t_f) is the final time dependent term of the performance index and * represents optimal values.

The steepest descend algorithms were used to obtain the optimal solution^[1].

5. Genetic algorithms solution.

The evolution and improvement of life occurs through the reproduction, when each individual contributes with its own genetic information building a new one^[2] with fitness to the environment and more surviving chances.

The genetic algorithms^[3] try to reproduce these natural mechanism into the computer software through the steps:

1. definition of relevant process variables.

2. sensibility analysis of each variable to define the ideal length of the alleles.
3. definition of total chromosome length.
4. creation of the randomic initial population that represents the total state space.
5. The performance index of each individual of the population is calculated, measured or searched in a numerical table. These index measure the fitness to the surroundings, and the weights in the reproduction.
- 6- trial of the individuals with best performance index to crossover, and after genetic mutation.
- 7-Calculus of performance index for each new individuals.
- 8- if the population converges, go to step 9 else go to step 6. Some convergence criteria would be defined.
- 9- choose the individuals with best performance and calculate the value of variables from chromosomes.

Some advantages of the genetic algorithms for this application were observed:

- quick convergence to search the best performance measure, when there are lots of states present in the model.

- the control system have to learn with the events. The updating parameters are the best values obtained in the calculation process.

- all possible states are represented by a small number of samples, through the crossover and mutation.

- multiprocessing with parallel execution in real time.

The execution of this algorithms need the polynomial representation of control function, and every parameter of the polinomials are coded into the chromosomes.

The performance index is calculated from the solution of the differential equations through 4th order Runge-Kutta method for each individual

Genetic algorithms is able to converge to the best fitness to maximize the performance index.

6.Conclusions.

In Figure 1 we present the results by the two differents techniques. We can conclude:

- the genetic algorithm attained the global optimum solution. Practically no differences were noted.

- the steepest descent methods have some problems with convergence.

- the deduction of the costate equations and hamiltonian could be very difficult, in general. Genetic algorithms don't need these development.

- the software is modular and structured to allow easy application to any process modelled with differential equations and a performance index.

Bibliografia.

- [1] Donald Kirk; *Optimal control theory*; Prentice-Hall Inc,1970.
- [2] Renato Basile & Luis Edmundo de Magalhães ; *Citologia e genética*; Editora Cultrix (1976)
- [3] D. E. Goldberg *Genetic Algorithms in Search, Optimization, and Machine Learning*; Addison-Wheesley (1989)
- [4] I.Queinnec; B.Dahhou; Optimization and control of a feedbatch fermentation process; *Optimal control applications & methods*, Vol. 15, 175-191 (1994) Wiley.
- [5] B.Valdman; N.Bojorge; E.Valdetaro; Controle adaptativo em uma fermentação alcoolica em batelada alimentada; *Instec*, Set 1992, pag 93
- [6] Sinclair; Kristiansen; *Fermentation kinetics and modeling*; New- York (1987).
- [7] B.McNeil; L.M.Harvey; *Fermentation a practical approah*; O'rl press (1990).
- [8] B.Volesky; J.Votruba; *Modeling and optimization of fermentation process*; Ed. Elsevier (1992).

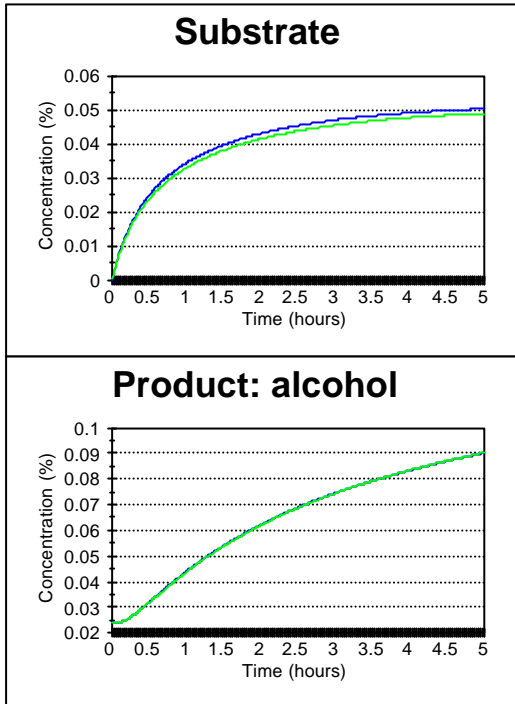


Fig. 1 Comparison between genetic algorithms and optimal control techniques.