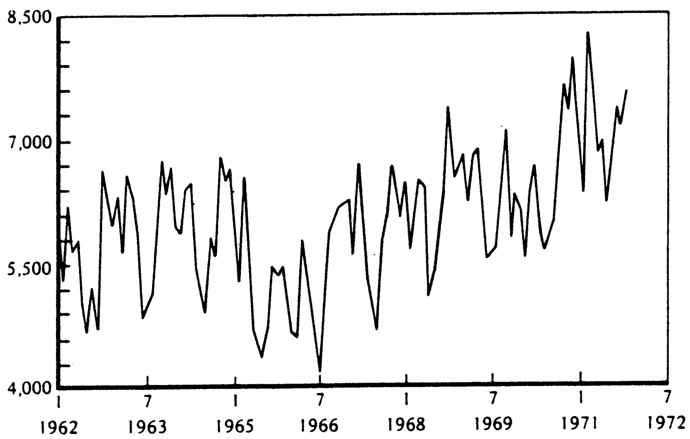
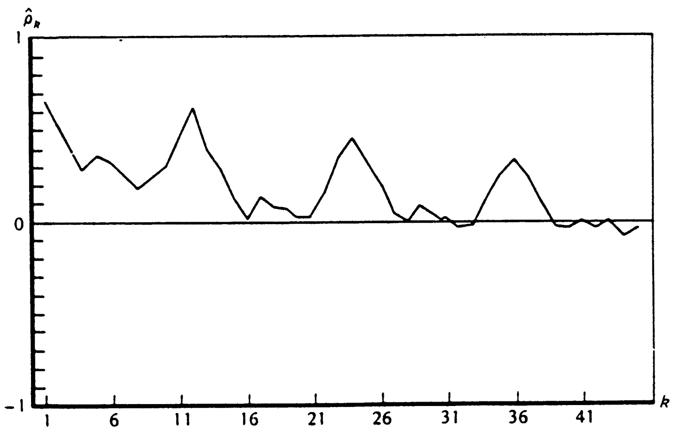


FIGURE 15.14
Hog production (in thousands of hogs per month). Time bounds: January 1962 to December 1971.

series model for hog production, since it accounts only for the annual cycle. We can complete this example by observing that the autocorrelation function in Fig. 15.16 declines only slowly, so that there is some doubt as to whether z_t is a stationary series. We therefore first-differenced this series, to obtain $w_t = \Delta z_t = \Delta(y_t - y_{t-12})$. The sample autocorrelation function of this series, shown in Fig. 15.17, declines rapidly and remains small, so that we can be confident that w_t is a stationary, nonseasonal time series.

FIGURE 15.15
Sample autocorrelation function for hog production series.



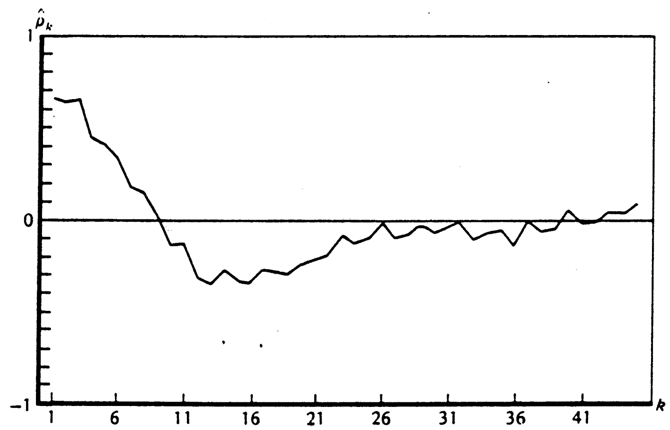


FIGURE 15.16
Hog production: sample autocorrelation function of $y_t - y_{t-12}$.

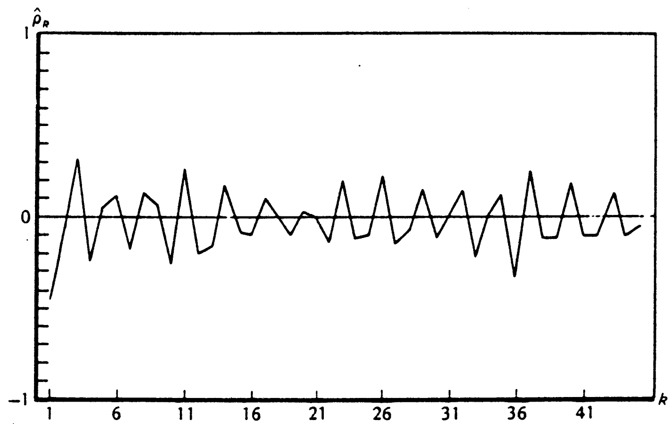


FIGURE 15.17
Hog production: sample autocorrelation function of $\Delta(y_t - y_{t-12})$.

