ECON 343

Lecture 9: Estimating and interpreting dynamic models of stationary time series

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Outline – Lecture 9

1. Error Correction Models ECM
2. Co-integration and ECM
3. Granger causality test
4. Vector Auto-Regression models (VAR) and Vector Error Correction Models (VECM)

1. Error Correction Models

- Belong to general ADL, (autoregressive distributed lag) models
- We allow for lags in the dependent variable, and also lags of one or more variables used to help predict
- An ADL(2, 2) of changes in the rate of inflation (infl) with changes in the rate of unemployment (unemp) as the lagged X variable:

$$\Delta infl_t = \beta_o + \beta_1 \Delta infl_{t-1} + \beta_2 \Delta infl_{t-2} + \beta_3 \Delta unem_{t-1} + \beta_4 \Delta uenmp_{t-2} + \varepsilon_t$$

$$Y_t = \beta_0 + \sum_{i=1}^{p} \beta_i Y_{t-i} + \sum_{j=1}^{q_1} \delta_{1j} X_{1,t-j} + \ldots + \sum_{l=1}^{q_k} \delta_{kl} X_{k,t-l} + u_t$$

Error Correction Models

- **A general statement for ADLs, with k X variables:**

$$Y_t = \beta_0 + \sum_{i=1}^{p} \beta_i Y_{t-i} + \sum_{j=1}^{q_1} \delta_{1j} X_{1,t-j} + \ldots + \sum_{l=1}^{q_k} \delta_{kl} X_{k,t-l} + u_t$$

- **Under what conditions can we make inference from estimates of this model? <u>Stationarity</u>**

Error Correction Models

- An ADL (1,1) model is written as:

$$y_t = \alpha + \beta x_t + \gamma x_{t-1} + \delta y_{t-1} + e_t$$

- For simplicity, the lower case denotes logarithmic variables
- The error correction model was popularised in the late 1970s by the work of Hendry and others in modelling consumption behaviour
- In such a model, the short-term dynamics of the variables in the system are influenced by deviations from the long-run equilibrium.

Error Correction Models

- The model can be developed by reference to the ADL model's long-run solution, which is given by:

$$y^* = \alpha/(1 - \delta) + (\beta + \gamma)/(1 - \delta)x^*$$

- The long-run effect is $(\beta + \gamma)/(1 - \delta)$
- In economic theory it is useful to posit a long-run proportional relationship between variables
- We could impose a long-run restriction of unity on the relationship between y and x
- This would imply a long-run elasticity of unity between y and x.

Error Correction Models

- **In our case the parameter restriction is:**

$(\beta + \gamma)/(1 - \delta) = 1$ **or** $\beta + \gamma = 1 - \delta$ **or** $\beta + \gamma + \delta = 1$

- **If we return to the original dynamic model described above, ignoring the random error term, we have**

$y_t = \alpha + \beta x_t + \gamma x_{t-1} + \delta y_{t-1}$

and subtracting y_{t-1} **from both sides, we obtain:**

$\Delta y_t = \alpha + \beta x_t + \gamma x_{t-1} - [1 - \delta] y_{t-1}$

Error Correction Models

- **If we now add and subtract βx_{t-1} to and from both sides we obtain:**

$$\Delta y_t = \alpha + \beta\Delta x_t + [\beta + \gamma]x_{t-1} - [1 - \delta]y_{t-1}$$

- **If we now impose the unit restriction**

$\beta + \gamma = 1 - \delta$ we obtain:

$$\Delta y_t = \alpha + \beta\Delta x_t + [1 - \delta]x_{t-1} - [1 - \delta]y_{t-1}$$

and re-arranging:

$$\Delta y_t = \alpha + \beta\Delta x_t - [1 - \delta][y - x]_{t-1}$$

Error Correction Models

- $\Delta y_t = \alpha + \beta\Delta x_t \; - [1 - \delta][y - x]_{t-1}$
- The final term is the error correction mechanism itself
- This ensures that the system under consideration adheres to its long-run equilibrium path.
- Any deviations or errors from it are corrected – hence the term error correction.
- When the equilibrium holds $y - x = 0$, but during periods of disequilibria, when x and y diverge from their equilibrium path, $y - x \neq 0$.
- The estimated coefficient $[1–\delta]$ is interpretable as a speed of adjustment parameter. A large value is indicative of a rapid response of the y variable to any disequilibrium shock.

Error Correction Models

- $\Delta y_t = \alpha + \beta \Delta x_t - [1 - \delta][y - x]_{t-1}$
- In order for convergence to occur δ must be less than 1. If δ is greater than 1, the system is explosive.
- If y > x in the last period, the disequilibria is positive, and y has moved above its steady state.
- The growth in y (Δyt) next period must fall to ensure a movement back to the equilibrium path. Provided $\delta < 1$ this will occur in this model.
- If y < x in the last period, the disequilibria is negative, and y has moved below its steady state. The growth in y (Δyt) next period must rise to ensure a movement back to the equilibrium path. Provided $\delta < 1$ this will also occur in this model. The condition $\delta < 1$ is necessary to ensure a stable model.

Error Correction Models

- $\Delta y_t = \alpha + \beta\Delta x_t - [1-\delta][y - x]_{t-1}$
- Equations like this were estimated extensively in the late 1970s and early 1980s on various data sets (e.g., consumption, imports, demand for money) to facilitate dynamic modelling
- OLS was invariably used in estimation but the problem was the error correction term itself
- unless the variables are stationary, the error correction term cannot be viewed as a stationary variable

2. Co-integration and ECMs

$$\Delta y_t = \alpha + \beta\Delta x_t - [1 - \delta][y - x]_{t-1} + v_t$$

$$\Delta y_t = \alpha + \beta\Delta x_t - \phi[y - x]_{t-1} + v_t$$

where ϕ is the speed of adjustment parameter.

- If yt and xt are integrated of order one (i.e., I(1)), and their differences are integrated of order zero. This does not represent too much of a problem.
- The problem comes in terms of the error correction mechanism variable [y – x].
- The only circumstances in which the [y – x] variable is stationary is if the two variables comprise a cointegrating vector and the resultant linear combination is I(0). This would mean that the ECM model is ‘balanced’ and contains a set of variables that are all integrated of the same order.

