Modelling of Rail Potential Rise and Leakage Current in DC Rail Transit Systems

J.G. Yu, B.Sc.
C.J. Goodman, M.A. Ph.D. C.Eng. MIEE.

1. INTRODUCTION

In designing DC railways and tramways, in which the running rails are used for traction current return, two problems associated with this return current must be addressed, viz. the rise of rail potential above earth and the effects of leakage currents, principally electrochemical corrosion. Rail potential distribution is basically a matter of current magnitude and conductor resistance; to what extent this appears as a potential with respect to earth is further influenced by earthing policies. Whilst the most immediate consideration regarding rail potential rise is the touch voltage hazard, it is also, of course, the driving force behind the leakage currents as it appears across the imperfect rail-earth "insulation". Although it is normal for running rails to be bonded together (at least for DC) in order to minimise the return resistance, substantial rail-to-earth voltages will still exist, and consequential stray currents will flow.

It is the intention of this paper to outline what can be done in the way of mathematical modelling of this situation, partly to provide some insight to the processes, and also to define a fundamental approach to the problem. This will reveal that, whist useful understanding is gained from a simple single conductor transmission line model, the modelling of any real situation will involve finite cell analysis using a multi-conductor structure [1].

In the following discussions, it is assumed that substation and train positions are known and that solutions have been obtained for the currents. It is envisaged that, ultimately, to obtain overall time averages of corrosion currents, the stray current modelling techniques will be linked to a multi-train simulator, such as that previously developed by Mellitt and Goodman [2].

2. TRANSMISSION LINE ANALYSIS

If the running rails are sufficiently frequently bonded, they may be considered as a single return conductor. Further, if the longitudinal rail resistance and the rail-earth conductance are assumed uniform, a very simple model results, which is nevertheless instructive in showing the distribution of stray currents.

Basic transmission line equations

Consider a section of transmission line of length l, excited by a finite current source of I_s with a shunt resistance R_s at one end (shunt energization), as shown in Fig.1. Assuming uniformity of the distributed parameters, i.e., G and R being constants, the current and potential of the rails have the following well known form of solution:

Mr. J.G.Yu and Dr. C.J.Goodman are with the School of Electronic and Electrical Engineering, University of Birmingham, B15 2TT.

$$i(x) = c_1 e^{\gamma x} + c_2 e^{-\gamma x} \qquad (1)$$

$$u(x) = - R_0 \, (c_1 e^{\gamma x} - c_2 e^{-\gamma x}) \qquad (2)$$

for $0<x<l$, where

$$\gamma = (RG)^{1/2}$$
$$R_0 = (R/G)^{1/2} \qquad (3)$$

$u(x)$= potential of the conductor (V);
$i(x)$= current in the conductor (A);
R= longitudinal resistance of the conductor (ohms/m);
G= leakage conductance between the conductor and the earth (S/m);
γ= propagation constant (m^{-1});
R_0= characteristic resistance of the transmission line (ohms).

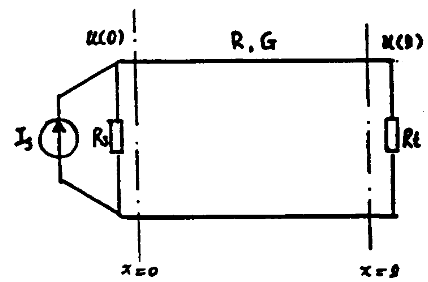


Fig.1 A section of transmission line under shunt energization

In practice, the transmission line considered here may represent a section of rails. The current source I_s may represent the current injected into the rails by a train, or current absorbed by a substation or a regeneratively braking train, depending on its sign. If the line under consideration extends to some distances beyond the section (0,l), R_s and R_t may stand for equivalent input resistances for these extensions. Note that the rails are not bonded to earth at any point in the section; if they were earthed at a substation, the corresponding shunt resistance would be zero, or very small. Earthing resistances at points beyond the section (0,l) (including the two terminal points) can be incorporated into R_s or R_t, depending on their locations.

The two constants c_1 and c_2 are determined by boundary conditions and solved as:

$$c_1 = \frac{k_t R_s I_s}{(R_s+R_0)\Delta} e^{-\gamma l} \qquad c_2 = - \frac{R_s I_s}{(R_s+R_0)\Delta} e^{\gamma l} \qquad (4)$$

and the current and potential are obtained by substituting (4) into (1) and (2):

$$i(x) = - \frac{R_s I_s}{(R_s+R_0)\Delta} [\, e^{\gamma(l-x)} - k_t e^{-\gamma(l-x)}] \qquad (5a)$$

$$u(x) = - \frac{R_0 R_s I_s}{(R_s+R_0)\Delta} [\, e^{\gamma(l-x)} + k_t e^{-\gamma(l-x)}] \qquad (5b)$$