15.3 TESTING FOR RANDOM WALKS

Do economic variables such as GNP, employment, and interest rates tend to revert back to some long-run trend following a shock, or do they follow random walks? This question is important for two reasons. First, if these variables follow random walks, a regression of one against another can lead to spurious results. [The Gauss-Markov theorem would not hold, for example, because a random walk does not have a finite variance. Hence ordinary least squares (OLS) would not yield a consistent parameter estimator.] Detrending the variables before running the regression will not help; the detrended series will still be nonstationary. Only first-differencing will yield stationary series. Second, the answer has implications for our understanding of the economy and for forecasting. If a variable like GNP follows a random walk, the effects of a temporary shock (such as an increase in oil prices or a drop in government spending) will not dissipate after several years, but instead will be permanent.

In a provocative study, Charles Nelson and Charles Plosser found evidence that GNP and other macroeconomic time series behave like random walks.[7] The work spawned a series of studies that investigate whether economic and financial variables are random walks or trend-reverting. Several of these studies show that many economic time series do appear to be random walks, or at least have random walk components.[8] Most of these studies use *unit root tests* introduced by David Dickey and Wayne Fuller.[9]

Suppose we believe that a variable Y_t, which has been growing over time, can be described by the following equation:

$$Y_t = \alpha + \beta t + \rho Y_{t-1} + \varepsilon_t \tag{15.34}$$

One possibility is that Y_t has been growing because it has a positive trend ($\beta > 0$), but would be stationary after detrending (i.e., $\rho < 1$). In this case, Y_t could be used in a regression, and all the results and tests discussed in Part One of this book would apply. Another possibility is that Y_t has been growing because it follows a random walk with a positive drift (i.e., $\alpha > 0$, $\beta = 0$, and $\rho = 1$). In this case, one would want to work with ΔY_t. Detrending would not make the series stationary, and inclusion of Y_t in a regression (even if detrended) could lead to spurious results.

One might think that Eq. (15.34) could be estimated by OLS, and the t statistic on $\hat{\rho}$ could then be used to test whether $\hat{\rho}$ is significantly different from 1. However, as we saw in Chapter 9, if the true value of ρ is indeed 1, then the OLS estimator is biased toward zero. Thus the use of OLS in this manner can lead one to incorrectly reject the random walk hypothesis.

Dickey and Fuller derived the distribution for the estimator $\hat{\rho}$ that holds when $\rho = 1$, and generated statistics for a simple F test of the random walk hypothesis, i.e., of the hypothesis that $\beta = 0$ and $\rho = 1$. The Dickey-Fuller test is easy to perform, and can be applied to a more general version of Eq. (15.34). It works as follows.

[7] C. R. Nelson and C. I. Plosser, "Trends and Random Walks in Macroeconomic Time Series: Some Evidence and Implications," *Journal of Monetary Economics*, vol. 10, pp. 139–162, 1982.

[8] Examples of these studies include J. Y. Campbell and N. G. Mankiw, "Are Output Fluctuations Transitory?," *Quarterly Journal of Economics*, vol. 102, pp. 857–880, 1987; J. Y. Campbell and N. G. Mankiw, "Permanent and Transitory Components in Macroeconomic Fluctuations," *American Economic Review Papers and Proceedings*, vol. 77, pp. 111–117, 1987; and G. W. Gardner and K. P. Kimbrough, "The Behavior of U.S. Tariff Rates," *American Economic Review*, vol. 79, pp. 211–218, 1989.

[9] D. A. Dickey and W. A. Fuller, "Distribution of the Estimators for Autoregressive Time-Series with a Unit Root," *Journal of the American Statistical Association*, vol. 74, pp. 427–431, 1979; D. A. Dickey and W. A. Fuller, "Likelihood Ratio Statistics for Autoregressive Time Series with a Unit Root," *Econometrica*, vol. 49, pp. 1057–1072, 1981; and W. A. Fuller, *Introduction to Statistical Time Series* (New York: Wiley, 1976).

TABLE 15.1
DISTRIBUTION OF F FOR $(\alpha, \beta, \rho) = (\alpha, 0, 1)$ IN $Y_t = \alpha + \beta t + \rho Y_{t-1} + \varepsilon_t$

Sample size N	Probability of a smaller value							
	.01	.025	.05	.10	.90	.95	.975	.99
25	.74	.90	1.08	1.33	5.91	7.24	8.65	10.61
50	.76	.93	1.11	1.37	5.61	6.73	7.81	9.31
100	.76	.94	1.12	1.38	5.47	6.49	7.44	8.73
250	.76	.94	1.13	1.39	5.39	6.34	7.25	8.43
500	.76	.94	1.13	1.39	5.36	6.30	7.20	8.34
∞	.77	.94	1.13	1.39	5.34	6.25	7.16	8.27
Standard error	.004	.004	.003	.004	.015	.020	.032	.058

Source: Dickey and Fuller, op. cit., Table VI, p. 1063, 1981.