Co-integration and ECMs

$\Delta y_t = \alpha + \beta \Delta x_t - \phi[y - x]_{t-1} + v_t$

- Granger's representation theorem states that if two (or more) variables are cointegrated, then there exists an error correction mechanism model representation for their relationship
- Indeed, if two variables are not cointegrated, then there cannot be a long-run relationship between the series and there can be no mechanism to ensure coherence to any long-run relationship, since one cannot exist. This then implies that ϕ=0. In other words:
- Ho: $\phi = 0$ [non-cointegration]
- Ha: $\phi < 0$ [cointegration]

Engle-Granger twp o stage procedure (Lecture 7)

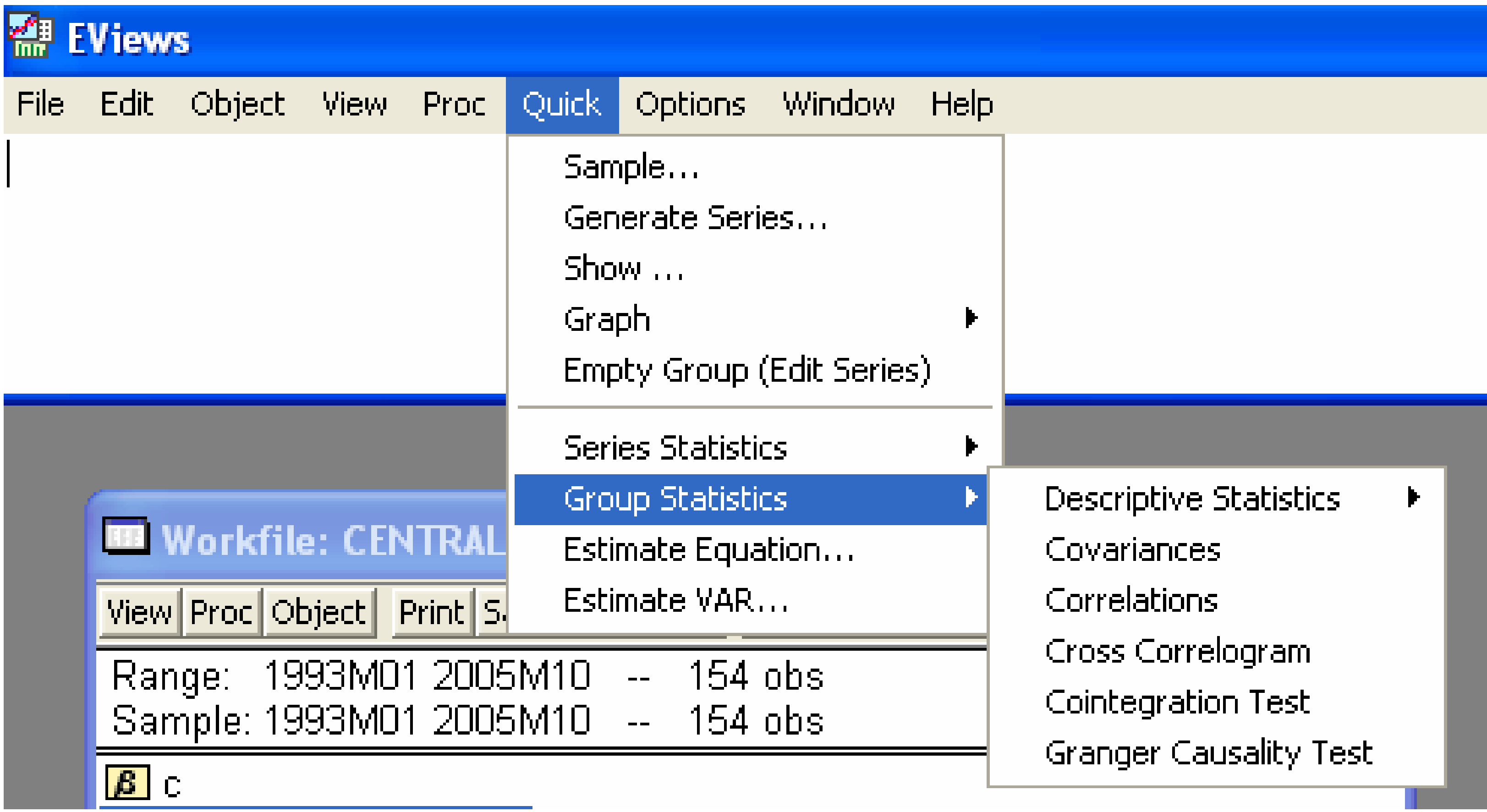
3. Granger Causality tests

- **The Autoregressive Distributed Lags ADL model forms the basis of the Granger causality test:**
- **If *X* and *Y* are stationary, then F-tests of lags of *X* in the *Y* equation and *Y* in the *X* equation can be thought of as testing the temporal causality, or, better, 'predictability'**

$$Y_t = \beta_0 + \sum_{i=1}^{p} \beta_i Y_{t-i} + \sum_{j=1}^{q} \delta_j X_{t-j} + u_{1t}$$

$$X_t = \beta_0 + \sum_{i=1}^{p} \beta_i Y_{t-i} + \sum_{j=1}^{q} \delta_j X_{t-j} + u_{2t}$$