where k_t and k_s are the reflection coefficients:

$$k_t = (R_t-R_0)/(R_0+R_t) \qquad k_s = (R_s-R_0)/(R_0+R_s) \qquad (6a)$$

and

$$\Delta = k_t k_s e^{-\gamma l} - e^{\gamma l} \qquad (6b)$$

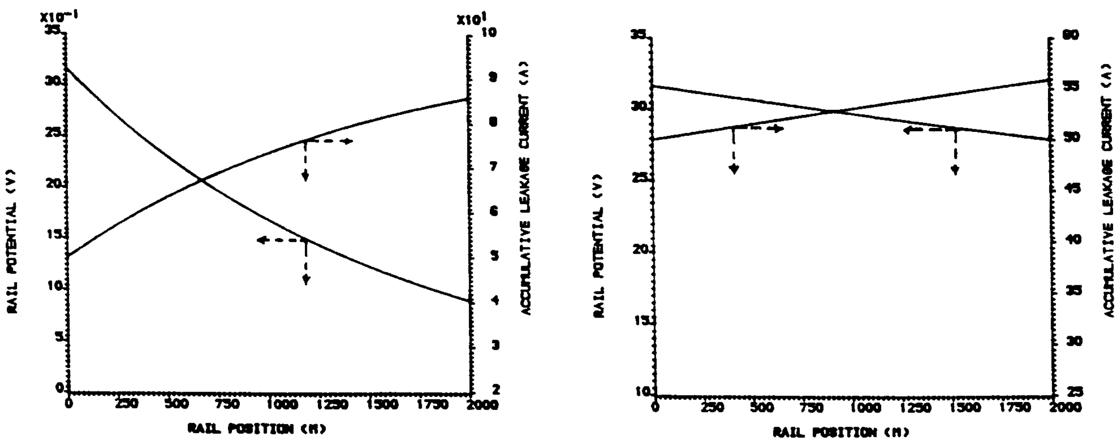
The leakage current per unit length along the conductor is obtained as

$$\delta(x) = - di(x)/dx = G.u(x) \qquad (7a)$$

Here $\delta(x)$ is directly proportional to u(x). The accumulative leakage current up to point x is defined as

$$DI(x) = \int_{-\infty}^{x} \delta(z)dz \qquad (7b)$$

Referring to Fig.1, it is obvious that $DI(0\_)=u(0)/R_s$, and therefore $DI(x)=DI(0\_)+i(0_+)-i(x)$ when $0<x<l$. Fig.2 shows a plot of the potential u(x) and accumulative leakage current DI(x) in a section of transmission line, with the same configuration as shown in Fig.1. Here the parameters used are: R=0.04 Ω/km, l=2.0 km, I_s=100 A, $R_s=R_0$, $R_t=R_0$. In Fig.2(a), G=10.0 S/km, u(x) and DI(x) varies exponentially with distance x within the section. In Fig.2(b), G=0.1 S/km, u(x) and DI(x) varies linearly with distance x within the section. In practice, the former represents a highly leaking track, e.g., in very wet conditions; the latter represents an insulated track in dry conditions. Without actual earthing, the rail potential will strongly depend on G, as the results illustrate.



(a) G=10.0 S/km (b) G=0.1 S/km

Fig.2 Potential variation and accumulative leakage current for a uniform transmission line, according to Eqs. (5b) and (7b). R=0.04 Ω/km; l=2.0 km, I_s=100 A, $R_s=R_t=R_0$.

It is demonstrated in the above that the potential and current of the running rails can be obtained analytically for uniformly distributed parameters. In practice, however, the two parameters R and G will vary. R is dependent on the number of rails serving as the return conductor, and G is dependent on the local ballast and soil conditions. In addition, multiple substations and trains are involved, and substation earthing conditions may need to be considered. It is very difficult or even impossible to obtain closed form solutions for such complicated situations [3]. Therefore, it is desirable to develop solutions using a circuit modelling approach, which is introduced in the next section.

3. CIRCUIT MODELLING

Finite cell modelling

The transmission line shown in Fig.1 can be divided into a number of sections longitudinally, called finite cells, as shown in Fig.3. The lengths of the cells are not necessarily equal. An equivalent π circuit is derived to represent the distributed parameters for each finite cell, and it is admissable for R and G to be different in different zones of the line. In this way, a network consisting of lumped elements is constructed. This is a simple ladder network and it can be easily solved by nodal voltage equations, which are written in matrix form as,

$$[G]\ [V] = [I_s] \tag{8}$$

where [G]= conductance matrix; [V]= voltage vector; $[I_s]$= injection current vector.

