ECON 343
Lecture 3 : The Capital Asset Pricing Model (CAPM)

Jad Chaaban

Spring 2005-2006

Outline – Lecture 3

- **PART I**
  - **Overview of the Capital Asset Pricing Model (CAPM)**
  - **CAPM: An empirical perspective**
- **PART II**
  - **Time series application of CAPM**
  - **Empirical application using Eviews: Estimating the market Beta for SOLIDERE's stock**

PART I

Public companies - Lebanon

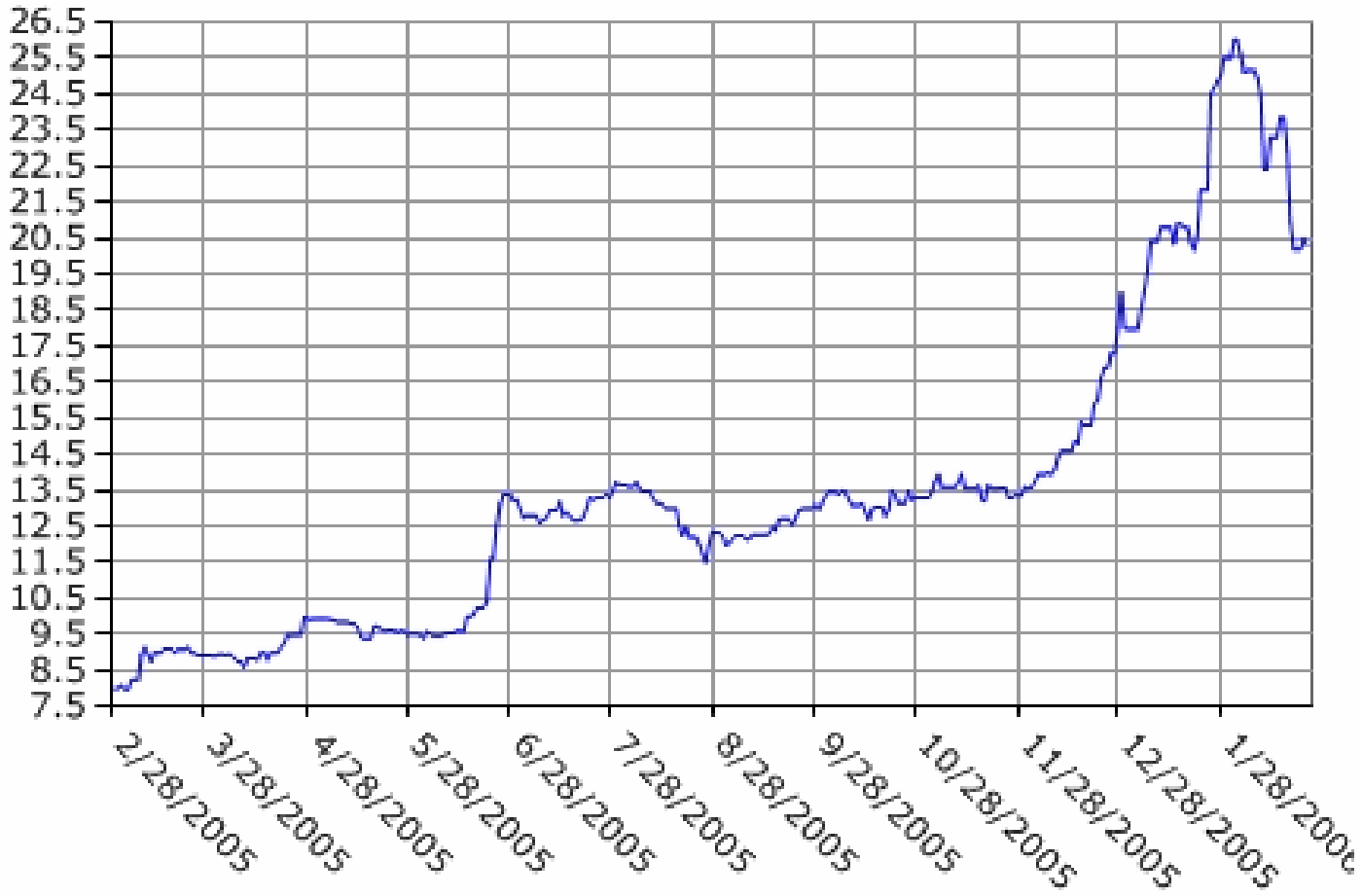
Index | Companies | News | Currency

Bahrain | Egypt | Jordan | Kuwait | Lebanon | Oman | Palestine | Qatar | Saudi Arabia | UAE

Company Name	Closing	%Change	Issued Shares	Market Cap
Bank of Beirut	12.5	▼ -1.96%	27,600,000	345,000,000
Banque Audi	78	▼ -11.26%	16,860,000	1,315,080,000
Banque Libanaise pour le Commerce	10.12	- 0.00%	10,080,000	102,009,600
BEMO	5.75	- 0.00%	16,000,000	92,000,000
BLOM Bank	85.5	▼ -11.40%	18,500,000	1,581,750,000
Bou Khalil Markets	1.75	- 0.00%	4,000,000	7,000,000
Byblos Bank	3.05	▼ -17.57%	204,898,162	624,939,394
Ciments Blancs - B	2.05	▼ -6.82%	9,000,000	18,450,000
Ciments Libanais	2.6	▼ -13.33%	234,192,509	608,900,523
Eternit	0.593	- 0.00%	9,187,500	5,448,188
Lebanon Holdings	4.75	- 0.00%	5,000,000	23,750,000
RYMCO	1.1	- 0.00%	25,000,000	27,500,000
Solidere	20.5	▼ -14.05%	165,000,000	3,382,500,000
Uniceramic	1.4	- 0.00%	11,000,000	15,400,000

Currency: US Dollars - USD

Stock Performance



Market Information

	Stock	Lebanon Index
Last Closing	USD 20.3	2326.96
YTD %	▲ 12.84%	▲ 19.06%
Weekly %	▼ -14.885%	▼ -13.57%
Year High	USD 26	2874.46
Year Low	USD 17.99	1954.47
52 Week High	USD 26	2874.46
52 Week Low	USD 7.86	857.12
BETA	1.1604	
Outstanding Shares	165,000,000	
Market Cap	USD 3,349,500,000	
Market Cap in $	USD $3,355,350,000	
% of Lebanon Market	25.35%	
% of Sector	60.50%	
Rank by Cap	3	

Overview of CAPM

- The capital asset pricing model (CAPM) of William Sharpe (1964) and John Lintner (1965) marks the birth of asset pricing theory (resulting in a Nobel Prize for Sharpe in 1990)
- Mainly used for:
  - estimating the cost of capital for firms
  - evaluating the performance of managed portfolios
  - evaluating the volatility of a given stock
- A simple framework for relating risk and expected returns of assets
- Theoretical appeal, but poor empirical performance

Assumptions of the model

- (A) Asset markets are in equilibrium.
  - A.1 Frictionless markets
  - A.2 Unlimited borrowing or lending at the risk-free interest rate
  - A.3 Divisible assets
  - A.4 Complete markets
  - A.5 Investors are price-takers
  - A.6 Taxes are zero
- (B) Mean-Variance portfolio selection
  - B.1 One-period investment horizon
  - B.2 Mean-Variance objective
- (C) Homogeneous beliefs: investors agree about means and variances

Logic of the model

- An investor selects a portfolio at time *t-1* that produces a stochastic return at *t*
- Investors are risk averse and, when choosing among portfolios, they care only about the mean and variance of their one-period investment return
- Investors choose "mean variance-efficient" portfolios, in the sense that the portfolios:

  1) minimize the variance of portfolio return, given expected return,

  2) maximize expected return, given variance

Some notations

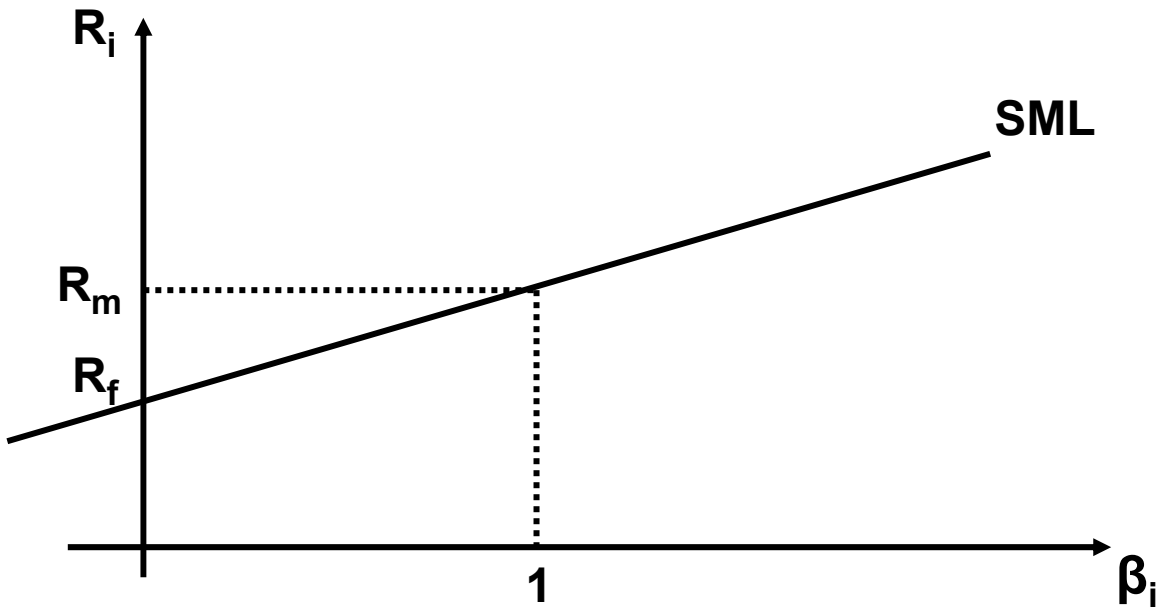
- R_i : return on asset *i*
- R_f: risk-free interest rate
- R_m: market rate of return

- CAPM relationship:

$$E(R_i) = R_f + [E(R_M) - R_f)]\beta_{iM},\ i = 1, \ldots, N.$$

- The expected return on any asset *i* is the risk-free interest rate, R_f, plus a risk premium, which is the asset's market beta, times the premium per unit of beta risk, $E(R_M)$-R_f

The Security Market Line (SML)



The model predicts that the average rates of return and beta coefficients for all assets, and all portfolios of assets, will be located along the SML

The Beta coefficient

- **The market Beta of asset *i*, is the covariance of its return with the market return divided by the variance of the market return**

$$\beta_{iM} = \frac{\text{cov}(R_i, R_M)}{\sigma^2(R_M)}$$

- **Since the market beta of asset *i* is also the slope in the regression of its return on the market return, a common (and correct) interpretation of beta is that it measures the sensitivity of the asset's return to variation in the market return**
- **The SML estimated via Ordinary Least Squares (OLS)**

CAPM: An empirical perspective

- Two aspects of empirical evaluation:
  1. Test whether theory should be rejected
  2. Provide information to aid financial decisions

- Methodology of the CAPM tests:
  1. Cross section: Examine average returns and beta coefficients
  2. Time series: Estimate beta-coefficients, one for each asset

CAPM: Cross-section test

- Estimating the Security Market Line SML
- SML: $R_i - R_f = (R_M - R_f)\beta_i$
- Define sample averages of excess returns:

$$\bar{z}_i = \overline{R}_i - \overline{R}_f$$

- These are the observed counterparts of $R_i - R_f$
- Let $\hat{\beta}_i$ represent the observed counterpart of β_i
- The cross-section empirical CAPM is now:

$$\bar{z}_i = \gamma_0 + \gamma_1 \hat{\beta}_i + \eta_i$$

where γ_0 and γ_1 are unobserved parameters and η_i is an unobserved random error, or disturbance, term

CAPM: Cross-section test

- CAPM predicts that $\gamma_0 = 0$ and that γ_1 equals the risk premium on the market portfolio
- The latter can be estimated by

$$\overline{R}_M - \overline{R}_f$$

- Empirical validation of CAPM:
  - Cross section evidence is mixed. Some studies find a positively sloped Security Market Line
  - Others cannot reject that the SML is flat
  - Most studies find that variables other than beta-coefficients are correlated with asset returns in cross sections

CAPM: Time-series test

- **Empirical model:** $R_{it} - R_{ft} = \alpha_i + (R_{Mt} - R_{ft})\beta_i + \varepsilon_{it}$
- **Define:** $z_{it} = R_{it} - R_{ft}$ **and** $z_{Mt} = R_{Mt} - R_{ft}$
- **Form the simple regression:**

$$z_{it} = \alpha_i + z_{Mt}\beta_i + \varepsilon_{it}$$

- **Assumption:**

$$E\left[\varepsilon_{it} \middle| z_{Mt}\right] = 0$$

- **CAPM implies that $\alpha_i = 0$ for all assets**
- **Test whether $\alpha_i = 0$ and whether the empirical model is compatible with the data**

CAPM: Time-series test

- **Fischer Black (1972) develops a version of the CAPM without risk-free borrowing or lending**
- **The Black version of the CAPM:**

$$\mu_i = \omega + (\mu_M - \omega)\beta_i$$

- **Empirical counterpart:**

$$R_{it} = \alpha_i + R_{Mt}\beta_i + \varepsilon_{it}$$

where

$$\alpha_i = \omega(1 - \beta_i)$$

The intercept (Jensen's alpha) measures abnormal performance

The Beta coefficient

- The market Beta answers the question: how much volatility can you expect from a given stock?
  - If a stock has a beta of 1, for instance, it means that over the past 60 months its price has gained 10% every time the market index has moved up 10%. It has also declined 10% on average when market declines the same amount. In other words, the price tends to move in synch with the S&P, and it is considered a relatively steady stock
- The more risky a stock is, the more its beta moves upward
  - A figure of 2.5 means a gain or loss of 25% every time the market gains or loses just 10%. Likewise, a beta of 0.7 means the stock moves just 7% when the index moves in either direction
- A low-beta stock will protect you in a general downturn, a high Beta means the potential for outsize rewards in an upturn.