Suppose Y_t can be described by the following equation:

$$Y_t = \alpha + \beta t + \rho Y_{t-1} + \lambda_1 \Delta Y_{t-1} + \varepsilon_t \tag{15.35}$$

where $\Delta Y_{t-1} = Y_{t-1} - Y_{t-2}$. (Additional lags of ΔY_t can be included on the right-hand side; the test is the same.) Using OLS, one first runs the unrestricted regression

$$Y_t - Y_{t-1} = \alpha + \beta t + (\rho - 1)Y_{t-1} + \lambda_1 \Delta Y_{t-1} \tag{15.36}$$

and then the restricted regression

$$Y_t - Y_{t-1} = \alpha + \lambda_1 \Delta Y_{t-1} \tag{15.37}$$

Then, one calculates the standard F ratio to test whether the restrictions ($\beta = 0$, $\rho = 1$) hold.[10] This ratio, however, is *not* distributed as a standard F distribution under the null hypothesis. Instead, one must use the distributions tabulated by Dickey and Fuller. Critical values for this statistic are shown in Table 15.1.

Note that these critical values are much larger than those in the standard F table. For example, if the calculated F ratio turns out to be 5.2 and there are 100

[10] Recall that F is calculated as follows:

$$F = (N - k)(\text{ESS}_R - \text{ESS}_{UR})/q(\text{ESS}_{UR})$$

where ESS_R and ESS_{UR} are the sums of squared residuals in the restricted and unrestricted regressions, respectively, N is the number of observations, k is the number of estimated parameters in the unrestricted regression, and q is the number of parameter restrictions.

observations, we would easily reject the null hypothesis of a unit root at the 5 percent level if we used a standard F table (which, with two parameter restrictions, shows a critical value of about 3.1), i.e., we would conclude that there is no random walk. This rejection, however, would be incorrect. Note that we fail to reject the hypothesis of a random walk using the distribution calculated by Dickey and Fuller (the critical value is 6.49).[11]

Although the Dickey-Fuller test is widely used, one should keep in mind that its power is limited. It only allows us to reject (or fail to reject) the hypothesis that a variable is *not* a random walk. A failure to reject (especially at a high significance level) is only weak evidence in favor of the random walk hypothesis.

Example 15.4 Do Commodity Prices Follow Random Walks? Like stocks and bonds, many commodities are actively traded in highly liquid spot markets. In addition, trading is active in financial instruments such as futures contracts that depend on the prices of these commodities. One might therefore expect the prices of these commodities to follow random walks, so that no investor could expect to profit by following some trading rule. (See Example 15.2 on daily hog prices.) Indeed, most financial models of futures, options, and other instruments tied to a commodity are based on the assumption that the spot price follows a random walk.[12]

On the other hand, basic microeconomic theory tells us that in the long run the price of a commodity ought to be tied to its marginal production cost. This means that although the price of a commodity might be subject to sharp short-run fluctuations, it ought to tend to return to a "normal" level based on cost. Of course, marginal production cost might be expected to slowly rise (if the commodity is a depletable resource) or fall (because of technological change), but that means the *detrended* price should tend to revert back to a normal level.

At issue, then, is whether the price of a commodity can best be described as a random walk process, perhaps with trend:

$$P_t = \alpha + P_{t-1} + \varepsilon_t \tag{15.38}$$

where ε_t is a white noise error term, or alternatively as a first-order autoregressive process with trend:

$$P_t = \alpha + \beta t + \rho P_{t-1} + \varepsilon_t \tag{15.39}$$

[11] For further discussion of the random walk model and alternative tests, see P. Perron, "Trends and Random Walks in Macroeconomic Time Series: Further Evidence from a New Approach," *Journal of Economic Dynamics and Control*, vol. 12, pp. 297–332, 1988, and P. C. B. Phillips, "Time Series Regression with Unit Roots," *Econometrica*, vol. 55, pp. 277–302, 1987.

[12] For a thorough treatment of commodity markets and derivative instruments such as futures contracts, see Darrell Duffie, *Futures Markets* (Englewood Cliffs, N.J.: Prentice-Hall, 1989), and John Hull, *Options, Futures, and Other Derivative Securities* (Englewood Cliffs, N.J.: Prentice-Hall, 1989).

