Notes in Macroeconometrics

Modelling The Long Run

Applying the Cointegration Methodology

Jarir Ajluni

August 2005

" The Long Run is a misleading guide to current affairs, in the Long Run we are all dead. Economists set themselves too easy, too useless task if in tempestuous seasons they can only tell us when the storm is long past, the Ocean will be flat".

J.M. Keynes

I. Intro

Economic theory suggest a specific nature -both in signs and magnitudes- of the relationships among economic and financial indicators, such a theoretical intuition is applied and tested on the data normally by standard econometric procedures such as OLS and IV regressions, that are relatively inadequate, firstly the dynamic nature of some theories denies marginal effects to be instantaneous and full, and secondly theory does not always suggest a stiff form of the relationships among variables and if it does, its unlikely to hold constantly. The theory of the ***Long Run*** (LR) allows for temporary deviations or disturbances from a defined 'equilibrium' that variables would eventually converge into.

From an econometric sight, in the literature since Nelson and Plosser (1982) showed most of the macroeconomic time series are non-stationary series integrated of order one, and then revolutionary findings of Granger and Newbold (1974) of "spurious regression" problem makes standard econometric procedures clearly inadequate. Modern developments in the literature; the evolution of cointegration in the work of Engle and Granger (1987) and then Johansen (1988), (1991), (1992) enables the applied researcher to properly investigate theory using actual data.

An Ideal overview of the strategies of empirical modelling of the LR is presented by leading authors in the field in the special volume 107 of the *Economic Journal*, introduced by Taylor and Dixon (1997), followed by the papers of Pesaran (1997), Granger (1997) and Harvey (1997).

Definition of Cointegration

Cointegration means that among non-stationary series integrated of order one $I(1)$ there exist a linear combination that is stationary $I(0)$ process, hence a presence of an equilibrium relation, two cointegrated variables X, Y that of an $I(1)$ order are said to be $CI(1,1)$.

Applications of Cointegration

Where does cointegration among variables exist? Or in other words where we suspect cointegration, since most macro and financial time series are $I(1)$ exhibiting a unit root as suggested by Nelson and Plosser (1982), and proved frequently using Augmented Dickey Fuller ADF test then technically motivation to test for cointegration exist among most variables, the point though is in selecting a number of variables relating to each other in a LR equilibrium relation to provide a valid motivation and interpretation for the cointegration test, all theories of the LR are relevant, more specifically examples below suggest –not exhaustively- what forms of relationships in economics that could be tested and was empirically tested in published studies using the cointegration methodology:

[1] Money demand equilibrium theory.

[2] Closed economy aggregate LR equilibrium in goods and financial markets: theory of IS-LM.

[3] Equilibrium theory of the open macroeconomy: the IS-LM-BP equilibrium as three equations are expected to hold in the LR requires cointegration test in a multi equation setting.

[4] Market Arbitrage: any form of a market arbitrage; from supply demand equilibrium, commodity price co-movements, but more commonly arbitrage conditions tested by

cointegration are the Purchasing Power Parity PPP, and the Uncovered Interest Parity UIP the PPP is a theory linking prices across economies through exchange rate movements as deviations from an equilibrium expected to hold in the LR, in other words $(s + p^* - p) \sim I(0)$ is supposed to be stationary, the UIP states interest rates converge into an equilibrium level through the exchange rate movements in the form of $\Delta s = r_t - r_t^* + \varepsilon_t$.

[5] Output co-movements and Growth theories: some growth theories emphasise on the common technological advancements that drives the output across economies to converge in the LR, meaning a suspicious existence of cointegration among output series of different economies.

[6] Permanent Income Hypothesis PIH: theory explaining Consumption to rely on the permanent income in the LR, meaning that $c_t - \beta y_t' \sim I(0)$.

[7] Balanced Growth Hypothesis: states that output, investment and consumption would not diverge from each other and would adjust to each other.

[8] Flexible Price Approach to the Exchange rate: a doctrine explaining the exchange rate by macroeconomic and monetary fundamentals, the exchange rate would be explained as in: $s = (m - m^*) + \beta(y - y^*) + \beta(r - r^*) + v_t$, where $v_t \sim I(0)$

[9] Stock Market Synchronisation: the phenomena of stock markets indices to exhibit co movements is explained by this proposition where testing for cointegration among stock market indices is motivated.

II. Single Equation setting

Starting from an economic doctrine –whatever it is- suggesting a relationship between a number variables in which one variable is endogenous and to be explained by the other explanatory variable(s) this leads to the of the most standard econometric specification:

$$y_t = \alpha + \beta_1 x_t + \varepsilon_t \quad ......(1)$$

Equation (1) is a traditional OLS specification, OLS is invalid as both variables y, x are non-stationary $I(1)$ series and the parameter β is 'spurious' and standard t-test tend to over reject the null implying a misleading significance as shown by Granger and Newbold (1974), since the relation is regarded to hold in the LRthen the temporary deviation from equilibrium is given by the error term:

$$\varepsilon_t = y_t - \alpha - \beta_1 x_t \quad ......(1.1)$$

now according to the LR theory, ε_t is a stationary series $\varepsilon_t \sim I(0)$.