PART II

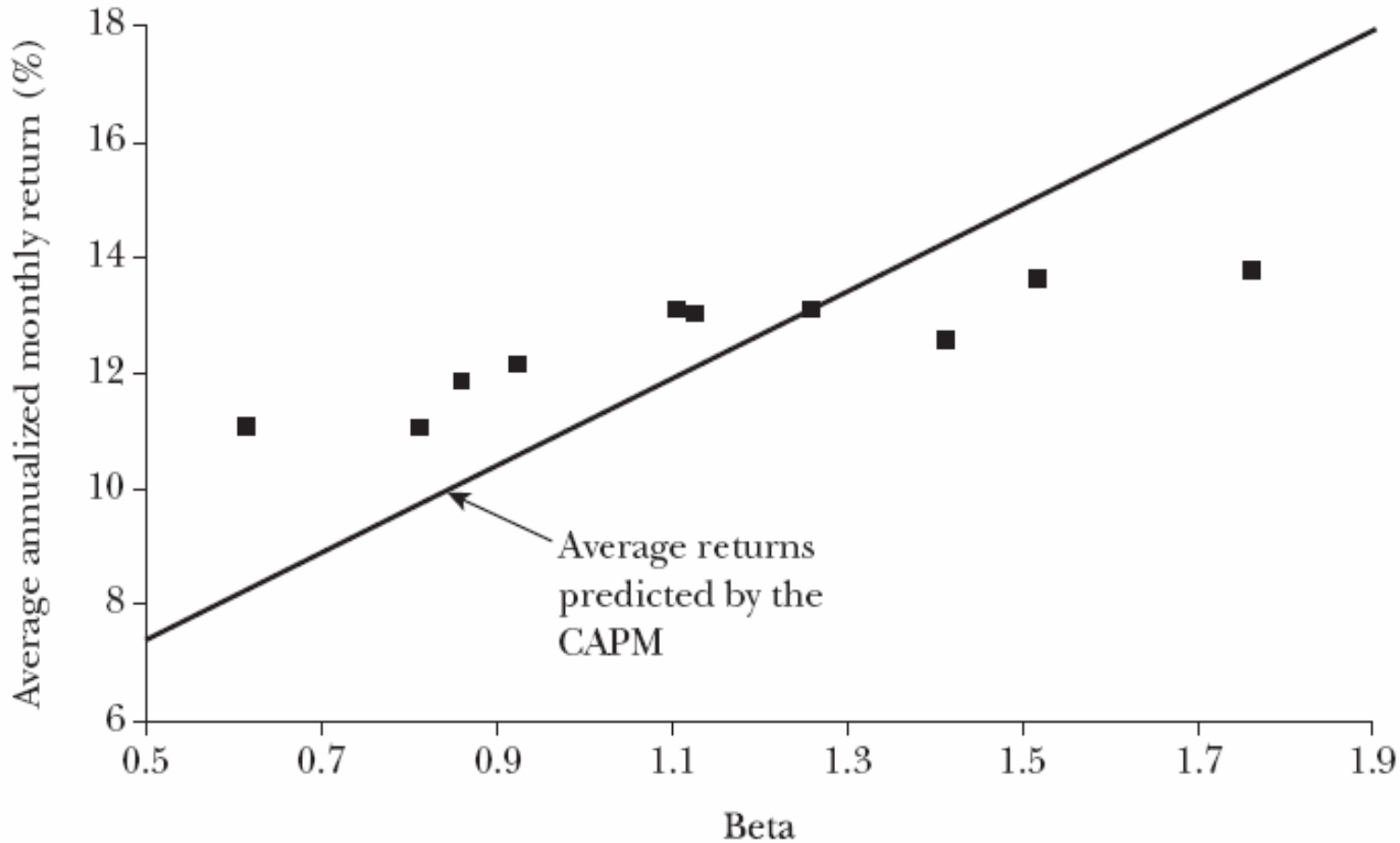
Issues in empirical implementation

- Risk-free rate R_f: average risk-free rate (typically proxied as the return on a one-month Treasury bill)
- Market portfolio return R_M
  - Traded financial assets
  - Depends on the geographic location (S&P 500 for US, CAC 40 for France, …)

- Time measurement of returns: daily, weekly, monthly, annually
- Depends on the frequency of observations and the behavior of investors

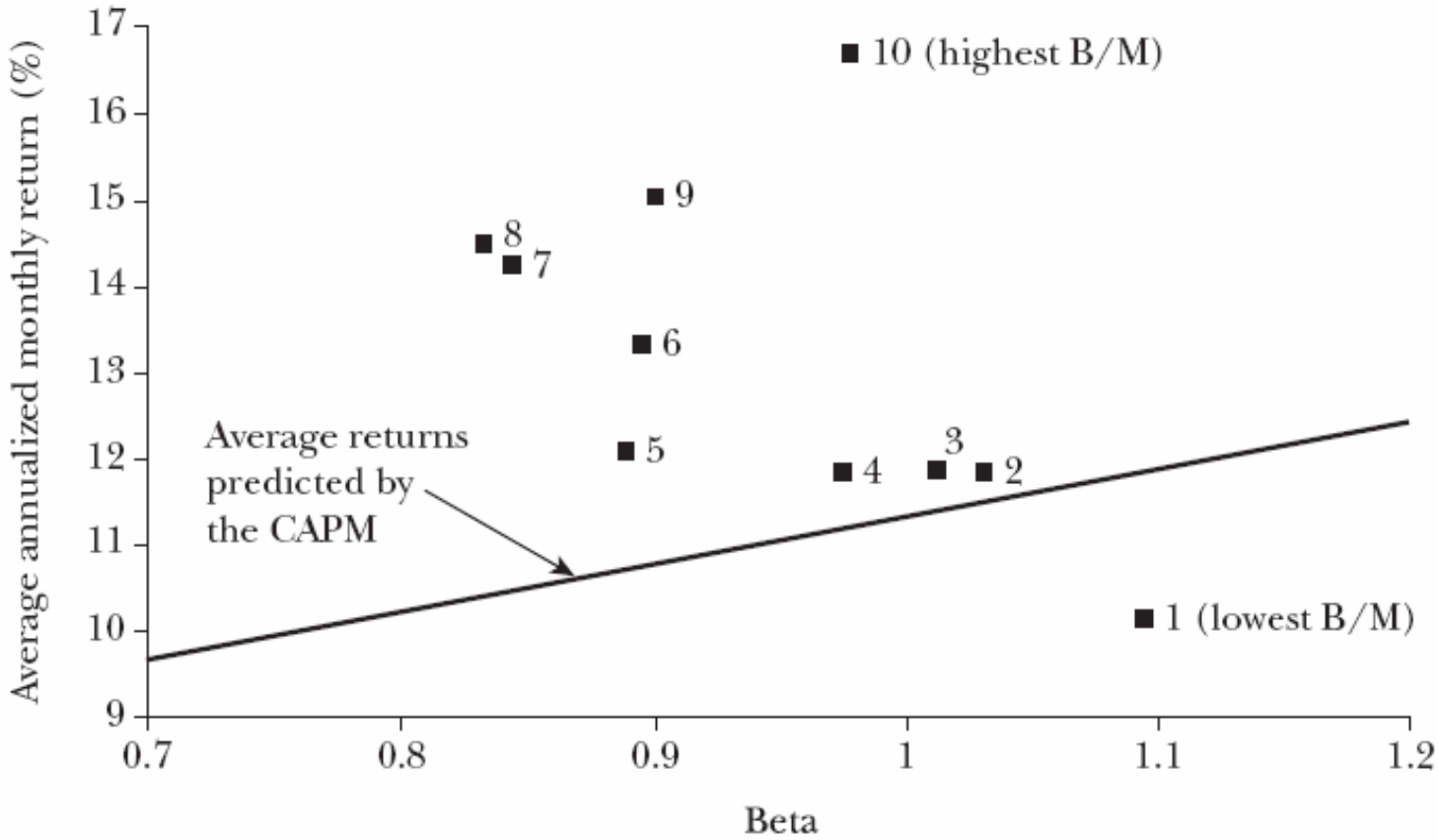
Actual relationship flatter than the predicted one - historical

Average Annualized Monthly Return versus Beta for Value Weight Portfolios Formed on Prior Beta, 1928–2003



Actual relationship flatter than the predicted one – Book/Market ratio

Average Annualized Monthly Return versus Beta for Value Weight Portfolios Formed on B/M, 1963–2003



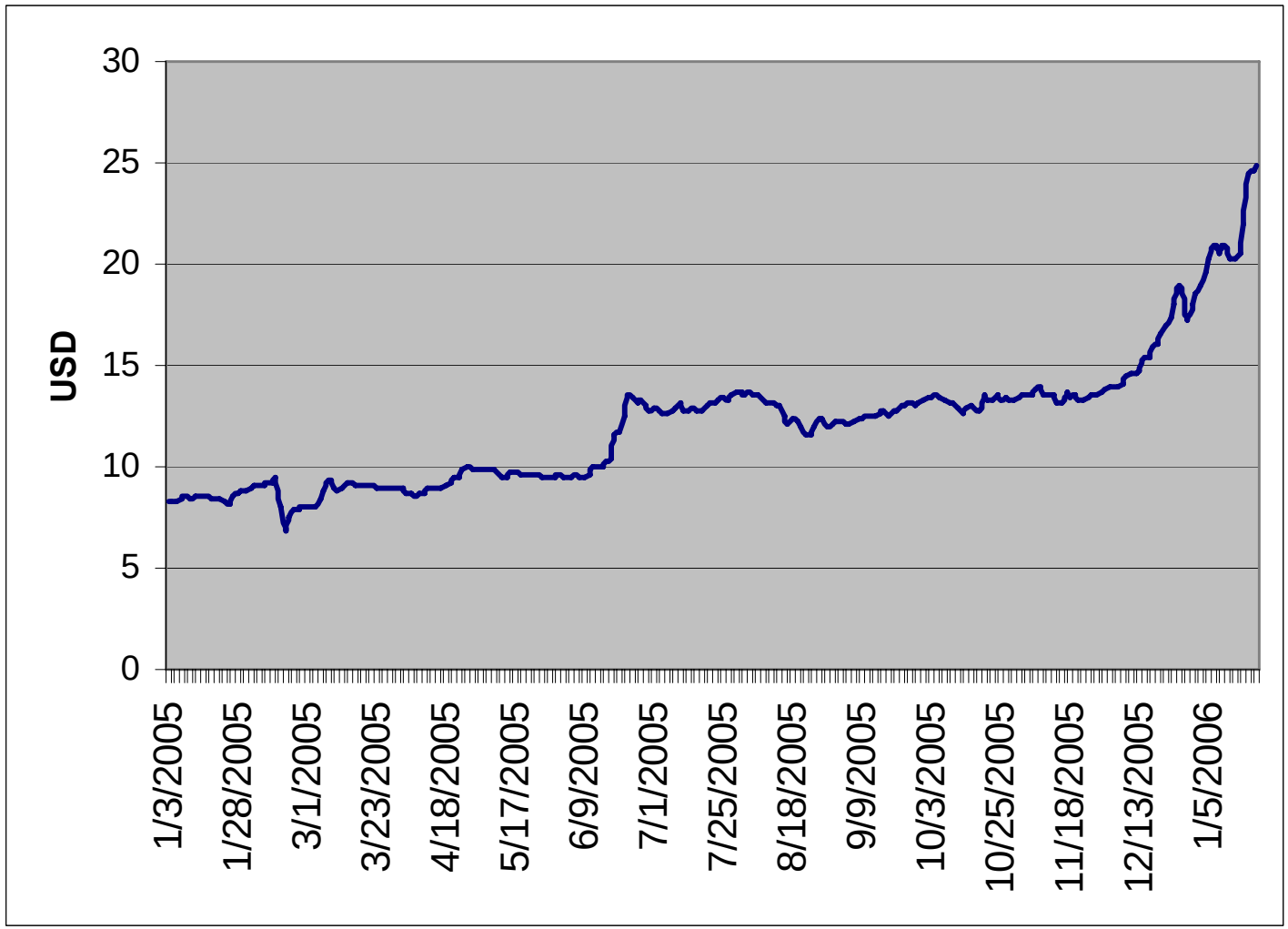
Application: SOLIDERE

- Questions to be answered:
  - How much volatility can we expect from SOLIDERE stock?
  - Is the SOLIDERE stock 'beating the market' (yielding abnormal returns)?
- Between January 2005 and January 2006: 130% increase in *SOLIDERE A* stock value
- *S&P 500 index* only increase 5% over the same period
- The *SC ARAB index* increased by 95% over the same period
- Oil price boom → high regional liquidity → increased financial investments in the region
- Post 9/11 restrictions on Arab capital mobility favored the region