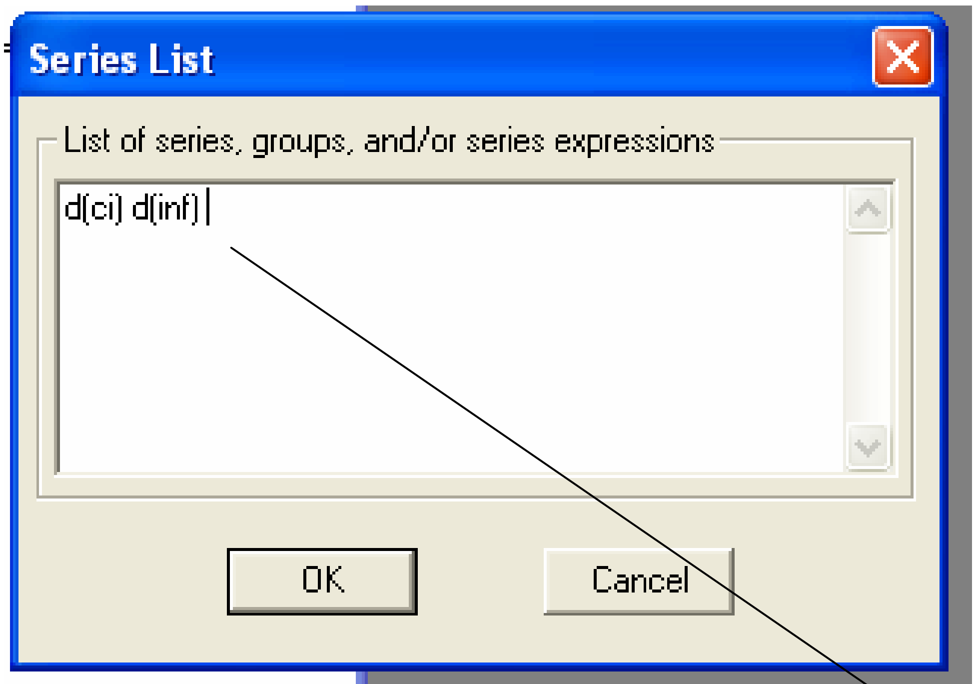
Granger Causality tests

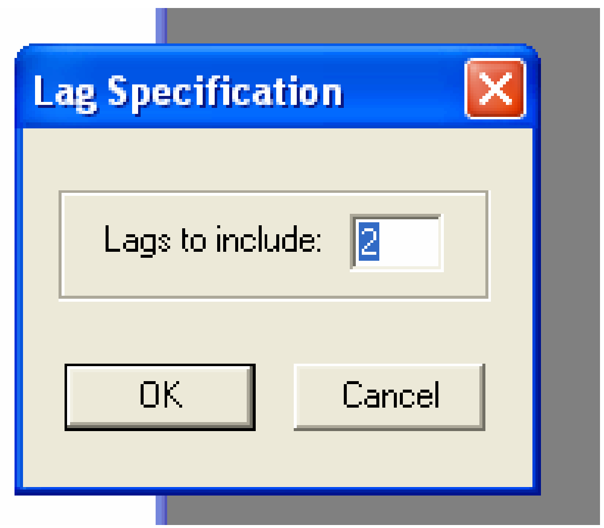


Granger Causality tests

Lebanese inflation rate and BDL Coincident Indicator

Does inflation Granger Cause the proxy for output?





First difference, you should know why

Granger Causality tests

Does inflation Granger Cause the proxy for output? YES

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Pairwise Granger Causality Tests
Date: 04/12/06 Time: 11:17
Sample: 1993M01 2005M10
Lags: 2

Null Hypothesis:	Obs	F-Statistic	Probability	
D(INF) does not Granger Cause D(CI)	141	4.93410	0.00854	→ **reject**
D(CI) does not Granger Cause D(INF)		1.63061	0.19961	→ **accept**

4. VAR and VECM models

- **A univariate autoregression is a single-equation, single-variable linear model in which the current value of a variable is explained by its own lagged values**
- **A VAR is an n-equation, n-variable linear model in which each variable is in turn explained by its own lagged values, plus current and past values of the remaining n - 1 variables.**
- **This simple framework provides a systematic way to capture rich dynamics in multiple time series, and the statistical toolkit that came with VARs was easy to use and to interpret**
- **If you have a number of stationary series that are likely to be interrelated, then it possible to estimate a set of ADL models, one for each of the series**

Inflation, oil and Composite Indicator : Granger Causality

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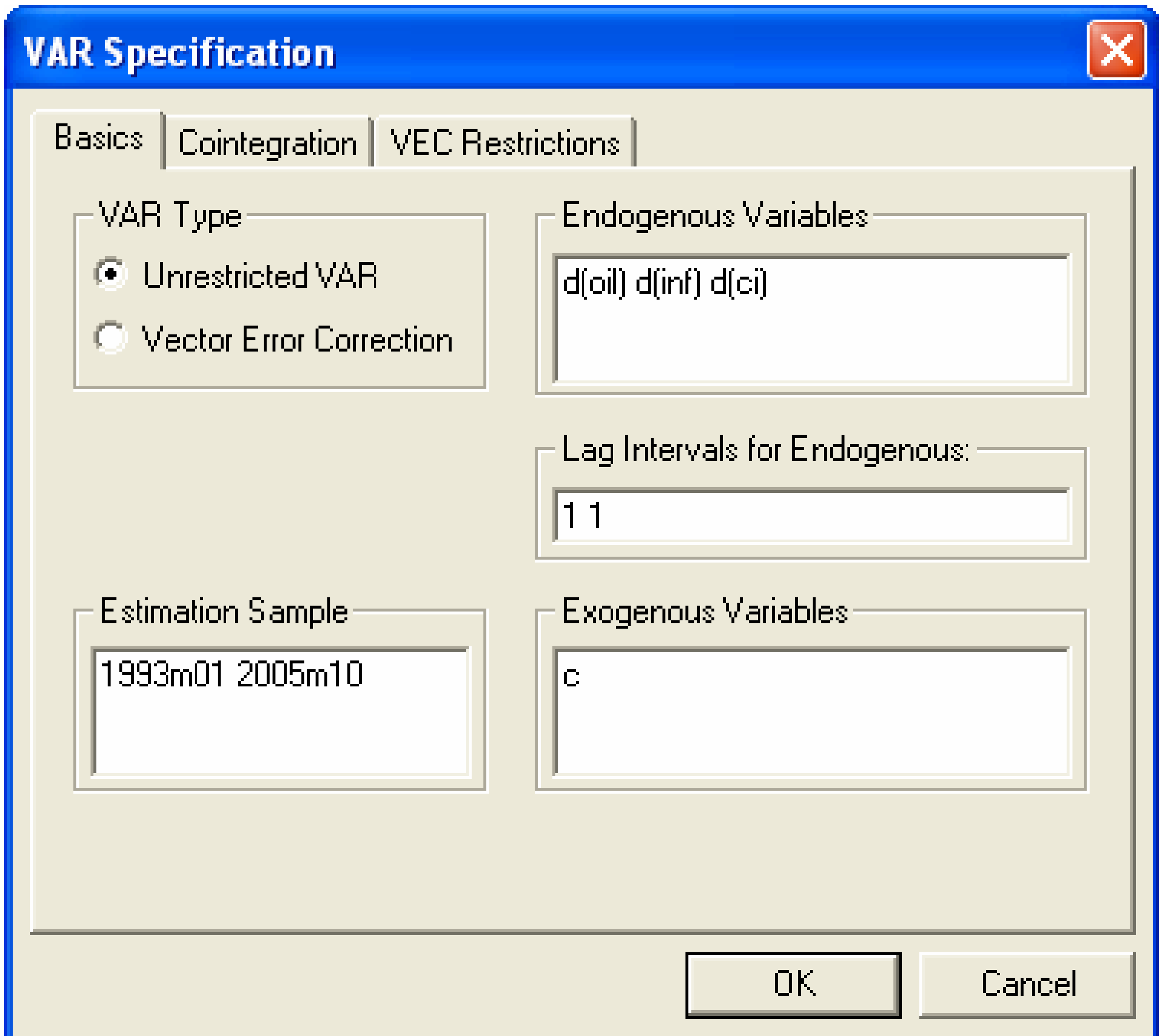
Pairwise Granger Causality Tests
Date: 04/12/06 Time: 12:00
Sample: 1993M01 2005M10
Lags: 1