Stationarity of ε_t implies that by regressing the error term on its own lag

$$\varepsilon_t = \phi \varepsilon_{t-1} + \upsilon_t \quad ....(2)$$

and since $\Delta\varepsilon_t = \varepsilon_t - \varepsilon_{t-1}$ then stationarity implies:

$$\Delta\varepsilon_t = (\phi - 1)\varepsilon_{t-1} + v_t, \qquad \phi < 1 \quad .... (2.1)$$

equation (2.1) could be seen also as Dickey-fuller test for the residuals, from equation (1) we get:

$$\varepsilon_{t-1} = y_{t-1} - \alpha - \beta_1 x_{t-1} \quad ....(3)$$

by substituting (3) in (2.1):

$\Delta\varepsilon_t = (\phi - 1)\,(y_{t-1} - \alpha - \beta x_{t-1}) + \nu_t, \quad \phi < 1$ .... (3.1)

the form above is of vital importance as once the model in (1) is estimated in differences

Taking the first difference of model (1) and substituting the term $\Delta\varepsilon_t$ given in (3.1) yields the model presented with the presence of cointegration.

$\Delta y_t = \alpha + \beta\,\Delta x_t + (\phi - 1)\,(y_{t-1} - \alpha - \beta x_{t-1}) + \nu_t$ .... (4)

Cointegration is a linear combination of non-stationary variables, this means that among non-stationary $I(1)$ variables there exists a linear stationary relationship that is $I(0)$; the same applies for higher orders of integration, generally any non-stationary variables of order $I(N)$ could be cointegrated by a linear relation integrated of the order $I(N-1)$.

Theoretically the above has a very sound meaning; the stationary 'cointegrating relation' is interpreted as 'the Long Run' relationship, the model in (4) is known as the "Engle-Granger representation", the components of the model in (4) is further interpreted as:

$(\phi - 1)$: The Cointegrating parameter.

$(y_{t-1} - \alpha - \beta x_{t-1})$: is the deviation from the LR equilibrium.

Engle-Granger Test Procedure
Engle & Granger (1987)

Implemented a procedure to test for the presence of a cointegration by testing the stationarity of the residuals, by testing for unit root using the standard Augmented Dickey-Fuller (ADF) test, this is quite straightforward, known as residual based cointegration test.

First step is to estimate a model using OLS as in model (1) above, according to Engle-Granger the usual presence of the trend in macroeconomic data suggest testing cointegration considering a deterministic trend, then the model in (1) is adapted as:

$y_t = \alpha + \beta' x_t + \delta t + \varepsilon_t$ ......(5)

estimating (5) and saving the residuals, the next step is to apply ADF test to the residuals:

$\Delta\varepsilon_t = \phi\varepsilon_{t-1} + \sum_{i=1}^{\rho} \phi_i\,\Delta\varepsilon_{t-i} + u_t$ .... (6)

then the hypothesis test of ADF would have the following meaning:

$H_0 : \phi = 1$ Unit Root: No Cointegration

$H_1 : \phi < 1$ Stationary residuals: Cointegration.

The above null represents a permanent non-reverting shock to the residuals, in other words deviation from equilibrium is permanent, implying no proof of a theoretical equilibrium in the data. The alternative of stationary residuals means that the deviation from equilibrium is temporary in the short run and eventually the residuals are 'mean-reverting'; meaning that shocks fades out eventually and equilibrium does exist, β would be interpreted as LR multiplier.

Engle and Granger (1982) implemented this procedure using the critical values of ADF test; they also showed that the small sample bias of β could be substantial.

BOX 1: Illustrative Example: Testing the Validity of the Purchasing Power Parity

Purchasing Power Parity PPP is an economic doctrine suggesting the price levels tend to converge into one level across countries through the exchange rate this is known as the Law of One Price (LOOP), measuring the price of an item in a foreign country in terms our domestic currency would yield the same price after considering the exchange rate, in an absolute sense PPP suggest:

$P = SP^*$, where p is price and the starred p refers to foreign price, S is the exchange rate defined as the price of one unit of foreign currency in terms of domestic currency.

then in log transformation it would become:

$p = s + p^*$ solving for the exchange rate: $s = p - p^*$.

The exchange rate reflects the difference in price levels in two countries, this is assumed to hold in the Long Run with a temporary deviation or *Disequilibrium*, this is economically explained as when two markets vary significantly in price level of an item this motivates speculative traders to buy from the foreign "cheaper" market and sell in the domestic market and gain profit, this market mechanism would lead to increased demand on the foreign currency that traders need to buy from abroad pushing its price up (exchange rate depreciation) and simultaneously increased demand in the foreign market would increase the price of the item traded, this process would continue until the possibility of profit is eliminated by rising foreign price and exchange rate depreciation, hence we reach an equilibrium of a one price.

The econometric formulation of PPP equation leads to the following model:

$$s_t = \alpha + \beta_1 p + \beta_2 p^* + \varepsilon_t \quad ...(1)$$

PPP expects $\beta_1 = 1, \beta_2 = -1$, with the intercept representing a 'premium'.