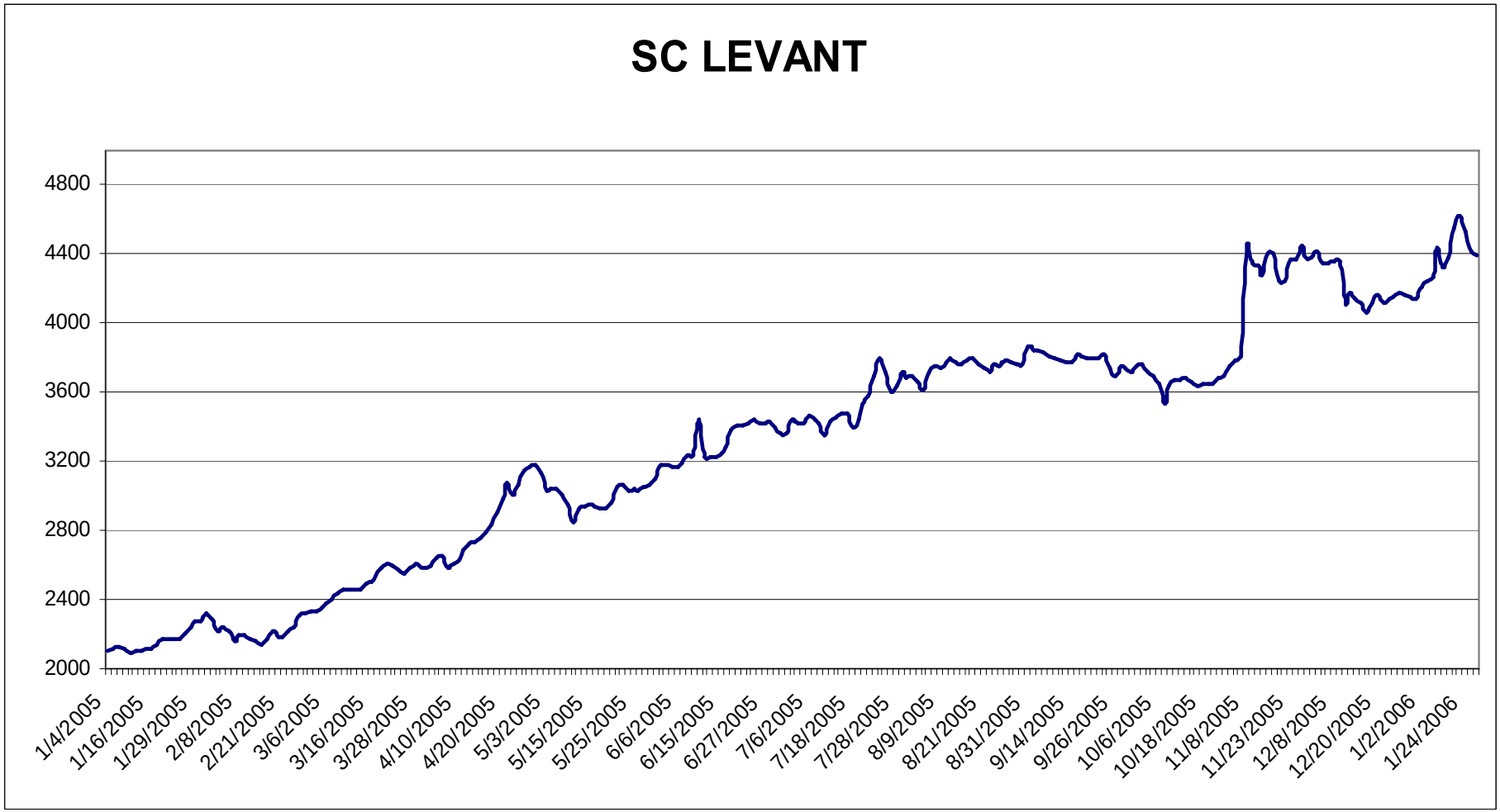
Historical returns on Real Estate Stocks and Bonds

	Period	*Compound annual return*	*Arithmetic mean return*	*Standard Deviation*	*Serial correlation*
CREF (commercial)	1969-87	10.80%	10.90%	2.60%	43.00%
REIT (commercial)	1972-99	14.20%	15.70%	15.40%	11.00%
I&S (commercial)	1960-69	8.70%	8.70%	4.90%	73.00%
C&S (residential)	1970-89	8.50%	8.50%	3.00%	17.00%
HOME (residential)	1947-89	9.80%	9.80%	4.70%	54.00%
HARRIS (residential)	1926-89	8.50%	8.50%	5.40%	55.00%
FARM (farmland)	1947-89	9.90%	9.90%	7.80%	64.00%
S&P 500	1928-00	10.46%	12.38%	20.02%	-5.00%
T. Bonds	1928-00	4.95%	5.21%	7.68%	16.00%
T.Bills	1928-00	3.97%	3.93%	3.18%	86.00%
Inflation rate	1928-00	3.21%	3.30%	3.05%	66.00%

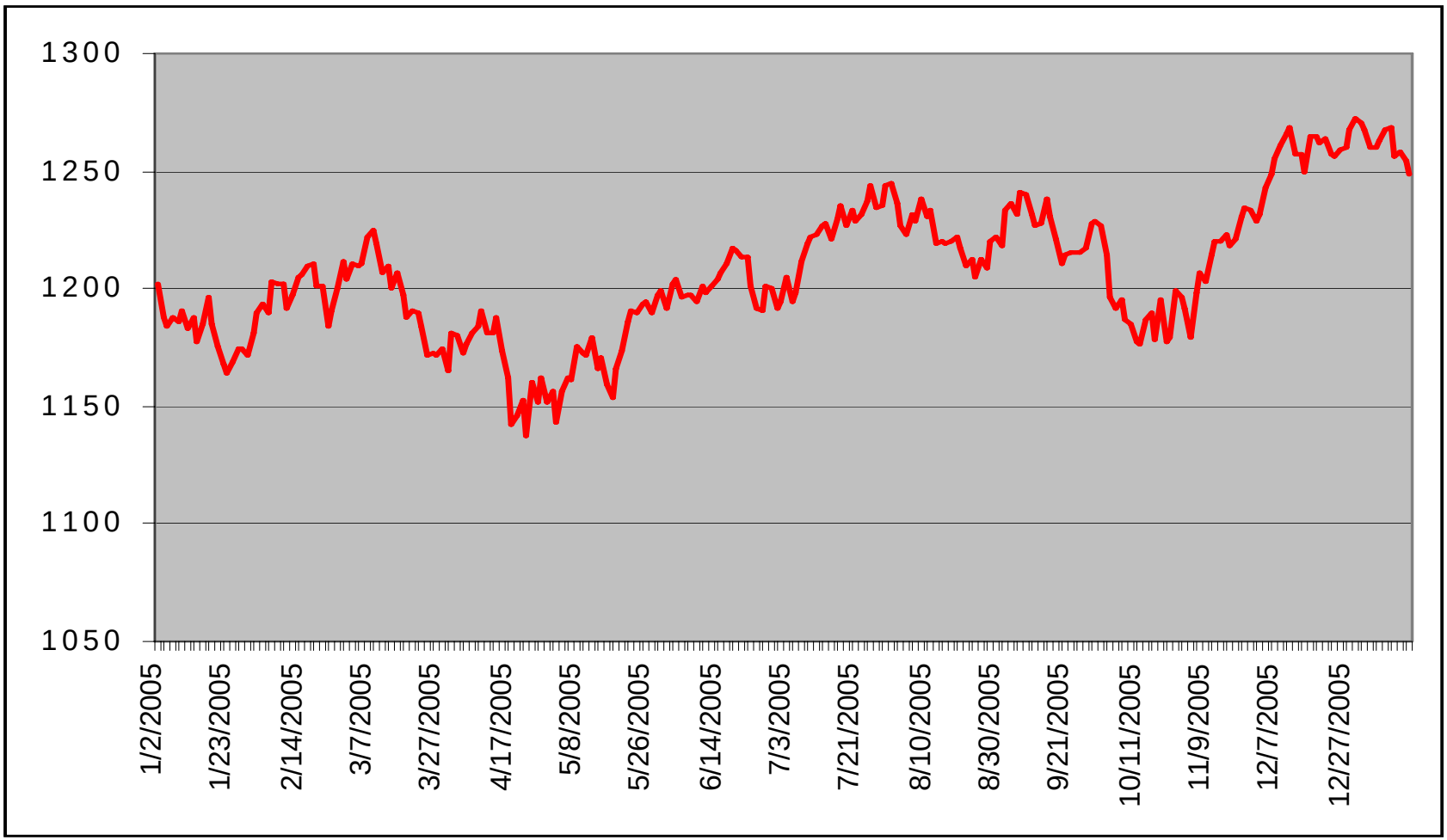
SOLIDERE A daily close, 2005



SC LEVANT daily close, 2005



S&P 500 daily close, 2005



Empirical model

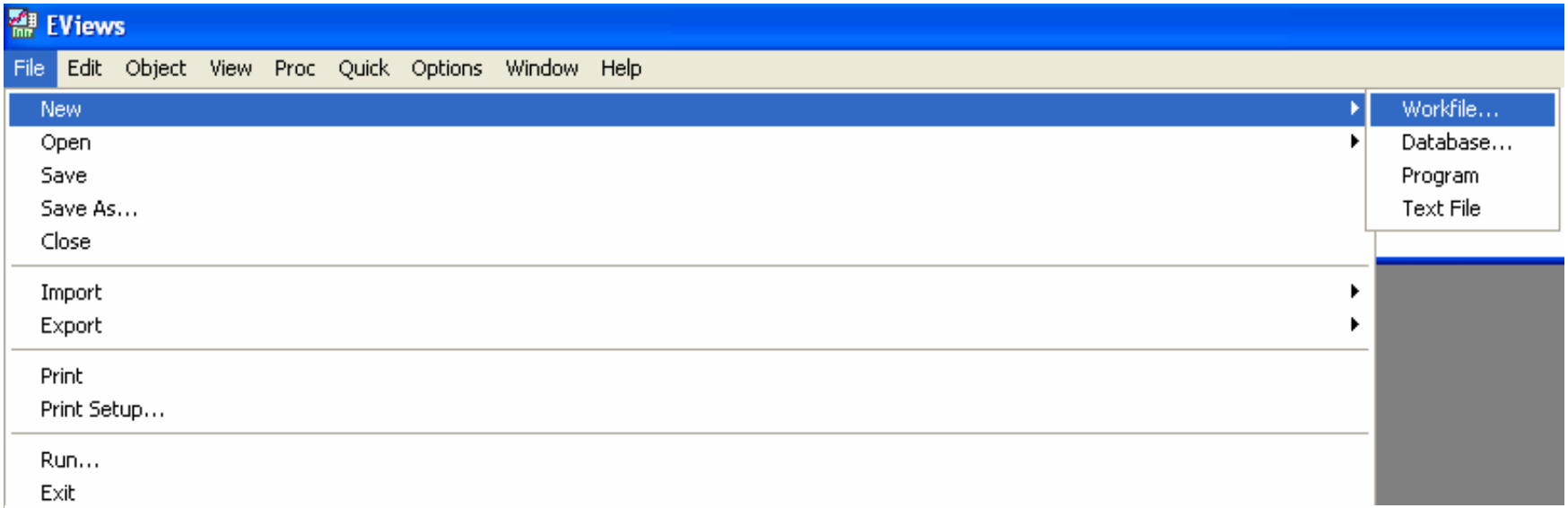
- Empirical model: $R_{it} - R_{ft} = \alpha_i + (R_{Mt} - R_{ft})\beta_i + \varepsilon_{it}$
- Define: $z_{it} = R_{it} - R_{ft}$ and $z_{Mt} = R_{Mt} - R_{ft}$
- Form the simple regression:

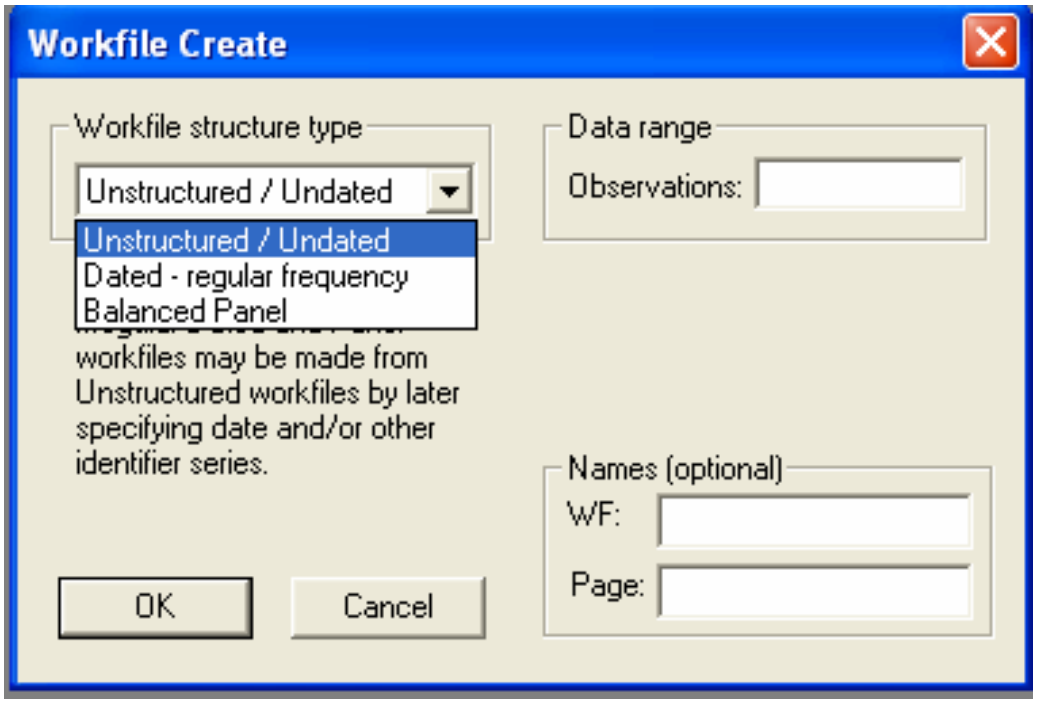
$$z_{it} = \alpha_i + z_{Mt}\beta_i + \varepsilon_{it}$$

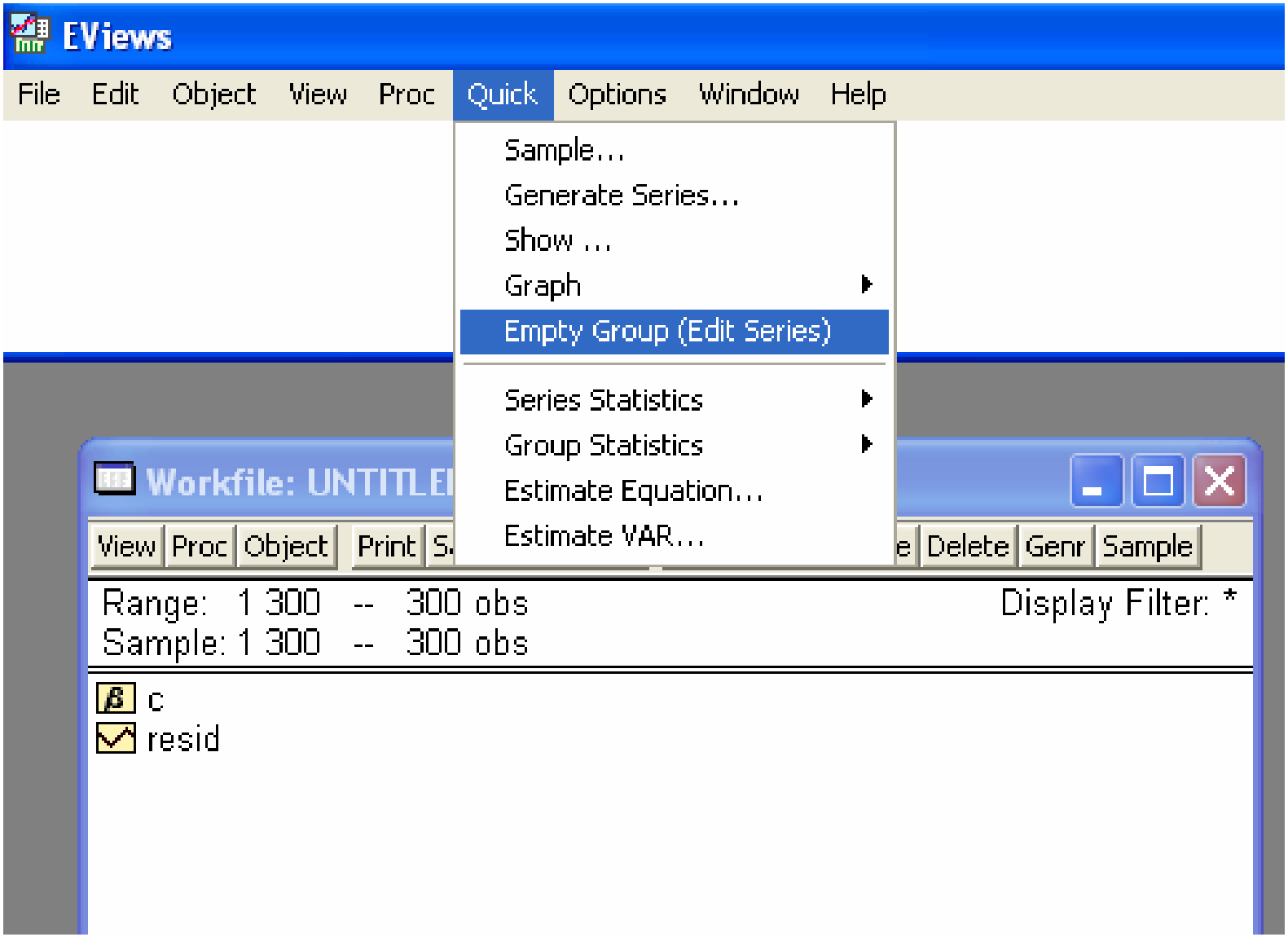
- R_i : continuously compounded rate of return on SOLIDERE A
  100 x log(pt/pt(-1))
- R_f: daily-adjusted 1-month US TB rate
  rf = - 100*(1-(1+rtb/100)^(1/260))
- R_M: market compounded rate of return (many options: S&P 500, SC Levant index, SC Arab index)
- Time frame: daily, weekly or monthly

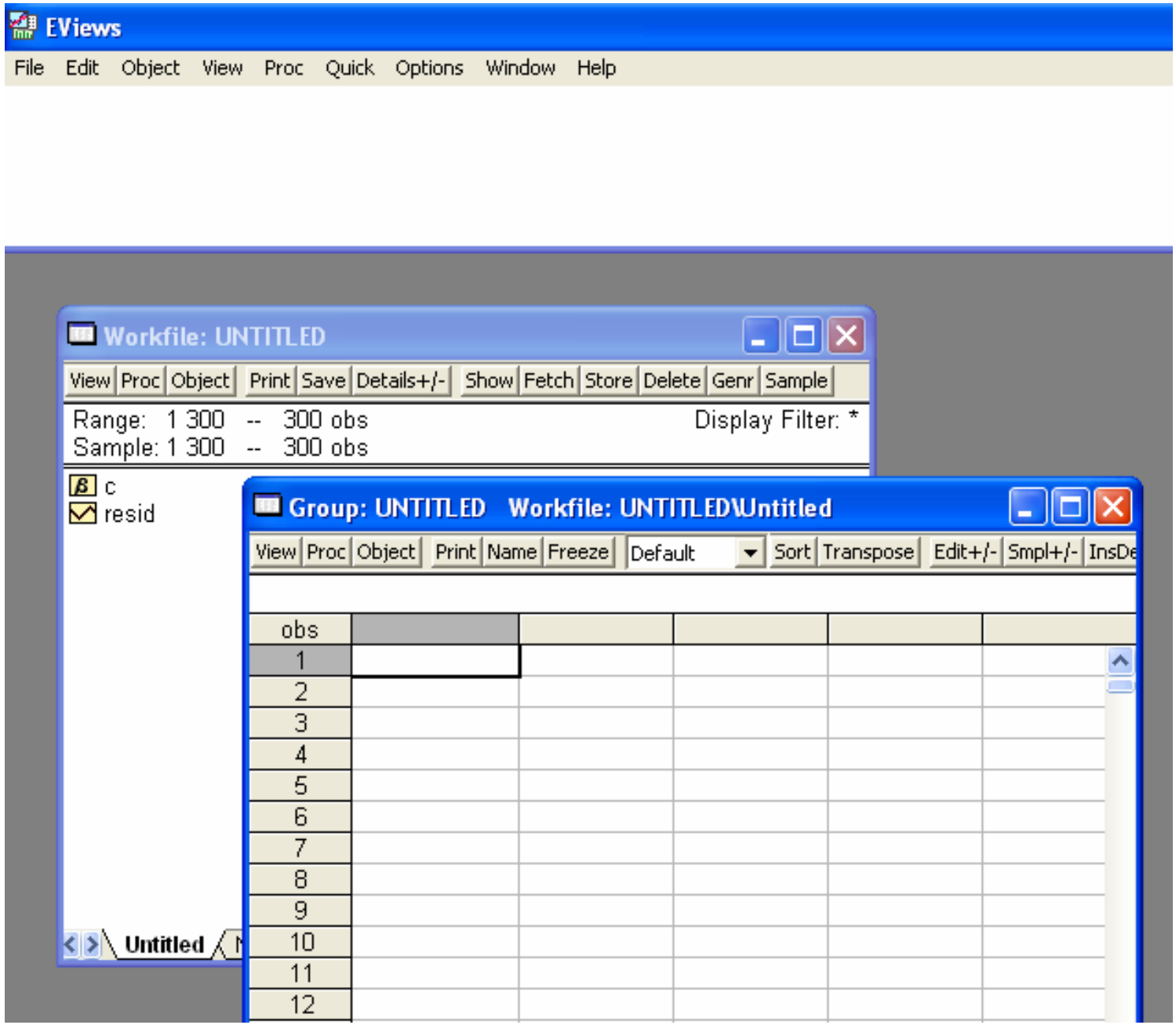
EViews application

- Open file and import data
- Compute basic summary indicators for each variables
- Estimate CAPM model
- Do regression diagnostics
- Test parameter restrictions
- Answer the primary questions


EViews
File Edit Object View Proc Quick Options Window Help
New
Open
Save
Save As...
Close
Import
Export
Print
Print Setup...
Run...
Exit
Workfile...
Database...
Program
Text File


Workfile Create
Workfile structure type
Unstructured / Undated
Unstructured / Undated
Dated - regular frequency
Balanced Panel
workfiles may be made from Unstructured workfiles by later specifying date and/or other identifier series.
Data range
Observations:
Names (optional)
WF:
Page:
OK
Cancel


EViews
File
Edit
Object
View
Proc
Quick
Options
Window
Help
Sample...
Generate Series...
Show ...
Graph
Empty Group (Edit Series)
Series Statistics
Group Statistics
Estimate Equation...
Estimate VAR...
Workfile: UNTITLE
View
Proc
Object
Print
Delete
Genr
Sample
Range: 1 300 -- 300 obs
Display Filter: *
Sample: 1 300 -- 300 obs
c
resid


EViews
File Edit Object View Proc Quick Options Window Help
Workfile: UNTITLED
View Proc Object Print Save Details+/- Show Fetch Store Delete Genr Sample
Range: 1 300 -- 300 obs
Sample: 1 300 -- 300 obs
Display Filter: *
c
resid
Untitled
Group: UNTITLED Workfile: UNTITLED\Untitled
View Proc Object Print Name Freeze Default Sort Transpose Edit+/- Smpl+/-
obs
1
2
3
4
5
6
7
8
9
10
11
12