With a suitable node numbering scheme as shown in Fig.3, the conductance matrix [G] can be formed as a tridiagonal one, which only needs to have 3 columns of computer storage, and Eq.(8) is solved very efficiently by LU decomposition or Gauss elimination [4]. As mentioned before, substations and trains are represented in the network by ideal current sources, therefore, Eq.(8) can cope very well with multiple substations and trains.

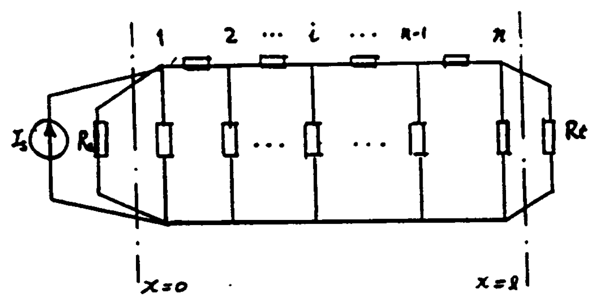


Fig.3 Finite cell model of the transmission line

Accuracy of finite cell modelling

The accuracy of finite cell modelling depends on the distributed parameters, the line length, the termination resistances and the number of cells used. Other parameters being fixed, the computation errors increase as the lengths of the cells increase. In fact, the variation of current and potential in a section of line is decided by the product of the propagation constant and line length, i.e., γl, as is seen from Eqs.(1) and (2). If $\gamma l<0.1$, the line is considered to be electrically "short", and the propagation effect is negligible. In practice, it can be shown that if the finite cell lengths are not greater than the critical "short" length, the errors produced by the finite cell model are contained withing 1% of the rigorous solution.

In the case of short transmission line, the exponential function can be simplified into

$$e^{x} = 1 + x \tag{9}$$

Then Eq.(5b) is simplified into

$$u(x) = -\frac{R_0 R_s I_s}{(R_s+R_0)\Delta}\left[\, 1-\gamma l+k_t(1+\gamma l) + (1-k_t)\gamma x \,\right] \qquad (10)$$

which suggests that the potential is a linear function of distance. This has been illustrated in Fig.2 (b).

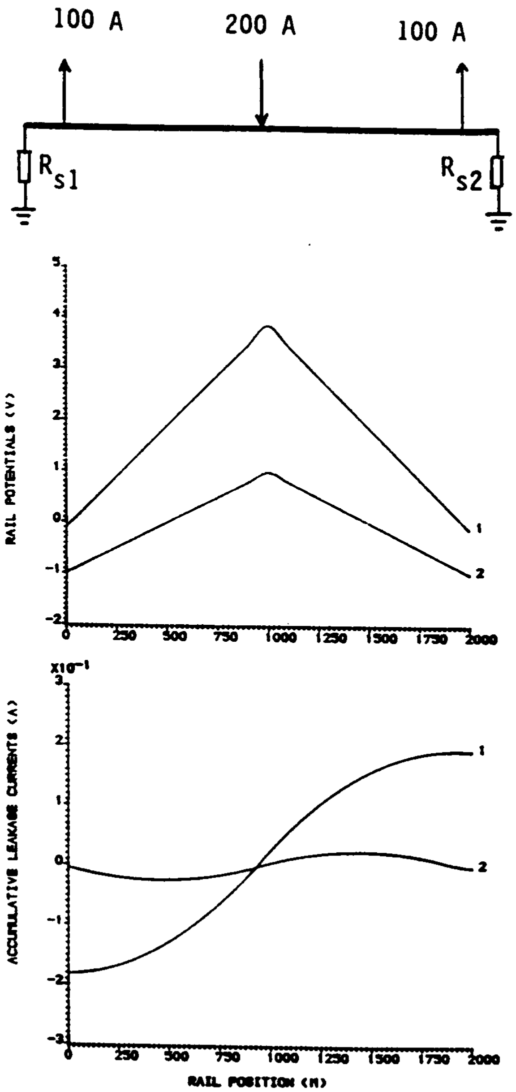


Fig.4 Potential variation and accumulative leakage current for a uniform line.

Curve 1 - $R_{s1}=R_{s2}=R_0$;

Curve 2 - $R_{s1}=R_{s2}=\infty$.