Now the statistical properties of the time series variables come to use, since we suspect that variables $s, p, p^* \sim I(1)$ are non-stationary variables with a unit root, and are probably cointegrated with the cointegration relation being suggested by the PPP. Here we can apply the Engle-Granger methodology once we test for a unit root for s, p, p^*.

We start with estimating (1) by OLS in logs and saving the residuals that are given by:

$$\varepsilon_t = s_t - \alpha - \beta_1 p - \beta_2 p^*$$

then investigating whether this residual is stationary or not, stationarity implies:

$$\Delta\varepsilon_t = (\phi - 1)\varepsilon_t + \upsilon_t$$
$$\varepsilon_t \sim I(0) \quad if \quad \phi < 1$$

we can represent the residual then as:

$$\Delta\varepsilon_t = (\phi - 1)(s_t - \alpha - \beta_1 p - \beta_2 p^*)_{t-1} + \upsilon_t$$

then ADF test is applied on ε_t

$$\varepsilon_t = \alpha + \beta\varepsilon_{t-1} + \sum_{i=1}^{\rho} \phi_i \Delta\varepsilon_{t-i} + \delta t + \nu_t$$

$H_0 : \beta = (\phi - 1) = 0$ presence of a unit root, deviations are permanent: PPP does NOT hold.

$H_1 : \beta = (\phi - 1) \neq 0$ stationary residuals deviations from equilibrium are temporary, PPP holds.

III. Multi-Equation setting

In a more sophisticated procedure where economic theory suggests equilibrium relationships among variables, there are several forms of these equilibrium relations to hold using a number of variables, hence a motivation for a multi equations system. In this setting a VAR is the first step treated as the regression model in the Engle-Granger methodology however in a VAR context incorporation of economic theory is different as the starting point is an unrestricted VAR while in the single equation setting the specification of the underlying regression model was fully motivated by theory more directly, in a VAR the role of theory would be in suggesting the number of cointegrating relations among the endogenous variables then the nature of this theoretical relationship would inform our Identification of the model, where restrictions on the cointegrating relations are imposed according to theory and statistically tested.

The VAR is known to be atheoretical, all variables depend on each other realisations and data is left to speak to itself, another main problem of VARs even if it is a Structural VAR well identified is the assumption of stationarity that does not hold in macroeconomic variables, a VAR in first differences is mis-specified due to the presence of cointegration as discussed below in the formation of a cointegrated VAR, the cointegration methodology of Johansen allows for testing the presence of equilibrium relations simultaneously across a group of variables, in macroeconomics the best example is the IS-LM framework, IS and LM as two equations are modelled dynamically using a VAR and then cointegration test is undertaken to prove whether IS-LM hold or not.

Modelling the LR for a number of equilibrium conditions would take the following steps:

[1] Econometric formulation of the theoretical equilibrium relationships.

Translating the theoretical model into a simple regression model, then to be augmented by lags to capture the dynamics of the model, Its of vital importance to understand the deviations from equilibrium once the equations are econometrically formulated, the deviations from equilibrium presented in the residual terms are used later in imposing the theoretical restrictions on the cointegrating relations.

Consider a static economic doctrines suggesting:

$$y_{1t} = \alpha_1 + \beta_{11} x_t + \ h_t + \varepsilon_{1t} \quad .....(7)$$

$$h_{2t} = \alpha_2 - \beta_{12} y_t + \varepsilon_{2t} \ .....\ (8)$$

By lagging the above models with lags of both the dependant and explanatory variables, also a deterministic trend is usually included, then it is direct to obtain disequilibrium deviations as:

$$\varepsilon_{1t} = y_{1t} - \alpha_1 - \beta_{11} x_t - \ h_t \quad .....\ (7.1)$$

$$\varepsilon_{2t} = h_{2t} - \alpha_2 + \beta_{12} y_t \ ......(8.1)$$

The importance of the above expressions is shown later as these expressions are used in normalising and identification of the cointegrating relations and before all theory provides an intuition on the number of cointegrating relations among the above variables.

[2] Testing the order of integration of the variables (testing for a Unit root using ADF).

Using ADF test for a unit root as in (6) applied individually for each variable is essential as we suspect variables to be integrated of order one as most macroeconomic variables according to Nelson and Plosser (1982), its also important to ensure that all the variables are integrated in the same order, some variables might be stationary and some variables are integrated of order two, the presence of such a variable enforce Multi-cointegration tests, a topic discussed at a later stage in a separate paper.

[3] Specification of an unrestricted VAR and determining the lag order.

Determination of the variables to order the VAR is a fundamental issue, the selection of the variables would change empirical results by inclusion or exclusion of a relevant variable, a lot of anomalies or puzzles in empirical research are due to VAR misspecification. In the case above we would define a vector $Z = (y, h, x)'$ where all variables are in logs, the VAR is:

$$Z_t = \alpha_{0t} + \Phi_1 \, Z_{t-1} + \varepsilon_t \quad .... \ (9)$$

Selecting the appropriate lag order to ensure that residuals are serially uncorrelated, normality of the residuals in the underlying VAR is also a concern, for these reasons the optimal lag order should be determined. The main lag selection statistics (criterions) are the Final Prediction Error (FPE), the Akaike Information Criterion (AIC), Schwarz Bayesian Criterion (SBC) and the Hannan-Quinn Information Criterion (HQIC), AIC and SBC are the most popularly applied, with usually the SBC favours a lower lag order the AIC is the typical standard for determination of the lag order.