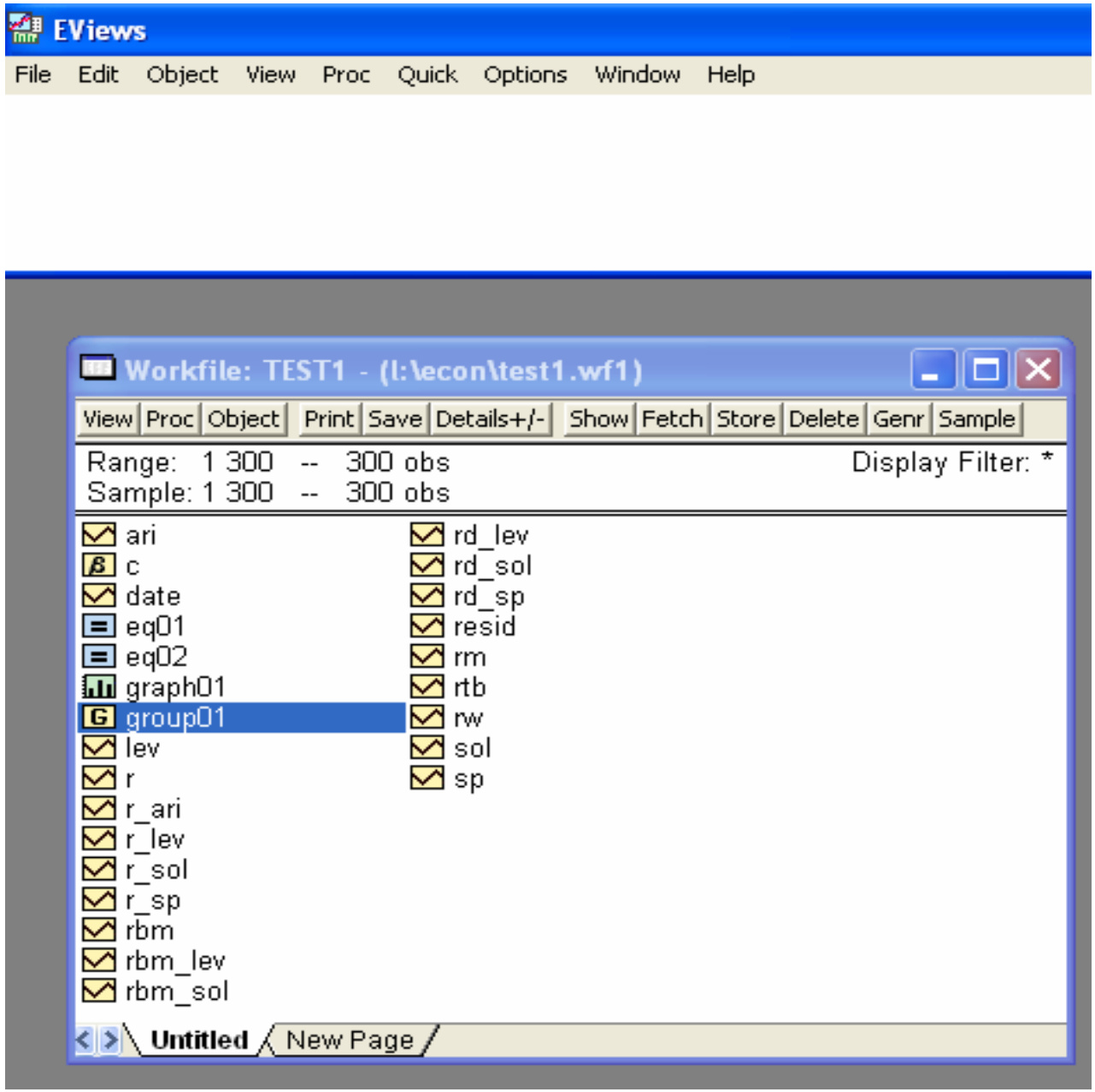
EViews - [Group: GROUP01 Workfile: TEST1\Untitled]

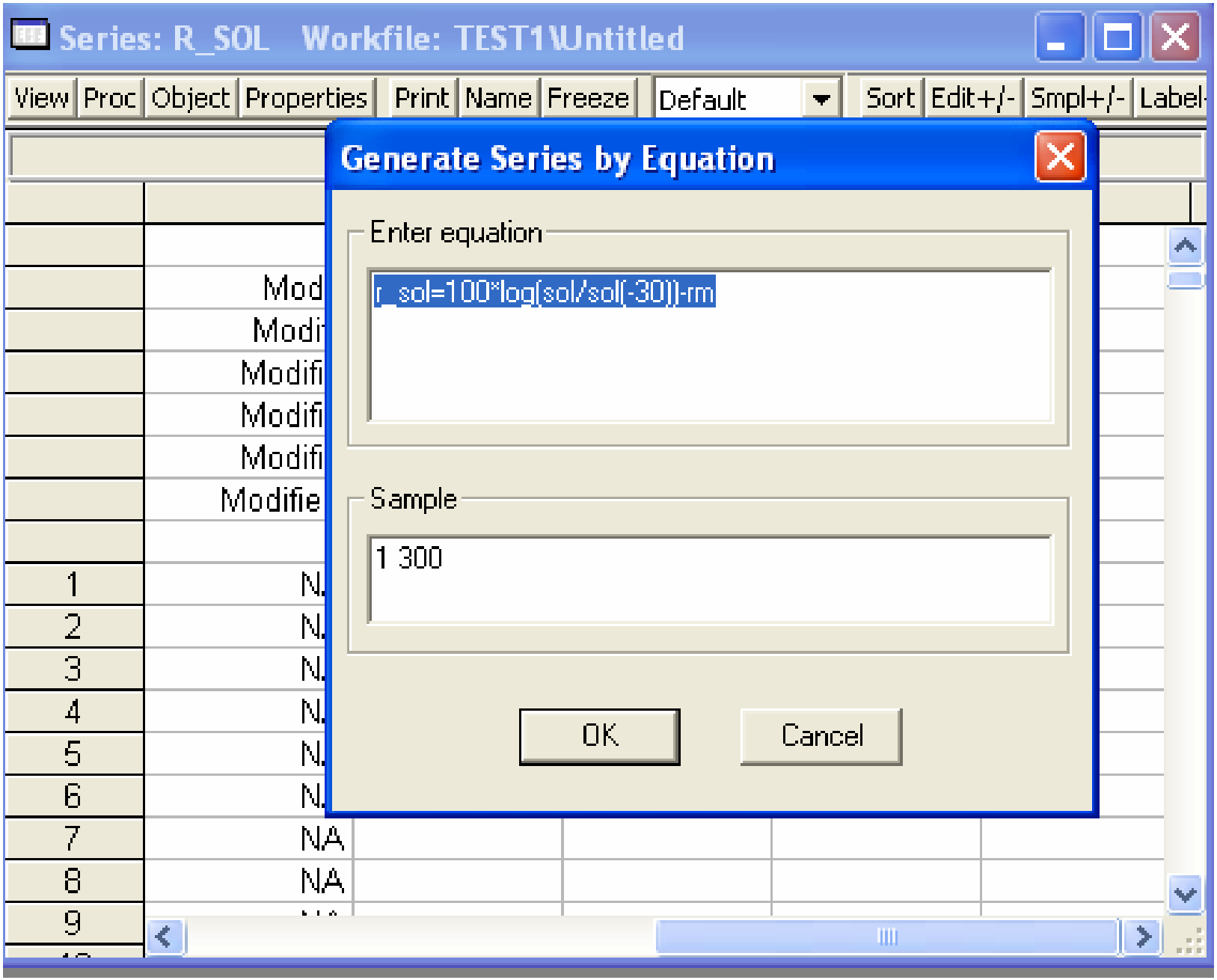
File Edit Object View Proc Quick Options Window Help

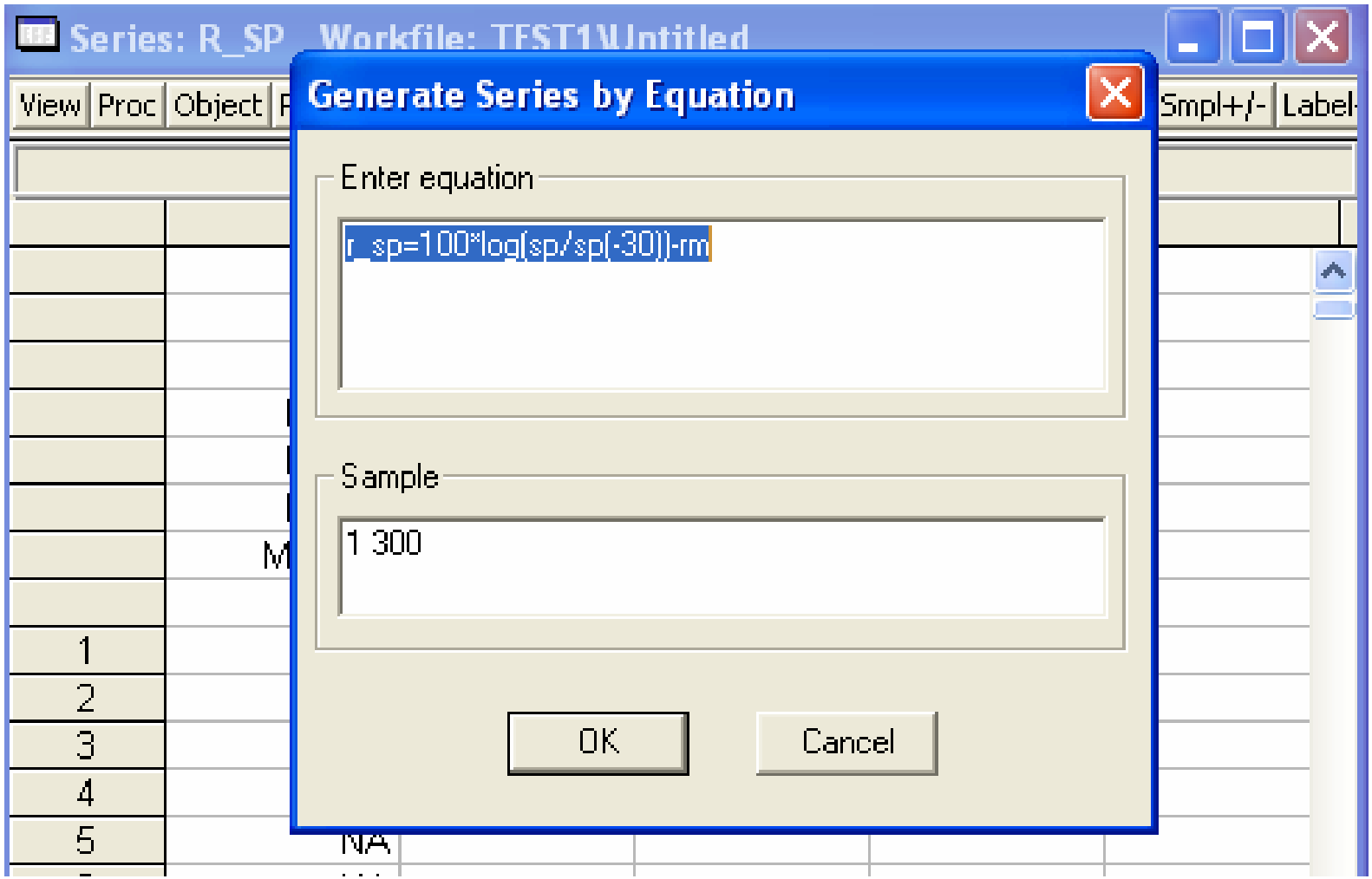
View | Proc | Object | Print | Name | Freeze | Default | Sort | Transpose | Edit+/- | Smpl+/- | InsDel | Title | Sample