Null Hypothesis:	Obs	F-Statistic	Probability
D(OIL) does not Granger Cause D(CI)	152	5.89322	0.01639
D(CI) does not Granger Cause D(OIL)		1.22184	0.27078
D(INF) does not Granger Cause D(CI)	142	10.2679	0.00168
D(CI) does not Granger Cause D(INF)		2.73380	0.10050
D(INF) does not Granger Cause D(OIL)	142	0.18775	0.66547
D(OIL) does not Granger Cause D(INF)		0.04484	0.83261

Inflation, oil and Composite Indicator: VAR



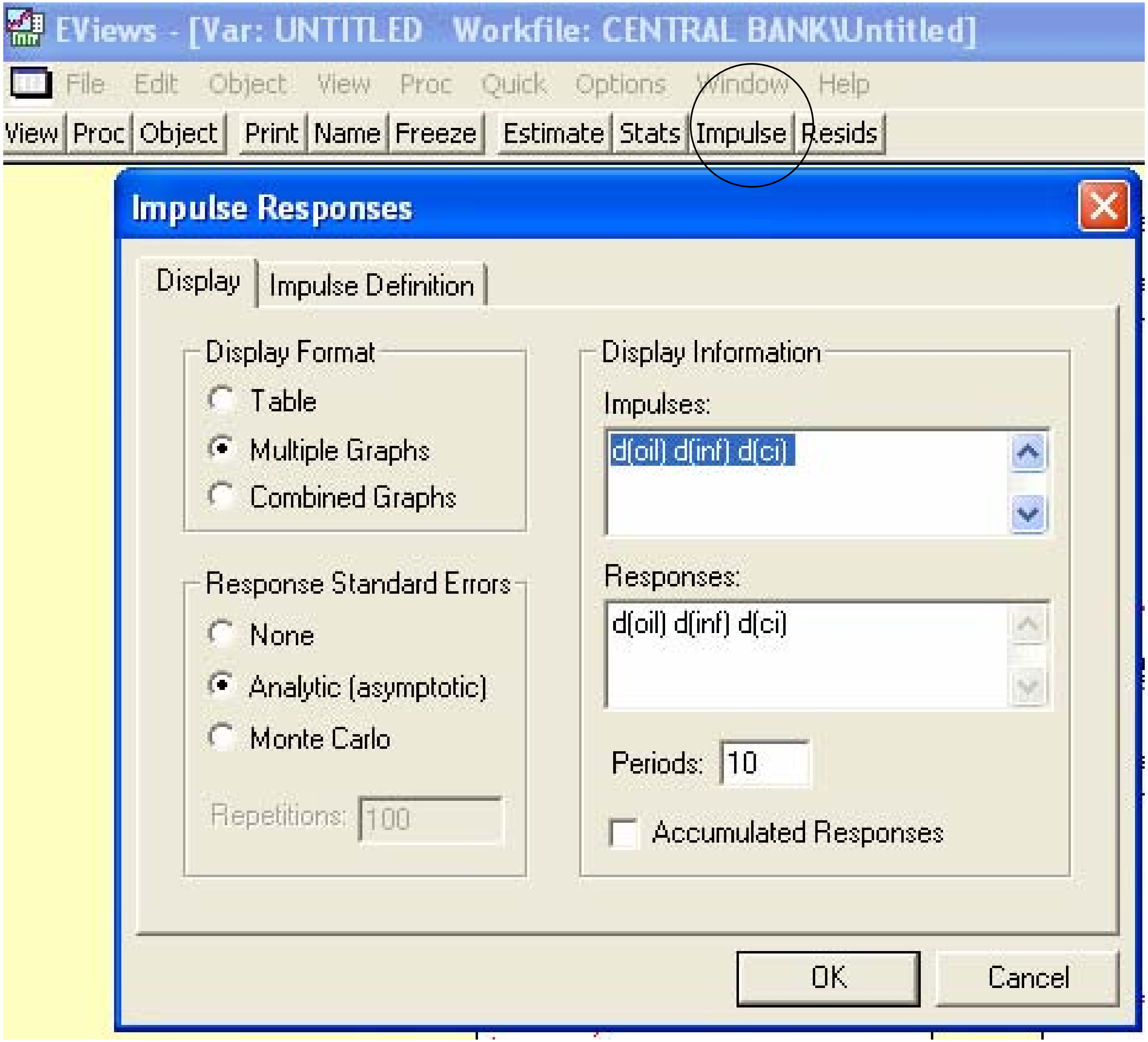
Vector Autoregression Estimates
Date: 04/12/06 Time: 11:57
Sample (adjusted): 1993M03 2004M12
Included observations: 142 after adjustments
Standard errors in ( ) & t-statistics in [ ]