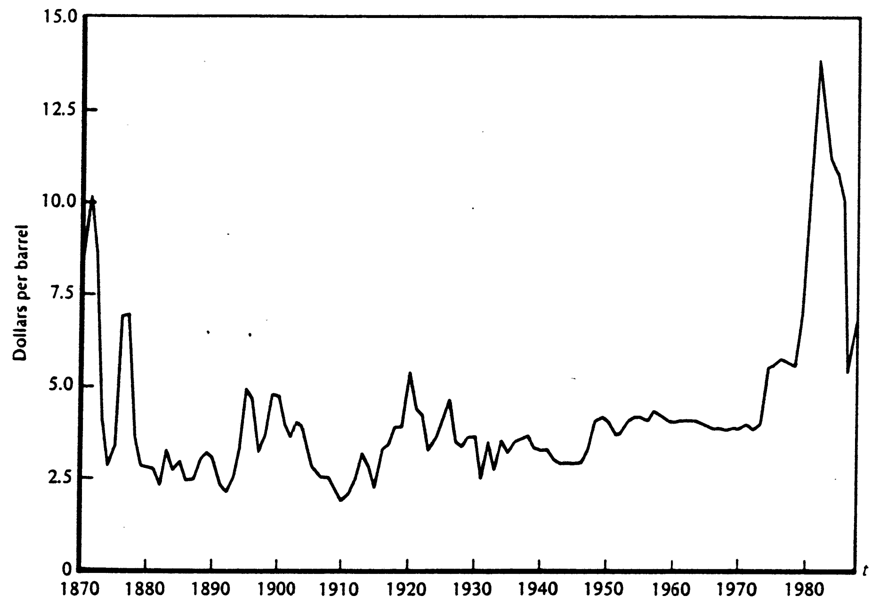


FIGURE 15.18
Price of oil (in 1967 constant dollars).

Since any reversion to long-run marginal cost is likely to be slow, we will only be able to discriminate between these two alternative models with data that cover a long time period (so that short-run fluctuations wash out). Fortunately, more than 100 years of commodity price data are available.

Figures 15.18, 15.19, and 15.20 show the real (in 1967 dollars) prices of crude oil, copper, and lumber over the 117-year period 1870 to 1987.[13] Observe that the price of oil fluctuated around $4 per barrel from 1880 to 1970 but rose sharply in 1974 and 1980–81 and then fell during the mid-1980s. Copper prices have fluctuated considerably but show a general downward trend, while lumber prices have tended to increase, at least up to about 1950.

We ran a Dickey-Fuller unit root test on each price series by estimating the unrestricted regression:

$$P_t - P_{t-1} = \alpha + \beta t + (\rho - 1)P_{t-1} + \lambda\, \Delta P_{t-1} + \varepsilon_t$$

and the restricted regression:

$$P_t - P_{t-1} = \alpha + \lambda\, \Delta P_{t-1} + \varepsilon_t$$

[13] The data for 1870 to 1973 are from Robert Manthy, *A Century of Natural Resource Statistics*, Johns Hopkins University Press, 1978. Data after 1973 are from publications of the Energy Information Agency and U.S. Bureau of Mines. All prices are deflated by the wholesale price index (now the Producer Price Index).

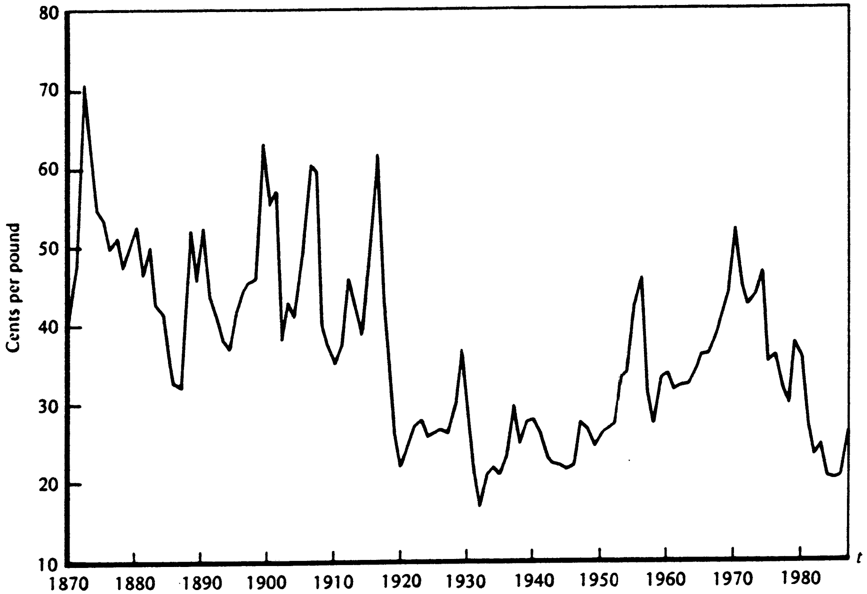


FIGURE 15.19
Price of copper (in 1967 constant dollars).

FIGURE 15.20
Price of lumber (in 1967 constant dollars).