For getting the lag order criterion statistic s the first step is to calculate the above criterions is to calculate the log likelihood of the underlying VAR as:

$$LL = \frac{1}{2} T \left[\ln\left|\hat{\Sigma}^{-1}\right| - K \ln(2\Pi) - K \right] \quad .... (9.1)$$

Where T is the total number of observations, K is the number of variables (equations), $\hat{\Sigma}^{-1}$ is the maximum likelihood estimate of $E(\varepsilon_t \varepsilon_t')$ the vector of residuals in (9) that have the dimension of $K \times 1$.

The LL can be obtained after fitting a VAR at any lag order level, the procedure is to run a number of VARs with lag order from one to a defined maximum subject to the data frequency and the sample size, in quarterly data for example a lag order of more than 12 (three years) would be too high for a sample of 100 observations, a lag order of 8 would be more suitable as a maximum lag order, the statistics provided below are obtained at every lag order and on the basis of their results the appropriate lag order could be selected, the Likelihood Ratio LR, AIC, FPE, SBC and HQIC are given in the following standard definitions:

$$LR = 2\left(LL_\rho - LL_{\rho-1}\right) \quad .... \ (9.2)$$

$$AIC = -2\left(\frac{LL}{T}\right) + \left(\frac{2P_\rho}{T}\right) \quad .... \ (9.3)$$

$$FPE = \left|\Sigma_\varepsilon\right| \left(\frac{T + \overline{m}}{T - \overline{m}}\right)^K \quad .... \ (9.4)$$

$$SBC = -2\left(\frac{LL}{T}\right) + \left(\frac{\ln(T)}{T} P_\rho\right) \quad .... (9.5)$$

$$HQIC = -2\left(\frac{LL}{T}\right) + \left(\frac{2\ln\{\ln T\}}{T} P_\rho\right) \quad .... \ (9.6)$$

Note that LL is the Log-Likelihood defined in (9.1), and P_ρ is the total number of parameters in the estimated model that depend on the number of lags in the model estimated, degrees of freedom for all the above statistics is equal to K^2, it is important to highlight that there are different versions of the formulas of the above statistics, full details on the above lag order selection criterions are out of scope of this paper and are not further discussed, it is very often

to select the lag order on the basis of the AIC in the literature, followed by the SBC that usually suggests a smaller lag order than AIC.
Eventually a lag order is selected according to the above criteria or out of empirical reasons; for example if using the properties of the data is also a factor (for quarterly data lag order of 4 is very reasonable), the VAR in (9) would be augmented by ρ lags and inclusion of a deterministic trend as given in equation (10) below:

$$Z_t = \alpha_{0t} + \sum_{i=1}^{\rho} \Phi_i \, Z_{t-i} + \delta \; t + \varepsilon_t \text{ .... } (10)$$

[4] The Vector Error Correction Model (VECM) representation.

Following the Engle-Granger representation theorem, if the $I(1)$ non-stationary variables are cointegrated then an error correction model represents these variables, in first differences the VECM would take the form of:

$$\Delta Z_t = \alpha_{0t} + \prod Z_{t-1} + \sum_{i=1}^{\rho-1} \Phi_i \, \Delta Z_{t-i} + \delta \; t + \varepsilon_t \text{ .... } (11)$$

Where $\prod = \left(I - \sum_{i=1}^{\rho} \Phi_i \right)$ is obtained since $\Delta Z_t = Z_t - Z_{t-1}$

$\prod$ is a square matrix with dimensions equal to the number of variables in the system and is known as **matrix of Long Run multipliers**, the 'dynamic' or 'impact parameters' matrix, it is clear that under the presence of cointegration the "stationary VAR" in differences is misspecified even if the variables are difference stationary, the difference is obvious in the lack of the matrix of LR multipliers.

[5] Testing for the number of LR relations:
Cointegration Ranks by applying Johansen reduced rank tests

Johansen (1995) showed that the number of cointegration relations is determined by the rank of the dynamic matrix Π with the dimension of $k \times k$ has a reduced rank in the presence of cointegration and has a rank of zero if there is no cointegration, hence the rank r of Π is $0 \le r \le m$, $m < k$.

Rank of a matrix is the number of linearly independent columns or rows, the linearly independent relations are the LR relationships among our variables expressed in the rank of the dynamic LR matrix, hence the importance and convenience of the Johansen approach, this is how Johansen procedure is clever! , how to determine this rank? Johansen introduced the well known ***'Trace'*** and ***'Maximal Eigenvalue'*** statistics of characteristic roots test for the number of cointegration relations and reported their critical values obtained using a Monte Carlo simulation.

The rank of a matrix is equal to its non-zero characteristic roots (or Eigenvalues), a characteristic root is defined as the scalar that makes the following equal to zero $(A - \lambda I)x = 0$.