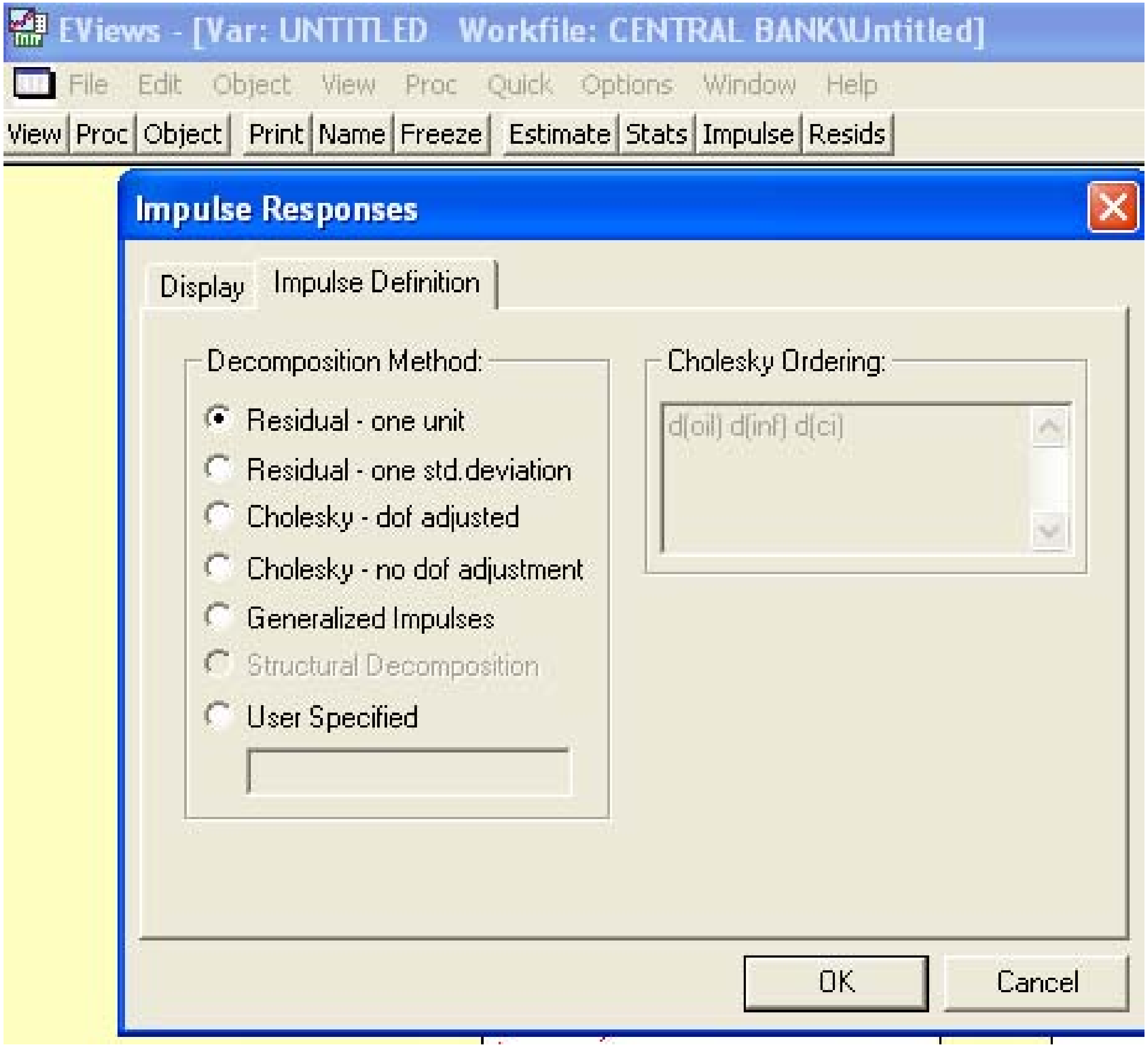
	D(OIL)	D(INF)	D(CI)
D(OIL(-1))	0.064879	0.131873	-0.421204
	(0.08657)	(1.02077)	(0.17645)
	[0.74941]	[0.12919]	[-2.38713]
D(INF(-1))	-0.001774	0.174784	-0.048331
	(0.00712)	(0.08391)	(0.01450)
	[-0.24930]	[2.08312]	[-3.33236]
D(CI(-1))	0.055584	0.688665	-0.496462
	(0.03564)	(0.42022)	(0.07264)
	[1.55962]	[1.63883]	[-6.83478]
C	0.130981	2.954022	1.092501
	(0.16720)	(1.97139)	(0.34077)
	[0.78339]	[1.49844]	[3.20598]
R-squared	0.023630	0.043857	0.303709
Adj. R-squared	0.002405	0.023072	0.288572
Sum sq. resids	516.7992	71846.01	2146.738
S.E. equation	1.935180	22.81717	3.944119
F-statistic	1.113301	2.109979	20.06433
Log likelihood	-293.2090	-643.5674	-394.3166
Akaike AIC	4.186042	9.120668	5.610093
Schwarz SC	4.269305	9.203931	5.693355
Mean dependent	0.163923	4.028169	0.568451
S.D. dependent	1.937511	23.08503	4.676108

Inflation, oil and Composite Indicator: Impulse responses

- *Impulse responses* trace out the response of current and future values of each of the variables to a one-unit increase in the current value of one of the VAR errors, assuming that this error returns to zero in subsequent periods and that all other errors are equal to zero

- The implied thought experiment of changing one error while holding the others constant makes most sense when the errors are uncorrelated across equations