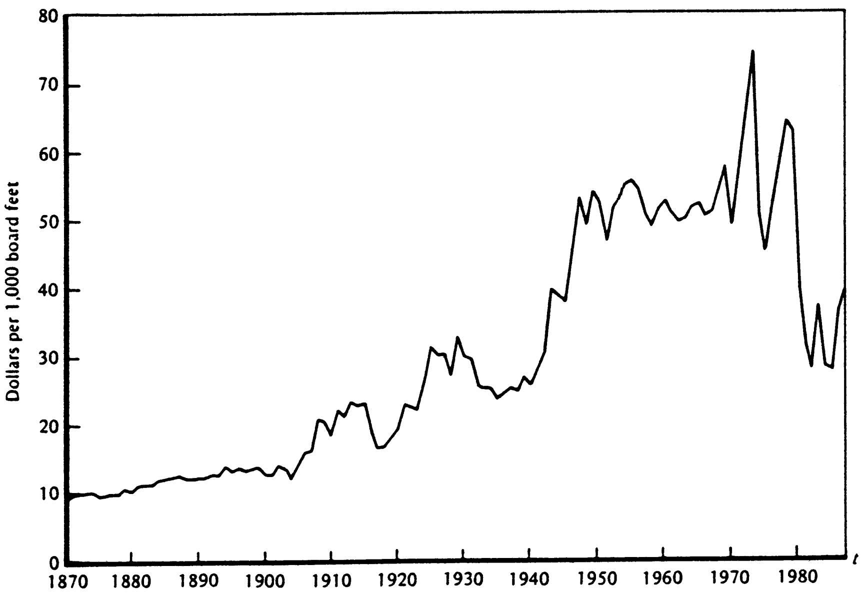


TABLE 15.2
DICKEY-FULLER TESTS

Commodity	α	β	$(\rho - 1)$	λ	ESS
Copper (unrestricted)	11.357 (3.228)	−.0446 (.0208)	−.2417 (.0631)	.0766 (.0941)	4,344.8
Copper (restricted)	−.1969 (.6098)			−.0440 (.0934)	4,913.5
Lumber (unrestricted)	.8825 (.8689)	.0660 (.0263)	−.1560 (.0515)	.1392 (.0958)	2,242.8
Lumber (restricted)	.2488 (.4291)			.0507 (.0938)	2,428.6
Oil (unrestricted)	.4366 (.2188)	.00895 (.00285)	−.2355 (.0459)	.2546 (.0848)	100.09
Oil (restricted)	−.0262 (.0973)			.1760 (.0918)	125.00

We tested the restrictions by calculating an F ratio and comparing it to the critical values in Table 15.1. Regression results (with standard errors in parentheses) are shown in Table 15.2.

In each case there are 116 annual observations. Hence, for copper the F ratio is $(112)(4{,}913.5 - 4{,}344.8)/(2)(4{,}344.8) = 7.33$. Comparing this to the critical values for a sample size of 100 in Table 15.1, we see that we can reject the hypothesis of a random walk at the 5 percent level. For lumber, the F ratio is 4.64, and for crude oil, it is 13.93. Hence we can easily reject the hypothesis of a random walk for crude oil, but we cannot reject this hypothesis for lumber, even at the 10 percent level.

Do commodity prices follow random walks? More than a century of data indicates that copper and crude oil prices are not random walks, but the price of lumber is consistent with the random walk hypothesis.[14]

15.4 CO-INTEGRATED TIME SERIES

Regressing one random walk against another can lead to spurious results, in that conventional significance tests will tend to indicate a relationship between the variables when in fact none exists. This is one reason why it is important to test for random walks. If a test fails to reject the hypothesis of a random walk, one can difference the series in question before using it in a regression. Since many economic time series seem to follow random walks, this suggests that one will typically want to difference a variable before using it in a regression. While this is

[14] The fact that copper and oil prices do not seem to be random walks does not mean that one can earn an unusually high return by trading these commodities. First, a century is a long time, so even ignoring transaction costs, any excess return from the use of a trading rule is likely to be very small. Second, the mean-reverting behavior that we have found may be due to shifts over time in the risk-adjusted expected return.

acceptable, differencing may result in a loss of information about the long-run relationship between two variables. Are there situations where one can run a regression between two variables in levels, even though both variables are random walks?

There are. Sometimes, two variables will follow random walks, but a *linear combination* of those variables will be stationary. For example, it may be that the variables x_t and y_t are random walks, but the variable $z_t = x_t - \lambda y_t$ is stationary. If this is the case, we say that x_t and y_t are *co-integrated*, and we call λ the *co-integrating parameter*.[15] One can then estimate λ by running an OLS regression of x_t on y_t. (Unlike the case of two random walks that are not co-integrated, here OLS provides a consistent estimator of λ.) Furthermore, the residuals of this regression can then be used to test whether x_t and y_t are indeed co-integrated.

The theory of co-integration, which was developed by Engle and Granger, is important for reasons that go beyond its use as a diagnostic for linear regression.[16] In many cases, economic theory tells us that two variables should be co-integrated, and a test for co-integration is then a test of the theory. For example, although aggregate consumption and disposable income both behave as random walks, we would expect these two variables to move together over the long run, so that a linear combination of the two should be stationary. Another example is the stock market; if stocks are rationally valued, the price of a company's shares should equal the present value of the expected future flow of dividends. This means that although dividends and stock prices both follow random walks, the two series should be co-integrated, with the co-integrating parameter equal to the discount rate used by investors to calculate the present value of earnings.[17]

Suppose one determines, using the Dickey-Fuller test described above, that x_t and y_t are random walks, but Δx_t and Δy_t are stationary. It is then quite easy to test whether x_t and y_t are co-integrated. One simply runs the OLS regression (called the *co-integrating regression*):

$$x_t = \alpha + \beta y_t + \varepsilon_t \tag{15.40}$$

and then tests whether the residuals, e_t, from this regression are stationary. (If x_t and y_t are not co-integrated, any linear combination of them will be nonstationary, and hence the residuals e_t will be nonstationary.) Specifically, we test the hypothesis that e_t is not stationary, i.e., the hypothesis of no co-integration.