Recall the VECM in (11), testing the number of cointegrating relations is made by obtaining the characteristic roots of matrix $\prod$ its rank would be known, the starting point is to find the characteristics roots its necessary to illustrate:

$$rank(\Pi) = \lambda_i > 0$$

$$|\Pi - \lambda I| = 0$$

The calculation of λ_i done by the software used to present the cointegration test result would perform the following steps as shown in Enders (2004)[1]:

Firstly, Estimate the underlying VAR as in (10) but in first differences:

$$\Delta Z_t = \alpha_{0t} + \sum_{i=1}^{\rho-1} \Phi_i \Delta Z_{t-i} + \delta\ t + e_{1t} \ \text{..........}\ (12)$$

Saving the residuals e_1 then a lagged VAR is to be estimated on the lagged differences given by:

$$Z_{t-1} = \alpha_{0t} + \sum_{i=2}^{\rho-1} \Gamma_i \Delta Z_{t-i} + \delta\ t + e_{2t} \ \text{..........}\ (13)$$

then the canonical correlation between the vectors of e_1 e_2 where the characteristic roots are found by solving:

$$\left|\lambda_i S_{22} - S_{12} S_{11}^{-1} S_{12}'\right| = 0 \ \text{.......}(14)$$

noting that: $S_{ij} = T^{-1} \sum_{i=j=1}^{T} \left(e_j e_i\right)'$

Once the characteristic roots λ_i are obtained by solving the above, the last step would be to derive the Maximum Likelihood estimates of the cointegrating vectors within Π, the n columns that would solve:

$$\lambda_i S_{22} \Pi_i = S_{12} S_{11}^{-1} S_{12}' \Pi_i \ \text{......}(15)$$

solving (15) yields the estimates for Π.

Using the estimated Characteristic roots (Eigenvalues) the following statistical tests for the cointegration rank (number of cointegration relations), Johansen (1991) (1995) introduced the following ***Trace*** and ***Maximal Eigenvalue*** statistics:

Trace Statistic: $\lambda_{trace} = -T \sum_{i=k+1}^{m} \ln\left(1 - \hat{\lambda}_i\right)$ ..... (16)

$H_0: rank \leq r$

$H_1: rank \neq r\ (other\ wise)$

Maximal Eigenvalue: $\lambda_{\max} = -T \ln\left(1 - \hat{\lambda}_{k+1}\right)$ ..... (17)

$H_0: rank = r$

$H_1: rank = r + 1$

[1] Appendix 6.2 in Enders (2004), provides an excellent in depth coverage of the topic.

[6] Altering the VECM: applying diagnostic tests on the model.

Diagnostic testing towards checking the validity of the VECM estimated that need to be more justified in the sense of double checking the lag order by applying a Lagrange-Multiplier LM test for serial correlation in the residual vector and more importantly in checking the normality of the residual vectors since the cointegration rank test relies heavily on the latter, the outcomes could inform the decision of the number of cointegrating relations in the model, despite the importance of the desired normality, in the case of excessive Skewness and Kurtosis the Trace Statistic test is more valid than the Maximal Eigenvalue, this is very useful when both tests suggest different cointegration ranks.

For the technically minded, the above tests are applied exactly as in the VAR framework with the VAR now being in differences with the estimated cointegrating relations as exogenous variables in the system.

[7] Normalisation and Identification of the cointegrating relations:

Once the cointegration rank is determined the VECM model is estimated by Maximum likelihood method imposing Johansen just identifying restrictions as shown in Johansen (1988), an identified model is when *number of restrictions on each of the cointegration relations is equal to the cointegration rank*, and the cointegrating relations should be normalised.

the VECM is given by:

$$\Delta Z_t = \alpha_{0t} + \prod Z_{t-1} + \sum_{i=1}^{\rho} \Phi_i \, \Delta Z_{t-i} + \delta \, t + \varepsilon_t \text{ .... (18)}$$

$$\prod_{k\times k} = \alpha_{k\times r} \beta'_{r\times k} \text{ .... (19)}$$

[8] Imposing and testing theory-based over-identifying restrictions on the cointegrated vectors.

The Initial results of the VECM are obtained with Johansen exact identification restrictions imposed in which these restrictions are not identical to what the theoretical framework is suggesting, then to utilise the methodology of the cointegrated VAR, the VECM is estimated with restrictions imposed by theory on the adjustment parameters matrix $\alpha_{K\times r}$ and/or on the cointegrating matrix of $\beta_{r\times K}$.

Recall equations (7.1) and (8.1), these expressions relate to a theory suggesting a specific restrictions, normalised respectively on *y* and *h*, going through the modelling of the VAR in equation (9) into a VECM the theory suggests two LR relationships meaning that rank is *r =2* and applying the theoretical restrictions from (7.1) (8.1) the cointegrating matrix would be:

$$\beta' = \begin{pmatrix} 1 & -\beta_{11} & -1 \\ \beta_{12} & 1 & 0 \end{pmatrix} \text{ .... (20)}$$

This approach was applied in several studies, in Garrat et al (2003) a model for the UK was tested by imposing and testing over-identifying restrictions on the cointegrating relations, same as in the PhD thesis of Papaikonomou (2003) applying the same methodology to identify the UK aggregate demand.