$R=0.04\ \Omega/km$; $G=0.1\ S/km$

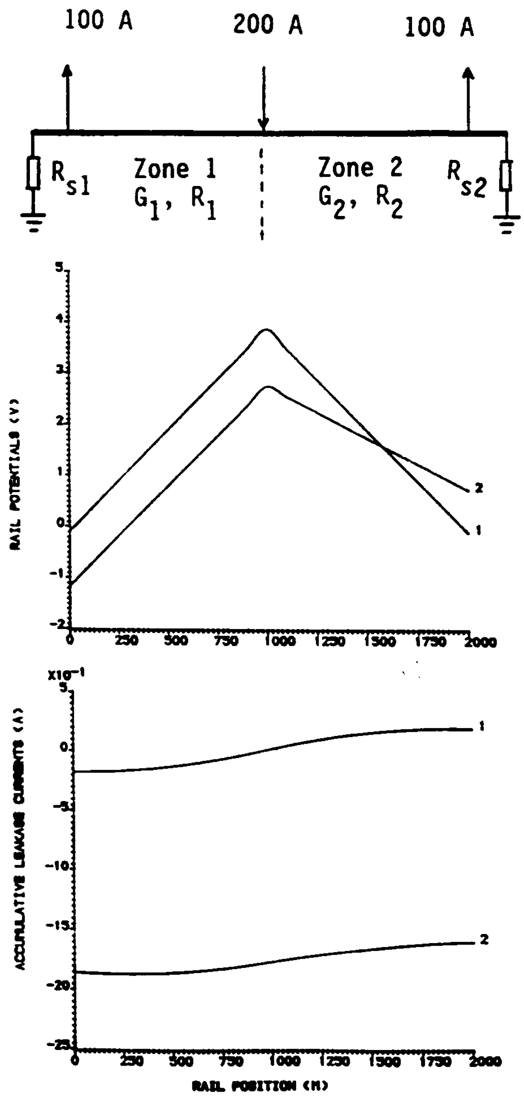


Fig.5 Potential variation and accumulative leakage current for a nonuniform line.

$R_{s1}=R_0(1)$, $R_{s2}=R_0(2)$

Curve 1 - $G_1=0.1$ S/km, $R_1=0.04\ \Omega/km$; $G_2=0.1$ S/km, $R_2=0.04\ \Omega/km$.

Curve 2 - $G_1=0.1$ S/km, $R_1=0.04\ \Omega/km$; $G_2=0.1$ S/km, $R_2=0.02\ \Omega/km$.

Examples

Fig.4 shows an example, in which a rail line with uniform parameters is energised at 3 points. At x=1 km, a train injects 200 A current into the rails. At x=0 km and x=2 km, 100 A currents return to each of the two substations. The rail extensions beyond the section [0,2] are represented by resistors R_{s1} and R_{s2} respectively. The rail potentials u(x) and the accumulative leakage currents DI(x) are plotted. In curve 1, the extensions are assumed infinitely long, therefore $R_{s1}=R_{s2}=R_0$. In curve 2, isolation gaps are assumed in the rails at the ends of the section, therefore $R_{s1}=R_{s2}=\infty$.

In Fig.5, the same line is considered. Here it is assumed that the line is divided into 2 zones, zones 1 and 2, each with different distributed parameters, G_1, R_1 and G_2, R_2 respectively. The dividing point is at x=1 km. The line is assumed to extend infinitely at both ends and therefore the extensions are represented by their respective characteristic resistances, $R_{s1}=R_0(1)$ and $R_{s2}=R_0(2)$. Curve 1 represents the situation of uniform parameters throughout the 2 zones. In curve 2, $R_1=2R_2$, which in practice may mean that more rails are used as the return conductor in zone 2. In the above examples, the finite cell length used is 100 meters.

4. CONCLUSIONS

In evaluating the rail potential rise and leakage current in DC powered electric railway lines, classical transmission line equations are limited to solving cases with uniform parameters. The circuit approach is more effective for modelling different and complicated situations. A finite cell model of transmission line under shunt energization at multiple points has been developed for this purpose. It is demonstrated that reasonably accurate results can be obtained if the finite cells are electrically "short". This finite cell approach, elaborated to represent many parallel conductors, forms the basis of a systematic approach to the representation of the highly complicated situations that arise in with real DC stray currents. Inevitably, computer solution of the resulting electrical network will be necessary and work is proceeding on the development of this software.

5. ACKNOWLEDGEMENT

The authors are grateful to Mr. R.W.Sturland and Mr. L.R.Denning for their contributions in discussions and for the constructive support from GEC-Alsthom Transmission &. Distribution Projects Ltd.

6. REFERENCES

[1]. Yu,J.G. Goodman,C.J. 'Computer simulation of stray currents in DC supplied rail transit systems and their corrosive effects'. To be presented at the International Conference on Transit 2020, IMechE, London, October 29-30, 1990.

[2]. Mellitt,B. Goodman,C.J. Arthurton,R.I.M. 'Simulator for studying operational and power-supply conditions in rapid transit systems'. Proc.IEE, vol.125, No.4, 1978, pp.298-303.

[3]. Starr,A.T. 'The nonuniform transmission line', Proc. IRE, vol.20, No.6, June,1932, pp.1052-1063.

[4]. Press,W.H. et al. 'Numerical recipes - the art of scientific computing', Cambridge University Press, Cambridge, 1987.