TABLE of CONTENTS

LIST of FIGURES

LIST of TABLES

LIST of EQUATIONS

PREFACE

The author had the distinct privileged of attending a water flooding course (IHDRC WaterFlooding, Dhahran, Saudi Arabia, September, 2000) given by Dr. Zaki Bassiouni (Louisiana State University, Baton Rouge, Louisiana).

In this document various results are presented from an application of aspects of water flooding to demonstrate the enhanced recovery that should be expected from a water flood implementation. Emphasis has been place on creating a time and/or pressure dimension which can be incorporated into incremental economics which should along with other considerations be used to justify any water flood project.

This document is also a work in progress.

WATER FLOODING CONCEPTS IN OIL AND GAS RESERVOIRS

Introduction

Some basic relationships and/or equations and/or concepts that are applied in petroleum reservoir water flooding engineering and planning are presented. These concepts are fundamentally theoretical but serve as a framework for planning and/or implementing a water flood project. This document is but a brief overview of the subject matter.

Objective

The objectives of this text are primarily to present various relationships and results that are used and considered in water flooding computations. Among the subjects and results are:

1. recovery factors for various water flood techniques
2. a working model that can be use for screening economics with respect to water flooding project considerations

I. Piston-Like Displacement

The fundamental concepts of production of an oil reservoir with no free gas and water injection in a “Piston-Like” displacement are as follow: [1]

Oil Production = Water Injection

1. the upswept region contains only irreducible water and oil
2. the swept region contains only residual oil and water

The assumptions are that the oil is swept by the invading water front, no oil flows behind the water front and that only residual oil remains behind the invading water front. The overall efficiency of the “piston-like” displacement process is characterized by the displacement efficiency or microscopic efficiency, E_D, and the sweep efficiency or macroscopic efficiency, E_V. E_D is the recovery efficiency in that portion of the reservoir rock invaded by the displacing fluid and E_V is that fraction of the reservoir rock actually invaded by the displacing fluid.

The overall or total recovery efficiency, E_r, is:

$$E_r = E_V \cdot E_D$$

Equation (1-0) *Total Recovery Efficiency*

The following characterizes the reservoir system:

$$MOS = S_{oi} - S_{or} = 1 - S_{wi} - S_{or}$$

Equation (1-1) *Movable Oil Saturation*

---

[1] See the list entitled Nomenclature in Appendix One towards the end of this document

$$N = V_p \cdot (1 - S_{wi}) / B_o$$

Equation (1-2) *Original Oil In Place (STB)*

$$V_p = A \cdot h \cdot \phi \ / \ 5.614583 \ (ft\text{^}3/bbl)$$

where A is in ft$^{\wedge 2}$, h is in ft and ϕ has units v/v.

$$V_p = 7758.358 \ (bbl/Acre-ft) \ A \cdot h \cdot \phi$$

where A is in Acres, h is in ft and ϕ has units v/v.

Equation (1-3) *Pore Volume (rbbl or rb)*

The displacement or microscopic recovery efficiency at break through is:

$$E_{D@BT} = \frac{S_{oi} - S_{or}}{S_{oi}} \quad \text{(\% of Oil In Place, N)}$$

Equation (1-4) *Displacement Efficiency at Break Through % of N*

$$E_{D@BT} = S_{oi} - S_{or} \quad \text{(\% of Pore Volume, } V_p \text{)}$$

Equation (1-5) *Displacement Efficiency at Break Through % of V_p*

$$V_i = V_D = V_p \cdot (S_{oi} - S_{or})$$

Equation (1-6) *Volume Injected at Break Through*

$$T_{BT} = V_i / Q_i$$

Equation (1-7) *Time to Break Through*

Where Q_i is the injection rate (bbl/d).

The "Piston-Like" displacement approach is an ideal one and yields greater recovery efficiencies that can be realized, hence it is useful only in an overall perspective sense and forms a basis for understanding more sophisticated water flooding models and/or calculations.

II. Buckley-Leverett Approach

The Buckley-Leverett approach is characterized by the frontal advance equation which essentially models the flood front and saturation profiles in the reservoir. Assumptions are that ahead of the water flood front only oil is moving and at the front there is a rapid increase in the displacing phase saturation. From the injection point to the water front a region of continuously increasing displacing phase saturation exists. At the injection point the oil saturation is at its residual value, S_{or}. Within the region of changing saturation behind the front both the oil and displacing phase, water, are assumed to flow simultaneously as related to their relative permeability relationships. After breakthrough there is a very rapid increase in the WOR which is followed by a time period of a slower increase in WOR. The Buckley-Leverett Approach further assumes two-phase flow of immiscible, incompressible fluids and a reservoir system of homogeneous permeability with negligible capillarity. In 1942 Buckley and Leverett also introduced the frontal advance equation. The Mobility Ratio is defined as follows:

$$M = \frac{\lambda_w}{\lambda_o} = \frac{k_w}{k_o} \cdot \frac{\mu_o}{\mu_w} = \frac{\lambda \ displacingfluid}{\lambda \ displacedfluid}$$

The value of M for the reservoir system qualifies the overall water flood efficiency. A general rule is for M<2 with M<1 being preferable for water flooding the reservoir.