Whether these restrictions are valid or not is tested by a rather simple log-likelihood ratio test with the null of validity against the alternative of restrictions to be invalid, the log-likelihood is assumed to have a Chi-square χ^2 distribution and is given by the following:

$$T \sum_{i=1}^{r} \left[\ln\left(1-\hat{\lambda}_i^*\right) - \ln\left(1-\hat{\lambda}_i\right) \right] \sim \chi^2 \quad \text{.........} \ (22)$$

Where $\hat{\lambda}^*$ is the Eigenvalues of the unrestricted model, (the initial estimation) and $\hat{\lambda}_i$ is the Eigenvalue of the restricted model, the degrees of freedom for this hypothesis testing are equal to the over-identifying restrictions.

[9] Testing Weak Exogeneity

An important issue in modern macroeconometric modelling is the fact that some variables are exogenous to the model, variables determined outside of the model do not have any adjustment to the endogenous variables in the model, weak exogeneity and its importance in identification was introduced in Johansen (1992) stressing the analysis of partial systems weakly exogenous variables are interpreted by Pesaran, Shin and Smith (2000) as *'Long Run Forcing'* variables, probably the best example of such a variable is the crude oil price included in the macroeconomic modelling framework of an open economy.

In the VAR considered in (9) and then the VECM of (11), suspicion that the variable x is weakly exogenous or long run forcing the partial systems of vector Z could be given by defining a vector $D=(y,h)'$, it is clear that vector $Z=(y,h,x)'=(D,x)'$, then the partial systems are:

$$\begin{bmatrix} \Delta D_t \\ \Delta x_t \end{bmatrix} = \begin{bmatrix} c_{X_h} \\ c_{X_G^{*\pm}} \end{bmatrix} \Psi_t + \begin{bmatrix} \alpha_{D_t} \\ \alpha_{X_t} \end{bmatrix} \beta' Z_{t-1} + \sum_{i=1}^{p-1} \begin{bmatrix} \Phi_i D_t \\ \Phi_i x_t \end{bmatrix} \Delta Z_{t-i} + \begin{bmatrix} \varepsilon_{D_t} \\ \varepsilon_{x_t} \end{bmatrix} \quad \text{..........} \ (23)$$

The condition of Weak exogeneity for variable x is provided in its adjustment coefficient being equal to zero $\alpha_{X_t}=0$, this also means lack of cointegration between variable x and lagged first differences of vector Z, define a vector $W_t=(x_t,\Delta y_{t-1},\Delta h_{t-1},\Delta x_{t-1})'$ among this vector there are no cointegrated relations (r =0), this exogeneity analysis is important in identification.

[10] Incorporation of Structural Breaks

Presence of structural breaks in the data effects the whole results of the model, breaks in every series are either jumps in the intercepts or a break in the slope coefficients, incorporating structural breaks in the analysis is usually done by splitting the data into periods or in constructing a Threshold Autoregressive model TAR, regarding the methodology of cointegrated VARs Hansen (2001) provide a generalisation of Johansen (1998), (1992) allowing for multiple cointegration ranks along the data period, and allowing for structural changes in the adjustment parameters α or in the cointegration parameters β.

BOX 2: Empirical Application: Testing The Long Run IS-LM Model

In a closed Economy setting, the equilibrium in the goods and financial markets is given by the IS -LM relations, that take the following general form:

$$y = -\alpha r + \delta(g - t) \qquad (IS)$$
$$(m - p) = \phi y - \lambda r \qquad (LM)$$

The above equations are static IS-LM model with y being log of output, r for interest rate, m for money, p is the price index, g the government expenditure and lastly t represents the revenues of taxation. The dynamic version of this model incorporates time and applying the cointegration methodology provide the assumption that the above hold in the Long Run with a temporary deviation or *Disequilibrium*, dropping the Government sector from the model, the direct econometric formulation of the above equations would be:

$$y_t = \beta_{11} r_t + \nu_{t1} \qquad (A)$$
$$(m - p)_t = \beta_{21} y_t + \beta_{22} r_t + \nu_{t2} \qquad (B)$$

Where the parameters β_{11}, β_{22}, are expected to have a negative sign and β_{21} to be positive, the temporary deviation from equilibrium is reflected in the residuals of the two models where both error terms are assumed by standard econometric techniques to be a serially uncorrelated white noise processes $\nu \sim N(\mu, \sigma^2)$.

The econometric formulation of a dynamic version of the model requires including lags, since both equations are simultaneous, this motivates using the multivariate estimation by employing a VAR containing all the variables in the two equations, Define a vector including R, M/P, P, Y the often used in the literature and called 'rumpy' (as a nickname), with all variables in natural logs except for the interest rate, the vector is then defined as: $Z = (R, (m - p), p, y)'$ using quarterly Korean data from 1960Q1 –2002Q4,

The ADF test is applied on the above variables showed that all are $I(1)$ which motivates testing for cointegration rank, by which our theory above indicate two LR relations, but first the lag order of the underlying VAR is determined by calculating the lag order statistics