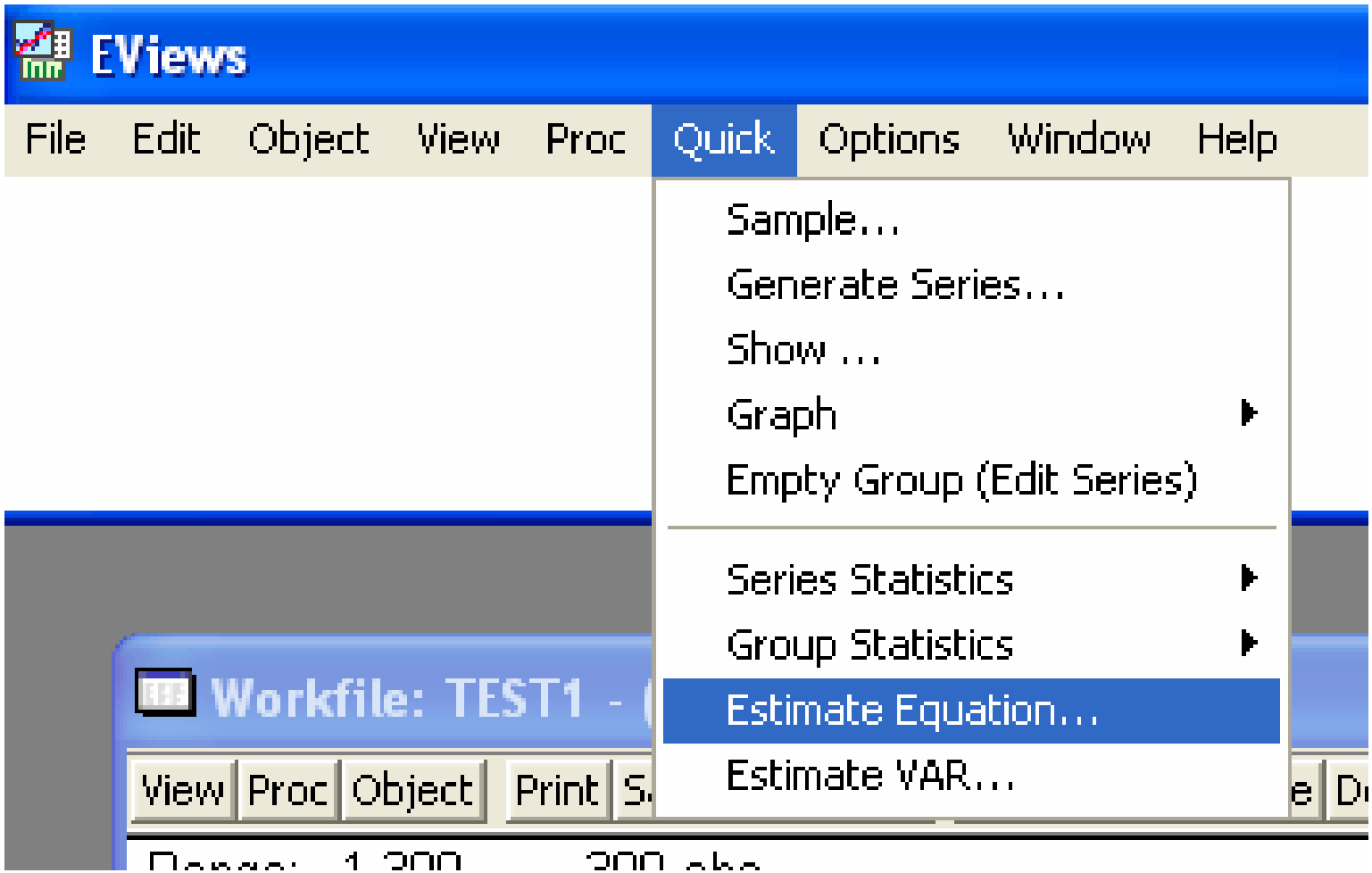
1/2/2005

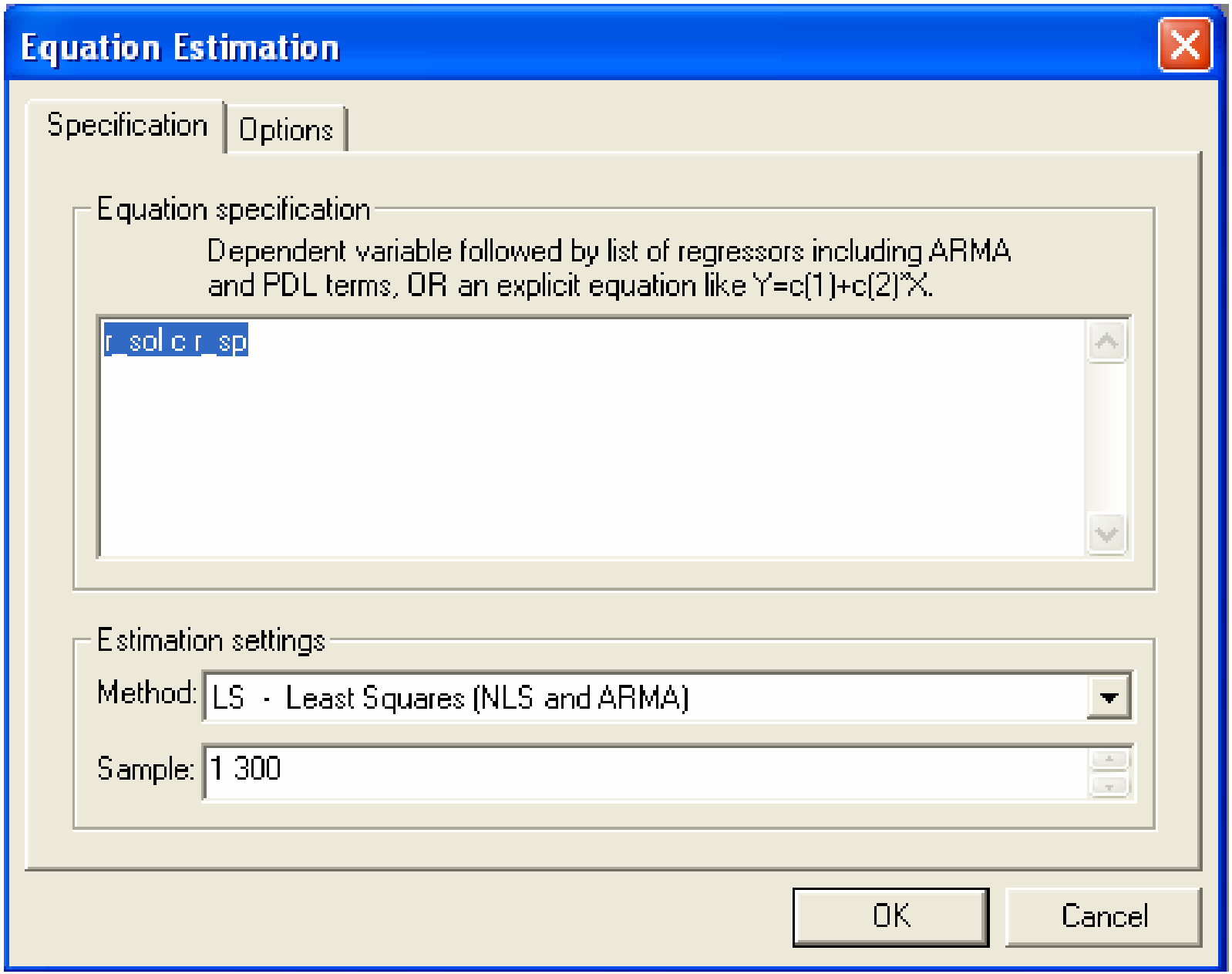
obs	DATE	LEV	ARI	SP	SOL	RTB
1	1/2/2005	2067.040	2676.090	1202.080	8.300000	1.910000
2	1/4/2005	2097.330	2690.850	1188.050	8.270000	2.010000
3	1/5/2005	2110.280	2703.380	1183.740	8.350000	2.000000
4	1/6/2005	2124.100	2710.320	1187.890	8.390000	1.990000
5	1/9/2005	2108.820	2683.870	1186.190	8.500000	1.980000
6	1/10/2005	2095.830	2674.020	1190.250	8.450000	2.000000
7	1/11/2005	2099.550	2694.060	1182.990	8.490000	1.990000
8	1/12/2005	2100.850	2725.490	1187.700	8.500000	1.970000
9	1/13/2005	2109.250	2726.950	1177.450	8.530000	2.010000
10	1/16/2005	2114.220	2715.410	1184.520	8.560000	2.000000
11	1/17/2005	2142.230	2708.520	1195.980	8.460000	1.960000
12	1/18/2005	2169.290	2711.010	1184.630	8.480000	1.900000
13	1/19/2005	2171.160	2747.540	1175.410	8.430000	1.830000
14	1/20/2005	2171.160	2747.390	1167.870	8.300000	1.960000
15	1/23/2005	2172.250	2747.800	1163.750	8.210000	1.970000
16	1/24/2005	2201.710	2760.540	1168.410	8.500000	2.090000
17	1/25/2005	2236.440	2764.720	1174.070	8.740000	2.120000
18	1/29/2005	2269.490	2822.440	1174.550	8.770000	2.110000
19	1/30/2005	2273.290	2825.890	1171.360	8.810000	2.060000
20	1/31/2005	2321.250	2825.660	1181.270	9.000000	1.990000

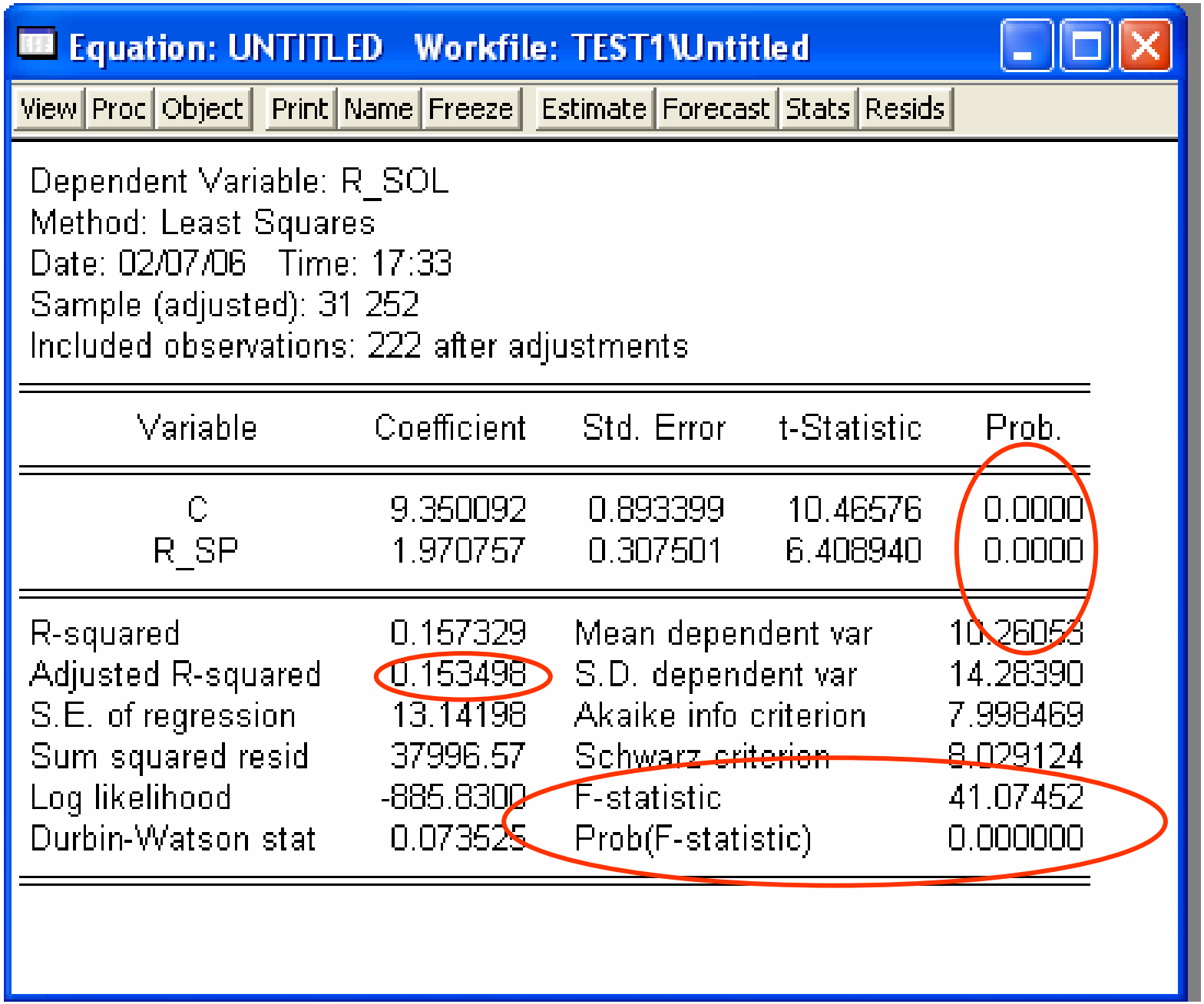

EViews
File Edit Object View Proc Quick Options Window Help
Workfile: TEST1 - (I:\econ\test1.wf1)
View Proc Object Print Save Details+/- Show Fetch Store Delete Genr Sample
Range: 1 300 -- 300 obs
Display Filter: *
Sample: 1 300 -- 300 obs
ari
c
date
eq01
eq02
graph01
group01
lev
r
r_ari
r_lev
r_sol
r_sp
rbm
rbm_lev
rbm_sol
rd_lev
rd_sol
rd_sp
resid
rm
rtb
rw
sol
sp
Untitled
New Page


Series: R_SOL Workfile: TEST1\Untitled
View
Proc
Object
Properties
Print
Name
Freeze
Default
Sort
Edit+/-
Smpl+/-
Label
Generate Series by Equation
Enter equation
r_sol=100*log(sol/sol(-30))-rm
Sample
1 300
OK
Cancel

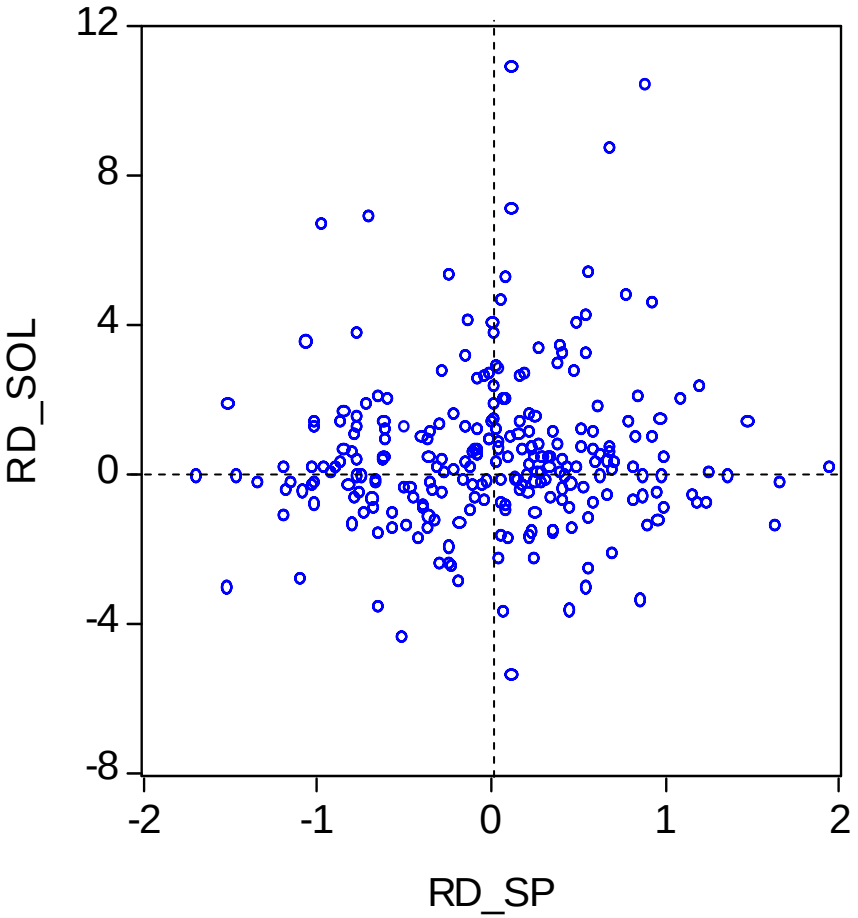

Series: R_SP Workfile: TEST1\Untitled
View
Proc
Object
Smpl+/-
Label
Generate Series by Equation
Enter equation
r_sp=100*log(sp/sp(-30))-rm
Sample
1 300
OK
Cancel
1
2
3
4
5
NA