A test of the hypothesis that e_t is nonstationary can be done in two ways. First, a Dickey-Fuller test can be performed on the residual series. Alternatively, one can simply look at the Durbin-Watson statistic from the co-integrating regression. Recall from Chapter 6 that the Durbin-Watson statistic is given by

$$\text{DW} = \frac{\Sigma(e_t - e_{t-1})^2}{\Sigma(e_t)^2}$$

If e_t is a random walk, the expected value of $(e_t - e_{t-1})$ is zero, so the Durbin-Watson statistic should be close to zero. Thus, one can simply test the hypothesis that DW = 0. For 100 observations, the critical values for this test are shown in Table 15.3.[18] For example, if after running the co-integrating regression we obtain a value of DW of .71, we could reject the hypothesis of no co-integration at the 1 percent level.

TABLE 15.3
CRITICAL VALUES FOR TEST OF DW = 0

Significance level, %	Critical value of DW
1	.511
5	.386
10	.322

Example 15.5 The Co-integration of Consumption and Income An interesting finding in macroeconomics is that many variables, including aggregate consumption and disposable income, seem to follow random walks. Among other things, this means that the effects of a temporary shock will not tend to dissipate after several years, but instead will be permanent. But even if consumption and disposable income are random walks, the two should tend to move together. The reason is that over long periods, households tend to consume a certain fraction of their disposable income. Thus, over the long term consumption and income should stay in line with each other, i.e., they should be co-integrated.

We will test whether real consumption spending and real disposable income indeed are co-integrated, using quarterly data for the third quarter of

[15] In some situations, x_t and y_t will be vectors of variables, and λ a vector of parameters; λ is then called the *co-integrating vector*. Also, we are assuming that x_t and y_t are both first-order homogeneous nonstationary (also called *integrated of order one*); i.e., the first-differenced series Δx_t and Δy_t are both stationary. More generally, if x_t and y_t are dth-order homogeneous nonstationary (integrated of order d), and $z_t = x_t - \lambda y_t$ is bth-order homogeneous nonstationary, with $b < d$, we say that x_t and y_t are *co-integrated of order d, b*. We will limit our attention to the case of $d = 1$ and $b = 0$.

[16] The theory is set forth in R. F. Engle and C. W. J. Granger, "Co-Integration and Error Correction: Representation, Estimation, and Testing," *Econometrica*, vol. 55, pp. 251–276, 1987.

[17] For tests of the present-value model of stock pricing, see J. Y. Campbell and R. J. Shiller, "Cointegration and Tests of Present Value Models," *Journal of Political Economy*, vol. 95, pp. 1062–1088, 1987. For other applications, see J. Y. Campbell, "Does Saving Anticipate Declining Labor Income? An Alternative Test of the Permanent Income Hypothesis," *Econometrica*, vol. 55, pp. 1249–1273, 1987, for a study of the co-integration of consumption and income; R. Meese and K. Rogoff, "Was It Real? The Exchange Rate–Interest Differential Relation over the Modern Floating-Rate Period," *Journal of Finance*, vol. 43, pp. 933–947, 1988, for a study of the co-integration of exchange rates and interest differentials.

[18] From R. F. Engle and C. W. J. Granger, op. cit., p. 269.

1950 through the first quarter of 1988. We first test whether each variable is a random walk, using the Dickey-Fuller test described in the previous section. For consumption, the unrestricted ESS is 21,203 and the restricted ESS is 22,737; with 151 observations, the F ratio is 5.32. Observe from Table 15.1 that with this value of F we fail to reject the random walk hypothesis, even at the 10 percent level. For disposable income, the unrestricted ESS is 40,418 and the restricted ESS is 42,594, so the F ratio is 3.96. Again we fail to reject the hypothesis of a random walk. (What about first differences of consumption and disposable income? We leave it to the reader to perform a Dickey-Fuller test and show that for first differences we can reject the random walk hypothesis.)

We next run a co-integrating regression of consumption C against disposable income YD.[19] The results are as follows (standard errors in parentheses):

$$C = \underset{(6.109)}{-133.82} + \underset{(.00346)}{.9651}\text{YD}$$

$$R^2 = .9981 \qquad s = 23.35 \qquad \text{DW} = .4936$$

We can use the Durbin-Watson statistic to test whether the residuals from this regression follow a random walk. Comparing the DW of .4936 to the critical values in Table 15.3, we see that we can reject the hypothesis of a random walk at the 5 percent level. The residuals appear to be stationary, so we can conclude that consumption and disposable income are indeed co-integrated.