EViews - [Var: UNTITLED Workfile: CENTRAL BANK\Untitled]
File Edit Object View Proc Quick Options Window Help
View Proc Object Print Name Freeze Estimate Stats Impulse Resids
Impulse Responses
Display
Impulse Definition
Display Format
Table
Multiple Graphs
Combined Graphs
Response Standard Errors
None
Analytic (asymptotic)
Monte Carlo
Repetitions: 100
Display Information
Impulses:
d(oil) d(inf) d(ci)
Responses:
d(oil) d(inf) d(ci)
Periods: 10
Accumulated Responses
OK
Cancel


EViews - [Var: UNTITLED Workfile: CENTRAL BANK\Untitled]
File Edit Object View Proc Quick Options Window Help
View Proc Object Print Name Freeze Estimate Stats Impulse Resids
Impulse Responses
Display
Impulse Definition
Decomposition Method:
Residual - one unit
Residual - one std.deviation
Cholesky - dof adjusted
Cholesky - no dof adjustment
Generalized Impulses
Structural Decomposition
User Specified
Cholesky Ordering:
d(oil) d(inf) d(ci)
OK
Cancel

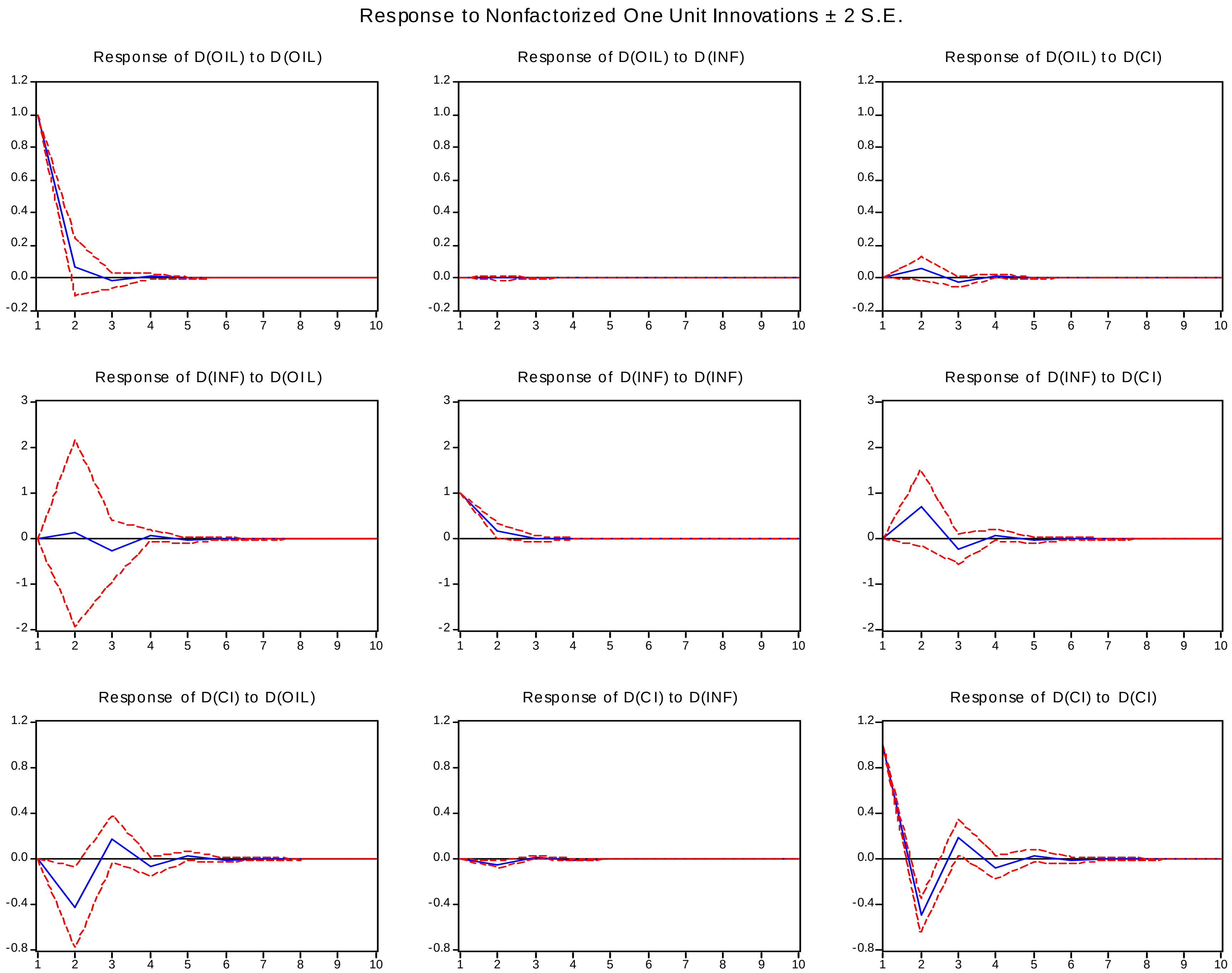
Some options for transforming the impulses:

- **Residual-One Unit: ignores the units of measurement and the correlations in the VAR residuals so that no transformation is performed.**

- **Residual-One Std. Dev. sets the impulses to one standard deviation of the residuals. This option ignores the correlations in the VAR residuals.**

- **Cholesky uses the inverse of the Cholesky factor of the residual covariance matrix to orthogonalize the impulses. This option imposes an ordering of the variables in the VAR and attributes all of the effect of any common component to the variable that comes first in the VAR system. Note that responses can change dramatically if you change the ordering of the variables. You may specify a different VAR ordering by reordering the variables in the Cholesky Ordering edit box.**

- **Generalized Impulses as described by Pesaran and Shin (1998) constructs an orthogonal set of innovations that does not depend on the VAR ordering. The generalized impulse responses from an innovation to the j-th variable are derived by applying a variable specific Cholesky factor computed with the j-th variable at the top of the Cholesky ordering.**


Response to Nonfactorized One Unit Innovations ± 2 S.E.
Response of D(OIL) to D(OIL)
Response of D(OIL) to D(INF)
Response of D(OIL) to D(CI)
Response of D(INF) to D(OIL)
Response of D(INF) to D(INF)
Response of D(INF) to D(CI)
Response of D(CI) to D(OIL)
Response of D(CI) to D(INF)
Response of D(CI) to D(CI)

4. VECM models

A vector error correction (VEC) model is a restricted VAR designed for use with nonstationary series that are known to be cointegrated. The VEC has cointegration relations built into the specification so that it restricts the long-run behavior of the endogenous variables to converge to their cointegrating relationships while allowing for short-run adjustment dynamics. The cointegration term is known as the *error correction* term since the deviation from long-run equilibrium is corrected gradually through a series of partial short-run adjustments.