EViews
File
Edit
Object
View
Proc
Quick
Options
Window
Help
Sample...
Generate Series...
Show ...
Graph
Empty Group (Edit Series)
Series Statistics
Group Statistics
Estimate Equation...
Estimate VAR...
Workfile: TEST1 -
View
Proc
Object
Print

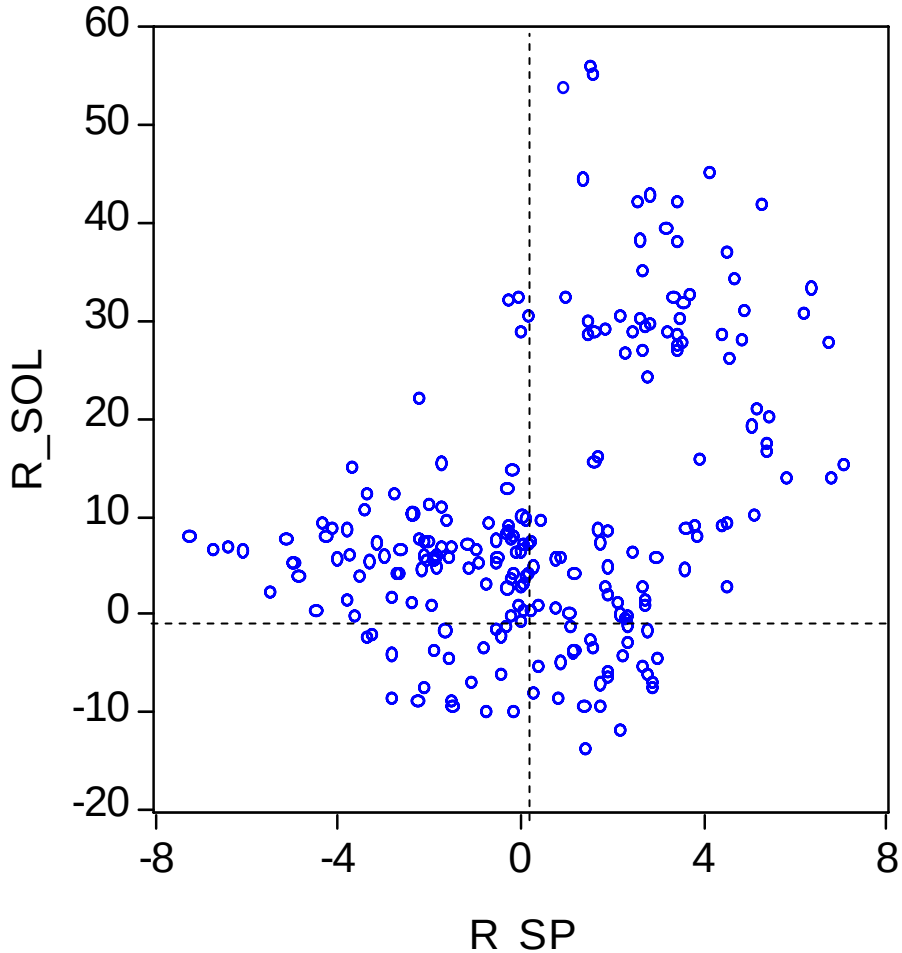

Equation Estimation
Specification
Options
Equation specification
Dependent variable followed by list of regressors including ARMA and PDL terms, OR an explicit equation like Y=c(1)+c(2)*X.
r_sol c r_sp
Estimation settings
Method:
LS - Least Squares (NLS and ARMA)
Sample:
1 300
OK
Cancel


Equation: UNTITLED Workfile: TEST1\Untitled
View Proc Object Print Name Freeze Estimate Forecast Stats Resids
Dependent Variable: R_SOL
Method: Least Squares
Date: 02/07/06 Time: 17:33
Sample (adjusted): 31 252
Included observations: 222 after adjustments
Variable Coefficient Std. Error t-Statistic Prob.
C 9.350092 0.893399 10.46576 0.0000
R_SP 1.970757 0.307501 6.408940 0.0000
R-squared 0.157329 Mean dependent var 10.26053
Adjusted R-squared 0.153498 S.D. dependent var 14.28390
S.E. of regression 13.14198 Akaike info criterion 7.998469
Sum squared resid 37996.57 Schwarz criterion 8.029124
Log likelihood -885.8300 F-statistic 41.07452
Durbin-Watson stat 0.073525 Prob(F-statistic) 0.000000