APPENDIX 15.1 The Autocorrelation Function for a Stationary Process

In this appendix we derive a set of conditions that must hold for an autocorrelation function of a stationary process. Let y_t be a stationary process and let L_t be any linear function of y_t and lags in y_t, for example,

$$L_t = \alpha_1 y_t + \alpha_2 y_{t-1} + \cdots + \alpha_k y_{t-k+1} \tag{A15.1}$$

Now since y_t is stationary, the covariances of y_t are stationary, and

$$\text{Cov}\,(y_{t+i}, y_{t+j}) = \gamma_{|i-j|} \tag{A15.2}$$

independent of t. Then, by squaring both sides of Eq. (A15.1), we see that the variance of L_t is given by

$$\text{Var}\,(L_t) = \sum_{i=1}^{k+1} \sum_{j=1}^{k+1} \alpha_i \alpha_j \gamma_{|i-j|} \tag{A15.3}$$

[19] For readers using Citibase data, the corresponding series are GC82 and GYD82.

If the α's are not all 0, the variance of L_t must be greater than 0 and therefore we must have, for all i and j,

$$\gamma_{|i-j|} > 0 \qquad \text{for } i = j \tag{A15.4}$$

Now, for n observations, write the covariances of y_t as a matrix:

$$\mathbf{\Gamma}_n = \begin{bmatrix} \gamma_0 & \gamma_1 & \gamma_2 & \cdots & \gamma_{n-1} \\ \gamma_1 & \gamma_0 & \gamma_1 & \cdots & \gamma_{n-2} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \gamma_{n-1} & \gamma_{n-2} & \gamma_{n-3} & \cdots & \gamma_0 \end{bmatrix} \tag{A15.5}$$

This matrix must be positive definite because the variance of L_t is always greater than zero. Note that

$$\mathbf{\Gamma}_n = \sigma_y^2 \begin{bmatrix} 1 & \rho_1 & \rho_2 & \cdots & \rho_{n-1} \\ \rho_1 & 1 & \rho_1 & \cdots & \rho_{n-2} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \rho_{n-1} & \rho_{n-2} & \rho_{n-3} & \cdots & 1 \end{bmatrix} = \sigma_y^2 \mathbf{P}_n \tag{A15.6}$$

where $\mathbf{P}_n$ is the matrix of autocorrelations, and is itself positive definite. Thus the determinant of $\mathbf{P}_n$ and its principal minors must be greater than 0.

As an example, let us consider the case of $n = 2$. The condition on the determinant of $\mathbf{P}_n$ becomes

$$\det \begin{bmatrix} 1 & \rho_1 \\ \rho_1 & 1 \end{bmatrix} > 0$$

which implies that

$$1 - \rho_1^2 > 0$$

or

$$-1 < \rho_1 < 1 \tag{A15.7}$$

Similarly, for $n = 3$, it is easy to see that the following three conditions must all hold:

$$-1 < \rho_1 < 1 \tag{A15.8}$$

$$-1 < \rho_2 < 1 \tag{A15.9}$$

$$-1 < \frac{\rho_2 - \rho_1^2}{1 - \rho_1^2} < 1 \tag{A15.10}$$

Sets of conditions can also be derived for $n = 4$, $n = 5$, etc., but it should become clear that as the number of observations n becomes large, the number of conditions that must hold also becomes quite large. Although these conditions can provide an analytical check on the stationarity of a time series, in applied work, it is more typical to judge stationarity from a visual examination of both the series itself and the sample autocorrelation function. For our purposes it will be sufficient to remember that for $k > 0$, $-1 < \rho_k < 1$ for a stationary process.

EXERCISES

15.1 Show that the random walk process with drift is first-order homogeneous nonstationary.
15.2 Consider the time series 1, 2, 3, 4, 5, 6, . . . , 20. Is this series stationary? Calculate the sample autocorrelation function $\hat{\rho}_k$ for $k = 1, 2, \ldots, 5$. Can you explain the shape of this function?
15.3 The data series for the prices of crude oil, copper, and lumber are printed in Table 15.4.

TABLE 15.4
PRICES OF CRUDE OIL, COPPER, AND LUMBER (in 1967 constant dollars)

