Fisher's Ideal Index

	Population	Increase per decade
1950	1,000	-
1960	2,000	2 times
1970	16,000	8 times
	Overall Increase	16 times

Arithmetic Mean = $\frac{2+8}{2}=5$

Is this typical increase per decade?

Ans: 1,000 $\rightarrow$ 5,000 $\rightarrow$ 25,000

No: increase was to 16,000 not 25,000

Try Geometric Mean

$$= \sqrt[n]{X_1 . X_2 . X_3 .... X_n}$$

i.e. $\sqrt{2.8} = \sqrt{16} = 4$

$1{,}000 \rightarrow 4{,}000 \rightarrow 16{,}000$

Total is correct!

<u>Conclusion</u>

If we want to average a number of ratios we should use the geometric mean.

Hence “Ideal Index” (Fisher):

$$=\sqrt{(\text{Lasperyes Index})(\text{Paasche Index})}$$

What do we mean by “Ideal”?

Fischer’s Tests

1. Identity Test

   Comparing one year with itself, index shows no change.

2. Proportionality Test

   When all prices move in proportion, so does index

3. Change of Units Test

   Index is invariant under any change of units of measurement

Let P_{st} be a price index which compares prices in year t with prices in year s

Then:

1. Identity Test

   $P_{tt} = 1$

2. Proportionality Test

   $P_{st} = \lambda$ when $P_t = \lambda\ P_s$ for each item

   (Index is λ when all prices rise by λ)

3. Change of units test

   P_{st} is invariant to changes of units

Lasperyes and Paasche pass each of these tests

4. Time Reversal Test

$$P_{st} = \frac{1}{P_{st}} \quad (s \neq t, \quad s \text{ and } t=0,1,2...)$$

or $$P_{st}.P_{ts} = 1$$

e.g

	Prices			Quantities		
	1950	1960	1970	1950	1960	1970
Good A	1.2	1.4	1.2	50	70	50
Good B	0.2	0.4	0.2	100	110	100

1970 is identical to 1950

Changes in 1960s reversed everything that happened in 1950s

Product of 1960 index using 1950 base and 1970 index using 1960 base should =1, since total change over 2 decades is zero

Paasche and Laspeyres Fail

Fisher Index passes

5. Factor Reversal Test

(Price Index)(Quantity Index) = Total Expenditure Index

e.g. Price index increases two fold
Quantity index increases three fold

Then total expenditure index should increase six fold

Paasche and Laspeyres fail; Fisher passes

Note:

If we use either

Laspeyres price index & Paasche quantity index

Or Paasche price index & Laspeyres quantity index

Then test is passed

These tests do not have the central theoretical importance claimed by Fisher

But Ideal index

- does make sense
- does pass the above tests
- can easily be computed

Hence this approach is a convenient tool for judging the comparative merits of the various formula used in practice.

Chain Index Numbers

Laspeyres index uses fixed weights of some base year. But weights can become out of date. e.g. P_{st} where t=2000 and s=1975 is using weights that are 25 years out of date.

Remedy: Use shifting weights in a ***chain linked index***

This constructs index out of product of short run indices.

Index number for period t based on period 0:

$$P_0^t = P_0^1 . P_1^2 . P_2^3 . P_3^4 ..... P_{t-2}^{t-1} . P_{t-1}^t$$

So given index up to *t-1* can update to *t* by multiplying by index for current period P_{t-1}^t

If we know value of index number in previous period, we can use chain index technique to compute value of number in current period

Example of Chain Linking

Year	Annual Price Index (P_{t-1}^{t})
1991	103.3
1992	103.0
1993	102.6
1994	102.2
1995	102.7

Compute $P_{90}^{95} = P_{90}^{91} . P_{91}^{92} . P_{92}^{93} . P_{93}^{94} . P_{94}^{95}$

= 103.3 . 103.0 . 102.6 . 102.2 . 102.7 = 114.6

Prices have risen by 14.6% between 1995 and 1990.