Graphs

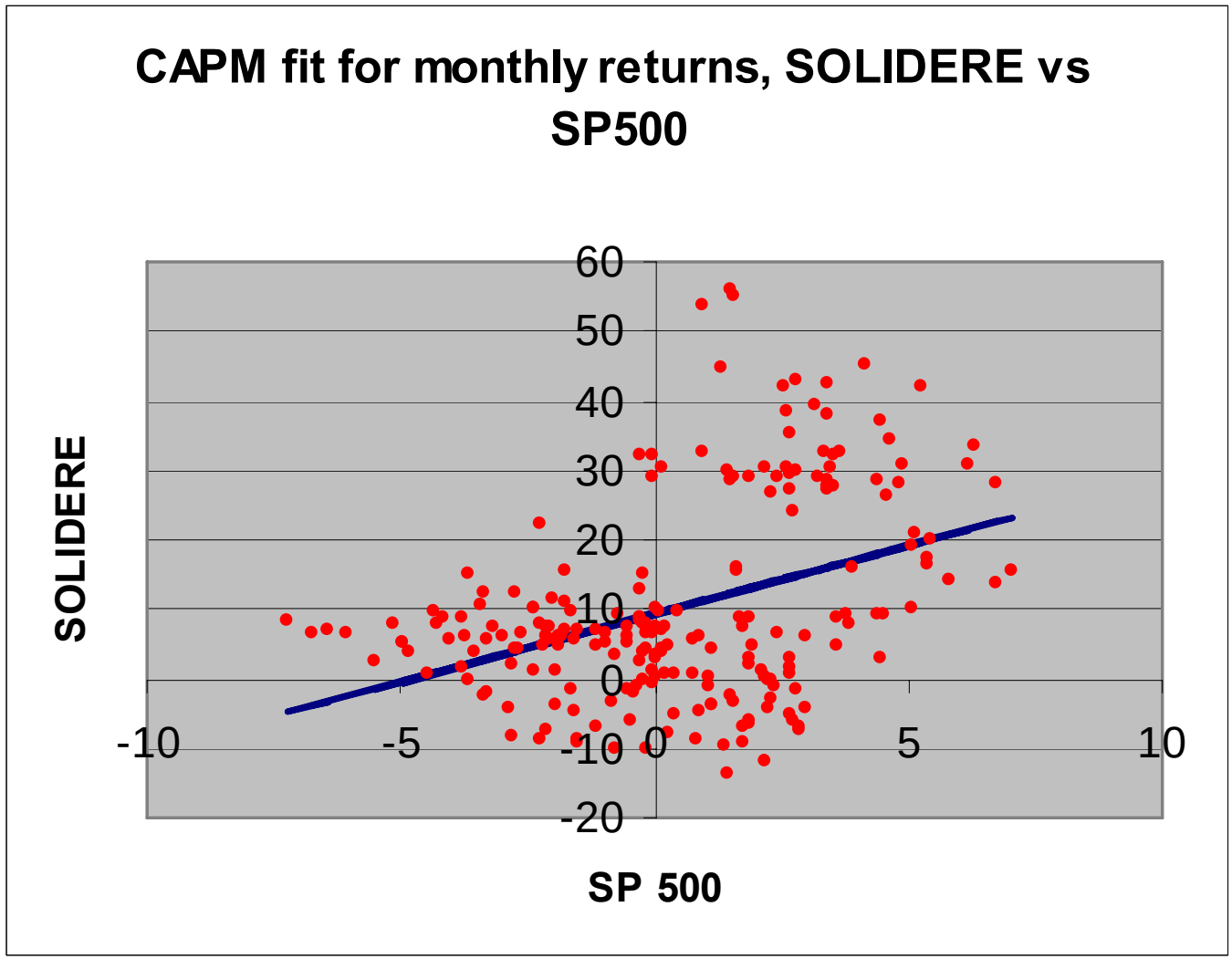


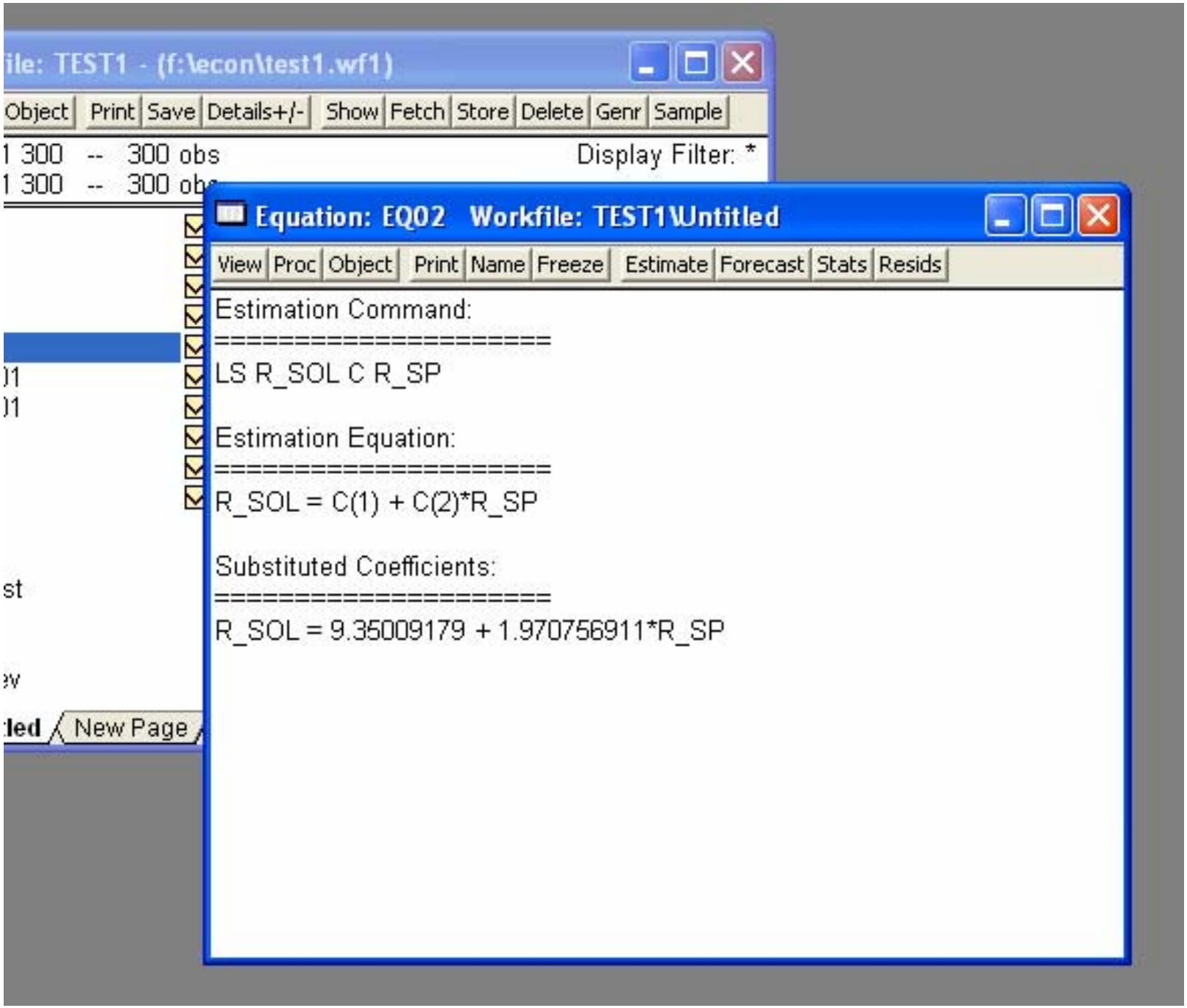
daily

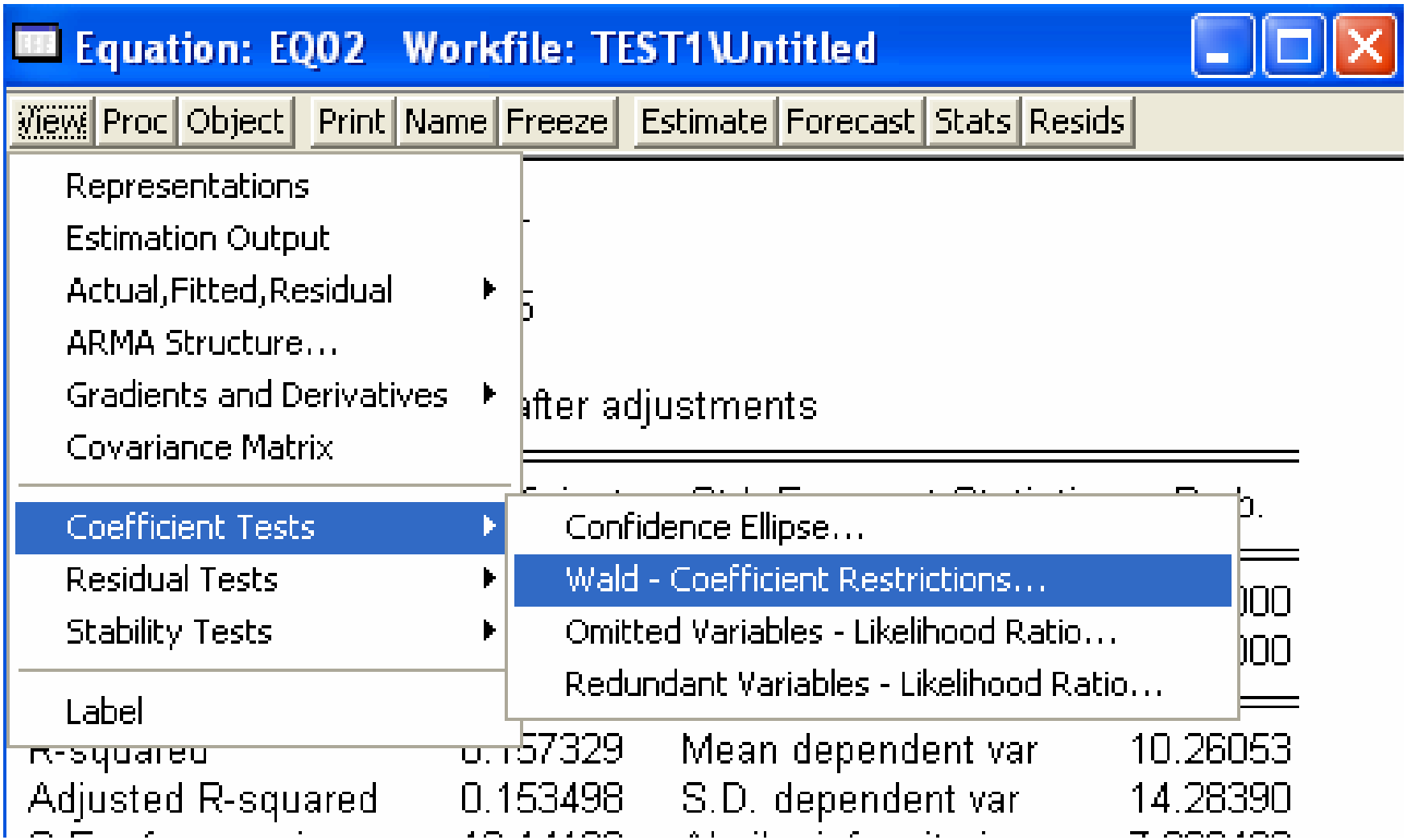


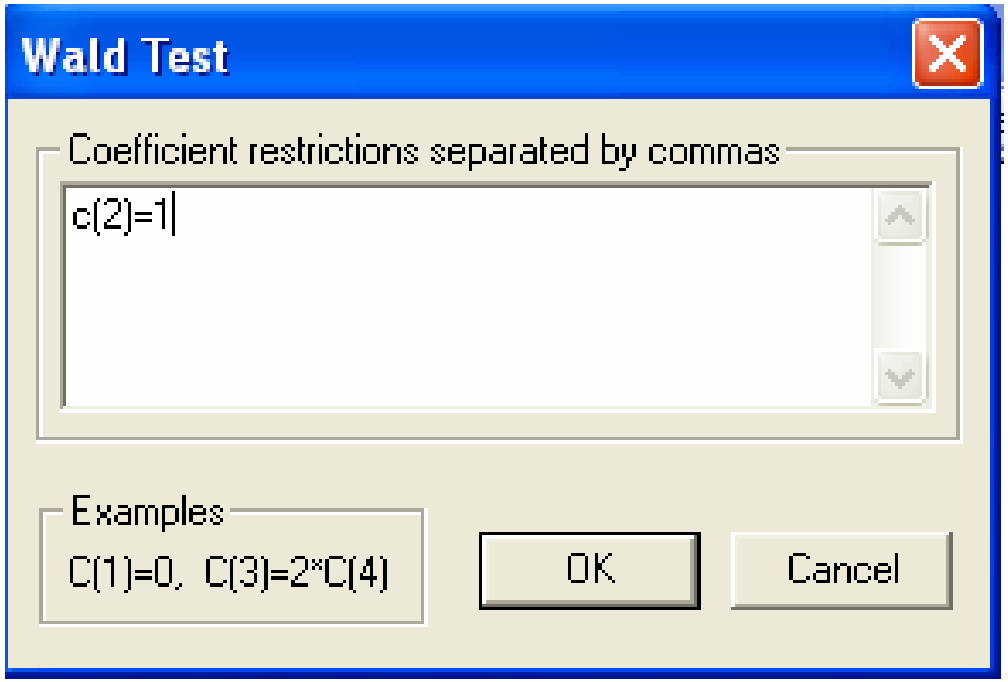
monthly

Graphs


CAPM fit for monthly returns, SOLIDERE vs SP500
SOLIDERE
SP 500
60
50
40
30
20
10
0
-10
-20
-10
-5
0
5
10


ile: TEST1 - (f:\econ\test1.wf1)
Object
Print
Save
Details+/-
Show
Fetch
Store
Delete
Genr
Sample
1 300 -- 300 obs
Display Filter: *
1 300 -- 300 ob
Equation: EQ02 Workfile: TEST1\Untitled
View
Proc
Object
Print
Name
Freeze
Estimate
Forecast
Stats
Resids
Estimation Command:
LS R_SOL C R_SP
Estimation Equation:
R_SOL = C(1) + C(2)*R_SP
Substituted Coefficients:
R_SOL = 9.35009179 + 1.970756911*R_SP
New Page


Equation: EQ02 Workfile: TEST1\Untitled
View
Proc
Object
Print
Name
Freeze
Estimate
Forecast
Stats
Resids
Representations
Estimation Output
Actual,Fitted,Residual
ARMA Structure...
Gradients and Derivatives
Covariance Matrix
Coefficient Tests
Residual Tests
Stability Tests
Label
Confidence Ellipse...
Wald - Coefficient Restrictions...
Omitted Variables - Likelihood Ratio...
Redundant Variables - Likelihood Ratio...
after adjustments
Mean dependent var 10.26053
Adjusted R-squared 0.153498
S.D. dependent var 14.28390

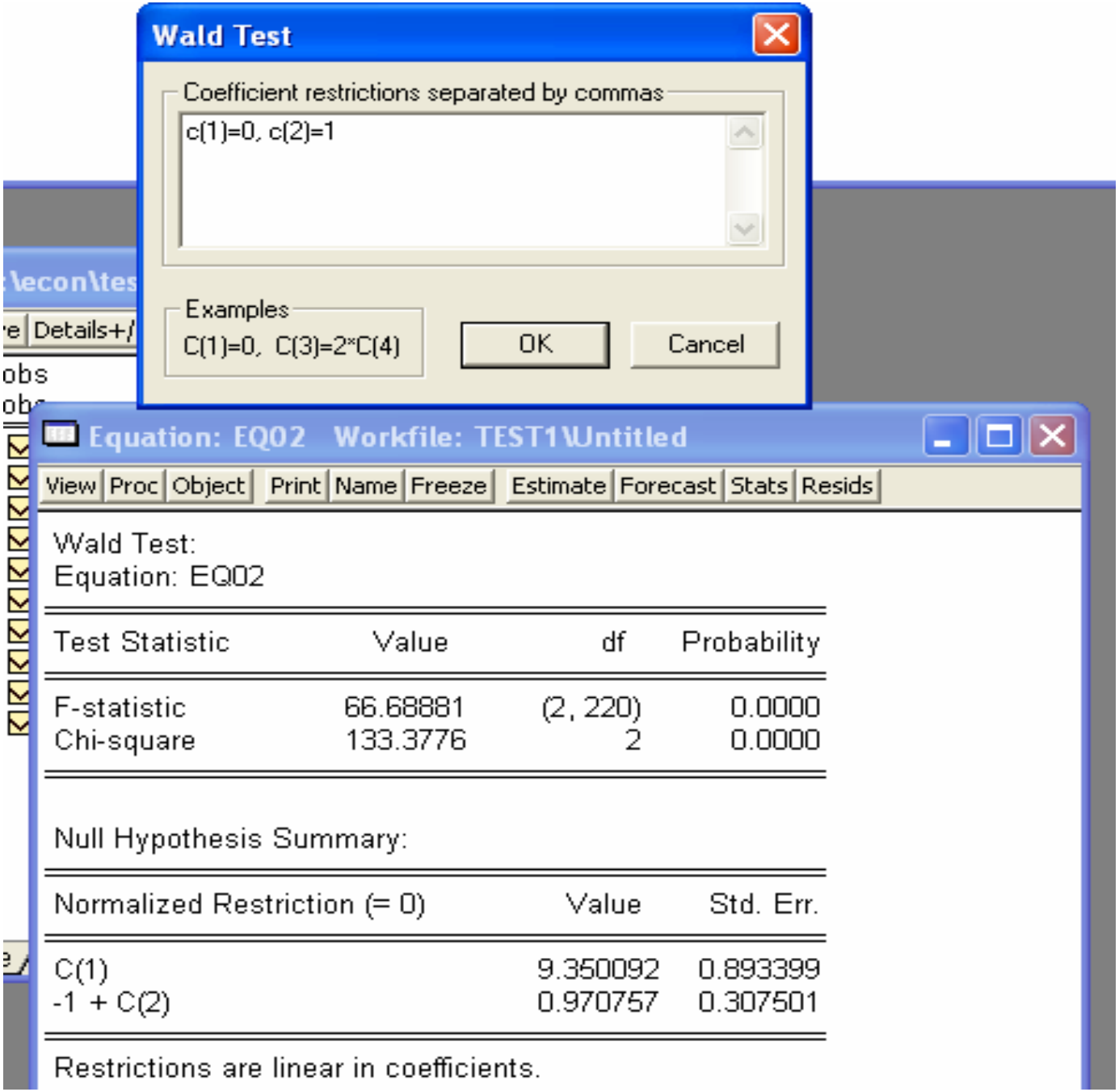


Equation: EQ02 Workfile: TEST1\Untitled

View | Proc | Object | Print | Name | Freeze | Estimate | Forecast | Stats | Resids

Wald Test:
Equation: EQ02

Test Statistic	Value	df	Probability
F-statistic	9.966148	(1, 220)	0.0018
Chi-square	9.966148	1	0.0016


Wald Test
Coefficient restrictions separated by commas
c(1)=0, c(2)=1
Examples
C(1)=0, C(3)=2*C(4)
OK
Cancel
Equation: EQ02 Workfile: TEST1\Untitled
View Proc Object Print Name Freeze Estimate Forecast Stats Resids
Wald Test:
Equation: EQ02
Test Statistic Value df Probability
F-statistic 66.68881 (2, 220) 0.0000
Chi-square 133.3776 2 0.0000
Null Hypothesis Summary:
Normalized Restriction (= 0) Value Std. Err.
C(1) 9.350092 0.893399
-1 + C(2) 0.970757 0.307501
Restrictions are linear in coefficients.

EXERCISE

1. The following table contains ten years of excess return data for the ordinary shares of ABC plc and the market portfolio. Discuss how could you use this information to obtain an estimate of the *beta-coefficient* for ABC plc's ordinary shares and calculate such an estimate.

Year	ABC plc	Market Portfolio
1	5.40%	3.50%
2	12.05	8.65
3	13.00	9.80
4	7.65	5.55
5	6.95	4.10
6	−2.65	−1.00
7	−3.25	−2.50
8	3.60	1.85
9	3.25	3.05
10	5.40	4.60

EXCEL: FORECAST formula

	A	B
1	**Known Y**	**Known X**
2	6	20
3	7	28
4	9	31
5	15	38
6	21	40
	Formula	**Description (Result)**
	=FORECAST(30,A2:A6,B2:B6)	Predicts a value for y given an x value of 30 (10.60725)