Obs.	Oil	Copper	Lumber	Obs.	Oil	Copper	Lumber
1870	8.64	41.61	9.13	1929	3.63	36.86	32.65
1871	10.16	47.54	9.70	1930	3.64	29.21	29.84
1872	8.35	70.64	9.75	1931	2.47	21.54	29.39
1873	4.24	61.57	9.98	1932	3.50	16.77	25.42
1874	2.81	54.68	9.93	1933	2.71	20.59	24.97
1875	3.37	53.37	9.45	1934	3.52	21.76	24.97
1876	6.90	49.60	9.60	1935	3.17	20.82	23.51
1877	6.95	51.15	9.74	1936	3.48	22.78	24.34
1878	3.74	47.17	9.75	1937	3.55	29.66	25.12
1879	2.84	49.50	10.43	1938	3.65	24.69	24.59
1880	2.80	52.68	10.09	1939	3.32	27.71	26.55
1881	2.74	46.39	10.90	1940	3.26	27.90	25.38
1882	2.26	49.85	11.11	1941	3.28	26.16	27.47
1883	3.27	42.64	11.05	1942	3.01	23.18	29.98
1884	2.66	41.67	11.79	1943	2.89	22.18	39.74
1885	2.95	35.62	12.02	1944	2.91	22.01	38.76
1886	2.42	32.53	12.32	1945	2.89	21.61	37.78
1887	2.44	31.96	12.44	1946	2.92	22.12	45.31
1888	2.97	52.03	12.03	1947	3.27	27.45	52.84
1889	3.18	45.61	12.03	1948	4.09	26.57	48.76
1890	3.00	52.41	12.28	1949	4.19	24.40	53.94
1891	2.33	43.75	12.19	1950	4.05	25.92	52.30
1892	2.08	41.26	12.60	1951	3.67	26.56	46.18
1893	2.44	38.18	12.55	1952	3.77	27.34	51.13
1894	3.24	36.84	13.72	1953	4.07	32.99	52.86
1895	4.90	41.43	13.07	1954	4.21	33.94	54.91
1896	4.58	44.17	13.67	1955	4.18	42.71	55.48
1897	3.17	45.42	13.17	1956	4.09	46.09	54.20
1898	3.64	46.00	13.32	1957	4.38	31.73	50.88
1899	4.80	63.20	13.68	1958	4.23	27.27	48.78
1900	4.72	55.17	12.59	1959	4.11	32.95	51.24

TABLE 15.4
PRICES OF CRUDE OIL, COPPER, AND LUMBER (in 1967 constant dollars)
(Continued)

Obs.	Oil	Copper	Lumber	Obs.	Oil	Copper	Lumber
1901	4.00	57.19	12.53	1960	4.07	33.83	52.57
1902	3.59	38.16	13.95	1961	4.10	31.64	50.89
1903	4.04	43.00	13.42	1962	4.10	32.28	49.43
1904	3.83	40.91	11.95	1963	4.11	32.38	50.03
1905	2.90	49.03	13.77	1964	4.09	33.79	51.58
1906	2.66	60.50	15.80	1965	3.99	36.23	52.09
1907	2.50	59.52	16.01	1966	3.89	36.27	50.46
1908	2.50	40.74	20.49	1967	3.90	38.20	51.17
1909	2.18	37.36	20.17	1968	3.83	40.78	53.99
1910	1.85	34.99	18.29	1969	3.90	44.60	57.65
1911	2.03	37.01	22.12	1970	3.89	52.26	49.09
1912	2.39	45.79	20.98	1971	4.00	45.13	57.88
1913	3.19	42.50	23.25	1972	3.84	42.49	65.20
1914	2.79	38.75	22.56	1973	3.99	43.73	74.21
1915	2.20	48.19	22.95	1974	5.56	46.81	50.57
1916	3.22	61.68	18.98	1975	5.64	35.11	45.27
1917	3.42	44.88	16.34	1976	5.82	36.22	52.35
1918	3.90	36.34	16.51	1977	5.71	32.48	57.54
1919	3.90	26.15	17.86	1978	5.61	30.23	64.23
1920	5.40	21.96	19.15	1979	6.98	37.77	62.77
1921	4.39	24.85	22.84	1980	10.34	35.99	41.17
1922	4.21	26.85	22.53	1981	13.94	27.22	32.09
1923	3.24	27.75	21.91	1982	12.34	23.37	27.71
1924	3.58	25.69	26.13	1983	11.26	24.80	37.21
1925	4.10	26.22	31.33	1984	10.86	20.76	28.27
1926	4.67	26.74	29.94	1985	10.15	20.47	27.58
1927	3.50	26.22	30.12	1986	5.46	20.91	35.70
1928	3.37	29.26	26.75	1987	6.57	25.61	39.91

(*a*) Calculate the sample autocorrelation function for each series, and determine whether they are consistent with the Dickey-Fuller test results in Example 15.4. Specifically, do the sample autocorrelation functions for crude oil and copper prices exhibit stationarity? Does the sample autocorrelation function for the price of lumber indicate that the series is nonstationary?

(*b*) How robust are the Dickey-Fuller test results to the sample size? Divide the sample in half, and for each price series, repeat the Dickey-Fuller tests for each half of the sample.

15.4 Go back to the data for the S&P 500 Common Stock Price Index at the end of the preceding chapter. Would you expect this index to follow a random walk? Perform a Dickey-Fuller test to see whether it indeed does.
15.5 Calculate the sample autocorrelation function for retail auto sales. (Use the data in Table 14.2 at the end of Chapter 14.) Does the sample autocorrelation function indicate seasonality?