Table 1: Lag Order Selection Statistics for the vector $Z = (R, (m - p), p, y)'$

Lags	LL	LR	AIC	SBC	FPE	HQIC
0	**-892.610**	-	10.934	11.010	6.589E-01	10.965
1	**339.533**	2464.300	-3.897	-3.519	2.400E-07	-3.743
2	**388.620**	98.174	-4.300	-3.620	1.600E-07	-4.024
3	**404.213**	31.186	-4.295	-3.312	1.600E-07	-3.896
4	**630.938**	453.450	-6.865	-5.580	1.200E-08	-6.343
5	**684.753**	107.630	**-7.32625***	**-5.73852***	**7.8e-09***	**-6.68169***
6	**697.230**	24.954	-7.283	-5.393	8.200E-09	-6.516
7	**712.485**	**30.51***	-7.274	-5.082	8.300E-09	-6.384
8	**724.418**	23.867	-7.225	-4.730	8.800E-09	-6.212

* Indicates the optimum lag order by each statistic where the statistic is at minimum except LR statistic, then a VAR as in equation (9) could be estimated of lag order set to 5 as clearly selected by all of the criterions except the Likelihood Ratio, since we are dealing with quarterly data a meaningful lag order is four (4 quarters = 1 year) that provide a logical reason behind the lag order selection, so we proceed with setting lag order equal to four.

BOX 2: Continued

Based on the VAR of the above, we proceed into testing the cointegration Rank based on a VAR of order 4 augmented by a deterministic trend as exogenous variable and an intercept, both unrestricted [which is case V in Pesaran, Shin and Smith (2000)] results shown below in Table 2

Table 2: cointegration rank Statistics

Rank	LL	Parameters	Eigen-values	Trace Statistic	5 % Critical Value	Maximal Eigenvalue	5 % Critical Values
0	674.8	56	-	*77.51*	54.64	40.4711	30.33
1	687.8	63	0.214	*37.04*	34.55	29.3184	23.78
2	**694.4**	**68**	**0.160**	**7.7202***	**18.17**	**7.0855**	**16.87**
3	698.5	71	0.041	*0.63*	3.74	0.6347	3.74
4	698.5	72	0.004				

In the above table the columns of ' Trace Statistics ' and ' Maximal Eigenvalue ' are the results of equations (16) (17), these results show that both the Trace and Maximal Eigenvalues statistics suggest that the null hypothesis of cointegration rank equal to 2 can not be rejected meaning that there exist two long run relations among the variables in the vector $Z = (R, (m-p), p, y)'$, the matrix of long run multipliers given in (19) is then of reduced rank equal to 2.

VECM Estimation Results:

We proceed into estimation of the unrestricted VECM given in equation (18) by the standard Johansen just identifying restrictions that would yield results for the long run multiplier matrix $\prod_{4\times4}$ since our vector consists of four variables (K=4), estimation results of the VECM are excluded and only the multiplier matrix is shown below:

Table 3: Estimates of the Long run multiplier Matrix* Π

	R	p	$(m-p)$	y
R	**4.41** (0.000)	**-0.23** (0.815)	**-1.62** (0.105)	**1.52** (0.129)
p	**-3.19** (0.001)	**2.76** (0.006)	**-1.99** (0.047)	**3.49** (0.000)
$(m-p)$	**-4.33** (0.000)	**0.07** (0.942)	**1.78** (0.075)	**-1.77** (0.077)
y	**-4.33** (0.000)	**0.08** (0.940)	**1.78** (0.076)	**-1.76** (0.087)

* probability p>|z|shown in brackets

Identification: Testing IS-LM

So far the ordering of the variables in the VAR/ VECM is not important, but for identification and normalisation of the long run equations the ordering of the variables would know be changed into $Z = (y, (m-p), p, R)'$ (nothing of the above would be different), the IS-LM equations are normalised below since the temporary deviations from equilibrium are derived from (A) and (B) using the expected signs of the coefficients, the model is normalised as:

$$v_{t1} = y_t + \beta_{11} r_t \qquad (A.1)$$

$$v_{t2} = (m-p)_t - \beta_{21} y_t + \beta_{22} r_t \qquad (B.1)$$

BOX 2: Continued

Equation (A.1) (A.2) are our Long Run cointegrating relations, where (A.1) is normalised on Output and reflects IS equation and (A.2) is normalised on real money balances and reflect the money market equilibrium of the LM equation, now Recall $\prod_{k\times k} = \alpha_{k\times r}\beta'_{r\times k}$ these two "cointegrating relations" by definitions provided in the texts are the rows of the cointegrating matrix, based on the theory we place restrictions on the parameters of the cointegrating matrix $\beta'_{r\times K}$ where (r =2) and (K=4) the cointegrating matrix is suggested by theory as:

$$\beta' = \begin{pmatrix} 1 & 0 & 0 & \beta_{11} \\ -\beta_{21} & 1 & 0 & \beta_{22} \end{pmatrix} \quad \text{..........} (C)$$

Exact identification implies imposing r^2 number of restrictions, number of restrictions equal to the cointegration rank on every cointegrating vector, in our case the rank is 2 hence the exactly identified model estimated by the Johansen methodology places two restrictions on each cointegrating vector, the theory outlined above and presented in (C) suggests imposing three restrictions on the first row and two restrictions on the second cointegrating vector, ending up with one over-identifying restriction.