To take the simplest possible example, consider a two variable system with one cointegrating equation and no lagged difference terms. The cointegrating equation is:

$$y_{2,t} = \beta y_{1,t} \tag{24.27}$$

The corresponding VEC model is:

$$\begin{aligned} \Delta y_{1,t} &= \alpha_1 (y_{2,t-1} - \beta y_{1,t-1}) + \epsilon_{1,t} \\ \Delta y_{2,t} &= \alpha_2 (y_{2,t-1} - \beta y_{1,t-1}) + \epsilon_{2,t} \end{aligned} \tag{24.28}$$

In this simple model, the only right-hand side variable is the error correction term. In long run equilibrium, this term is zero. However, if y_1 and y_2 deviate from the long run equilibrium, the error correction term will be nonzero and each variable adjusts to partially restore the equilibrium relation. The coefficient α_i measures the speed of adjustment of the *i*-th endogenous variable towards the equilibrium.

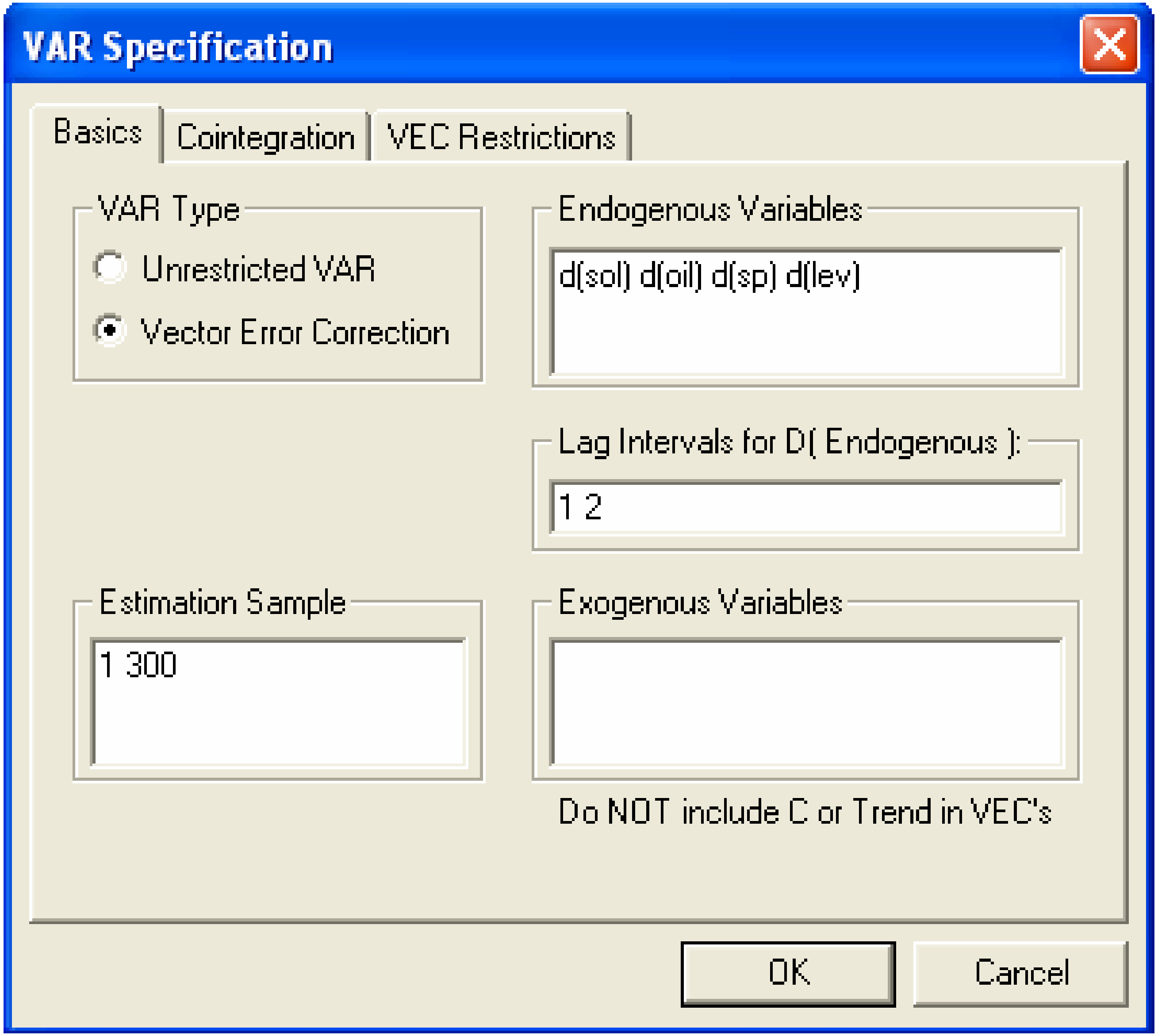
Johansen Cointegration Test

Date: 04/12/06 Time: 12:24
Sample (adjusted): 1993M07 2004M12
Included observations: 138 after adjustments
Trend assumption: Linear deterministic trend
Series: D(CI) D(OIL) D(INF)
Lags interval (in first differences): 1 to 4