The above restrictions are tested by the log-likelihood test of equation (21) with degrees of freedom equal to the number of over-identifying restrictions, in which is equal to 1 in our application, with the null hypothesis of " Validity " of restrictions.

Table 4: Estimates of Beta β subject to restrictions

Equation/Variable	y	p	$(m-p)$	R	*Constant*	*Trend*
IS	1	0	0	- 0.129 (0.000)	4.029	-0.065
LM	0.369 (0.001)	1	0	-0.058 (0.001)	1.025	-0.067

LR test of over-identifying restrictions: $\chi^2 = 4.216 \ (0.121)$

The over-identification restrictions suggested by theory cannot be rejected, concluding that the framework of IS-LM is valid as a long run cointegrating relationship between non-stationary variables, in brief IS-LM hold and their estimates are provided in Table 4 where all the estimated coefficients are significant with correct signs once compared with the coefficients in (C), the coefficients in the IS relation show the LR elasticity of the negative effect of a monetary policy shock on output reflecting the downward slope of the IS curve, results from the LM relation show the effect of a deflationary monetary policy on real money balances, second the LM relation shows the high income elasticity of demand for money of the Korean economy.

Data Sources: International Financial Statistics, IMF provided by the ESDS International Data Services, series 34…ZF, 35…ZF, 60…ZF, 63…ZF, 99B..ZF (Korea country code is 542)
STATA 8.2 Do File for the above empirical application is attached for free downloading, however due to copyright regulation data cannot be provided, though it is available through ESDS.

IV. Concluding remarks

The equilibrium concept in economics is regarded as a framework for most of the modelling practice, disequilibrium disturbances and short run dynamics are essential for the discussion of policy intervention when modelling the open macroeconomy, the methodology introduced in this paper serves only as a procedure of adopting techniques introduced and applied in the literature, the steps of modelling the LR are incomplete, the issue of structural breaks and its effects on the cointegration rank, adjustment parameters and the trends of the time series data are not covered, also a possibility to test for a non-linear adjustment process that is supported by theory would be also a compelling addition, further analysis as the innovation accounting of shocks in estimating of the Impulse Response Functions and construction of Variance Decompositions are clearly applicable further steps in the analysis, measuring the length of the Long run is also being examined by the measures of the persistence of shocks.

Nevertheless there is no promised assurance that in the Long Run a proposed theory would definitely hold, in the case of theory based restrictions being invalid then the suggested identification is rejected and the theory has failed, yet such an outcome is not necessarily a disappointment to the applied researcher, as theories are not meant to be exactly spot on the data, all the advancements of the science of econometrics would improve the robustness of econometric estimation but offers no guarantee that a given theory have to hold, as Koop (2000) notes in his directions on how to undertake an empirical investigation, a study showing that a theory does not hold is as valid as a study that provide outcomes that are in line with or supportive to the theory.

References

[1] Engle, R.F. and Granger, C. (1987), “Cointegration and Error Correction: Representation, Estimation and Testing”, *Econometrica*, 55, pp. 251-276.

[2] Garratt, A. Lee, K. Pesaran, H and Shin, Y, (2002), “A Structural Macroeconomic Model of the UK”, Book under preparation.

[3] --, (2003), “A long Run Structural Macroeconometric Model of the UK”, *Economic Journal*, 113, 412-455.

[4] Granger, C. (1997), “On Modelling the Long Run in Economics”, *Economic Journal*, 107, pp 178-91.

[5] Harvey, A. (1997), “Trends, Cycles and Autoregressions”, *Economic Journal*, 107, pp 192-201.

[6] Hansen, P (2001), “Structural Changes in The Cointegrated Vector Autoregressive Model”, *Department of Economics, Brown University*, working paper # 00-20.

[7] Johansen, S, (1988),“Statistical Analysis of Cointegration Vectors,” *Journal of Economic Dynamics and Control*, 12, pp 231-254.

[8] Johansen, S, (1992),“Cointegration in Partial Systems and the Efficiency of Single-Equation Analysis,” *Journal of Econometrics*, 52, 389-402.

[9] Johansen, S., (1995) “Likelihood-Based Inference in Cointegrated Vector Autoregressive models”, Oxford, UK: Oxford University Press.

[10] Koop, G. (2000), “Analysis of Economic Data”, John Whilley publishing.

[11] Nelson, C.R. and C.I. Plosser, (1982), “Trends and Random Walks in Macroeconomic Time Series: Some Evidence and Implications”, *Journal of Monetary Economics*, vol. 10, pp.139-162

[12] Pesaran, M.H. (1997), "The Role of Economic Theory in Modelling the Long Run", *Economic Journal*, 107, pp 178-91.

[13] Pesaran, H. Shin, Y and Smith, R. (2000), "Structural Analysis of Vector Error Correction Models with Exogenous I(1) Variables", *Journal of Econometrics,* 97 pp. 293-343.

[14] Papaikonomou, D, (2003), "Identifying UK aggregate Demand and aggregate Supply Relations Within the Long-Run Structural VAR Framework", *PHD thesis, Department of economics, University of Leicester.*

[15] Enders, William (2004), "Applied Time Series Econometrics", 2nd edition, John Whilley publishing.