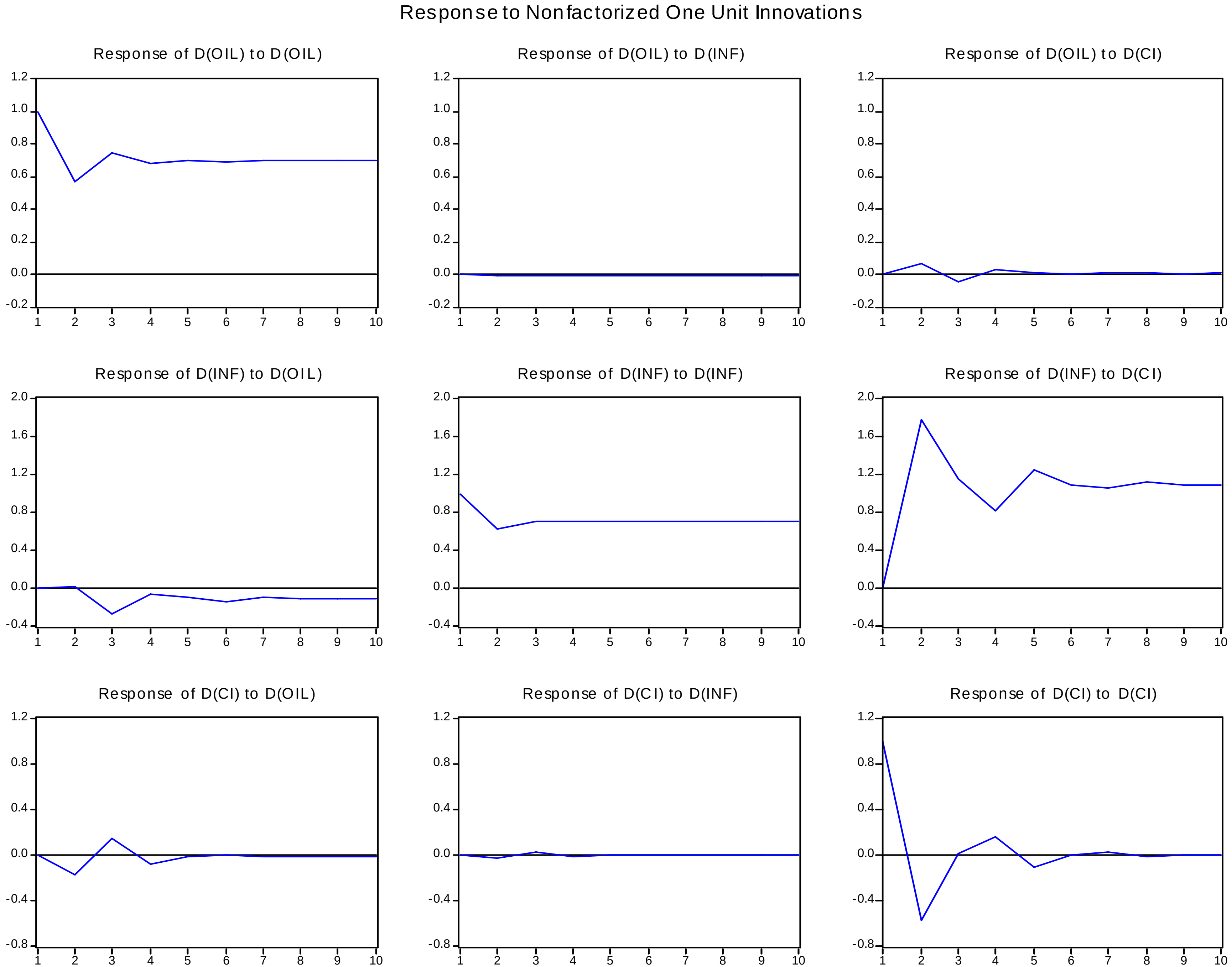
Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None *	0.307143	102.6223	29.79707	0.0000
At most 1 *	0.221945	51.98576	15.49471	0.0000
At most 2 *	0.118165	17.35347	3.841466	0.0000

Trace test indicates 3 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values


VAR Specification
Basics
Cointegration
VEC Restrictions
VAR Type
Unrestricted VAR
Vector Error Correction
Endogenous Variables
d(sol) d(oil) d(sp) d(lev)
Lag Intervals for D(Endogenous):
1 2
Estimation Sample
1 300
Exogenous Variables
Do NOT include C or Trend in VEC's
OK
Cancel

Error Correction:	D(OIL,2)	D(INF,2)	D(CI,2)
CointEq1	0.000623	0.040380	-0.026050
	(0.00125)	(0.01476)	(0.00205)
	[0.49900]	[2.73528]	[-12.6905]
D(OIL(-1),2)	-0.436389	-0.027899	-0.147680
	(0.07817)	(0.92484)	(0.12860)
	[-5.58242]	[-0.03017]	[-1.14840]
D(INF(-1),2)	-0.012034	-0.368004	-0.029374
	(0.00681)	(0.08061)	(0.01121)
	[-1.76616]	[-4.56511]	[-2.62061]
D(CI(-1),2)	0.021347	-1.042044	0.246240
	(0.05070)	(0.59980)	(0.08340)
	[0.42105]	[-1.73733]	[2.95253]
C	-0.055407	-0.061325	0.090669
	(0.19831)	(2.34618)	(0.32623)
	[-0.27940]	[-0.02614]	[0.27793]
R-squared	0.212350	0.165780	0.771877
Adj. R-squared	0.189183	0.141244	0.765168
Sum sq. resids	753.8821	105519.7	2040.106
S.E. equation	2.354411	27.85462	3.873084
F-statistic	9.166356	6.756630	115.0427
Log likelihood	-318.2619	-666.6318	-388.4461
Akaike AIC	4.585275	9.526692	5.580796
Schwarz SC	4.689841	9.631258	5.685362
Mean dependent	-0.038468	-0.028369	0.089722
S.D. dependent	2.614694	30.05816	7.992419


Response to Nonfactorized One Unit Innovations
Response of D(OIL) to D(OIL)
Response of D(OIL) to D(INF)
Response of D(OIL) to D(CI)
Response of D(INF) to D(OIL)
Response of D(INF) to D(INF)
Response of D(INF) to D(CI)
Response of D(CI) to D(OIL)
Response of D(CI) to D(INF)
Response of D(CI) to D(CI)
1.2
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
2.0
1.6
1.2
0.8
0.4
0.0
-0.4
-0.8
1 2 3 4 5 6 7 8 9 10