Equation (2-0) *Mobility Ratio*

$$f_w = \frac{1}{1 + \frac{k_o}{k_w} \cdot \frac{\mu_w}{\mu_o}}$$

Equation (2-1) *Frontal Advance Equation (Simple Form)*

$$f_w = \frac{1}{1 + \frac{\mu_w}{\mu_o} \cdot \alpha \cdot e^{\beta \cdot S_w}}$$

where $\frac{k_o}{k_w} = \alpha \cdot e^{\beta \cdot S_w}$ and α and β are determined from linear regression.

Equation (2-2) *Frontal Advance Equation (Simple Form) Regression*

The water oil ratio in the reservoir is:

$$WOR = \frac{f_w}{1 + f_w}$$

Equation (2-4) *Water Oil Ratio*

$$WOR_{BT} = \frac{f_{wBT}}{1+f_{wBT}}\left(\frac{B_o}{B_w}\right)$$

Equation (2-5) *Water Oil Ratio at Break Through*

$$\frac{\partial f_w}{\partial S_w} = \left(\alpha\cdot\beta\cdot\frac{\mu_w}{\mu_o}e^{\beta\cdot S_w}\right)\Big/\left(1+\alpha\cdot\frac{\mu_w}{\mu_o}e^{\beta\cdot S_w}\right)^2$$

Equation (2-6) *Derivative of f_w with respect to S_w*

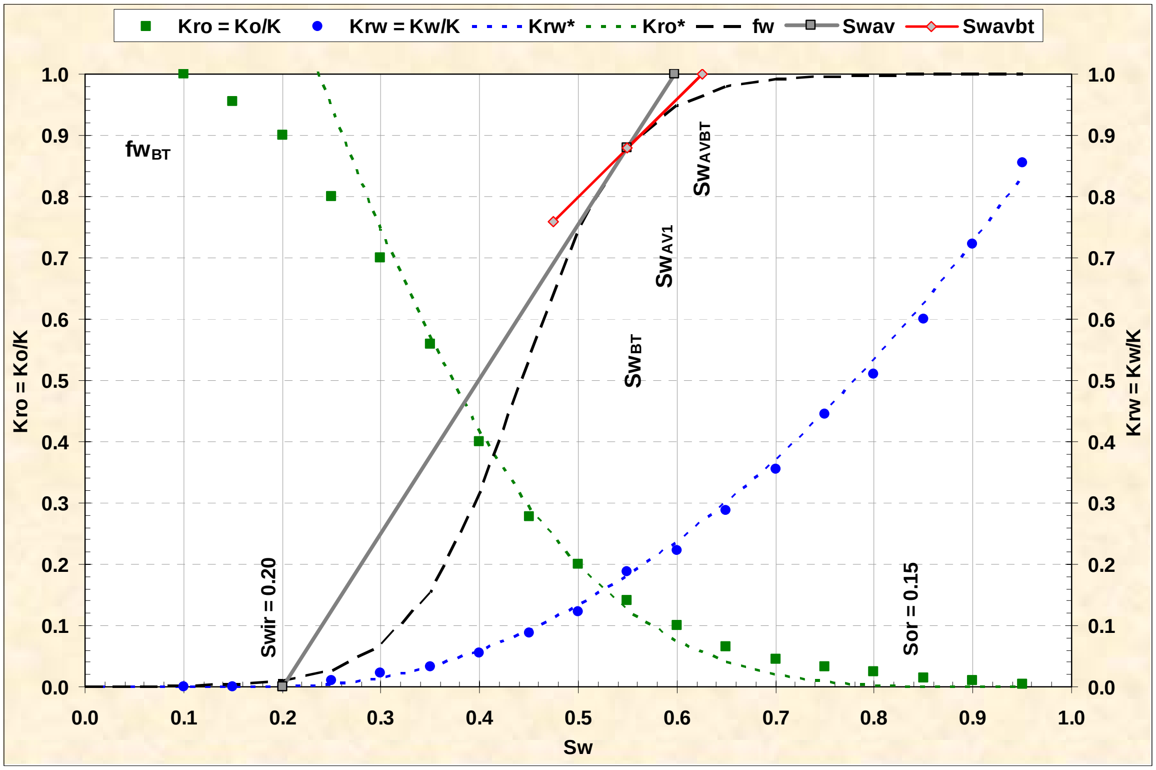

Figure 2 - A – Sw vs. Kro, Krw & fw

Figure 2-A depicts various parameters, e.g. f_w vs. S_w, f_{wBT}, S_{wBT}, S_{wAVBT} for the oil and water relative permeability relationship shown.

$$S_{wAVBT} = S_{wBT} + \left(\frac{\partial f_w}{\partial S_w} @\ f_{wBT}\right)^{-1}$$

Equation (2-7) *S_w Average at Break Through*

$$N_{pBT} = \left(S_{wAVBT} - S_{wi}\right) \cdot V_p \Big/ B_{oi}$$

Equation (2-8) *Cumulative Oil Production at Break Through*

Volume injected at break through:

$$V_{iBT} = \left(S_{wAVBT} - S_{wi}\right) \cdot V_p$$

Equation (2-9) *Cumulative Volume Injected at Break Through*

The estimated time to break through is:

$$T_{BT} = V_{iBT} \Big/ Q_i$$

Equation (2-10) *Estimated Time to Break Through*

$$E_{rBT} = \left[\frac{S_{wAVBT} - S_{wi}}{1 - S_{wi} - S_{or}}\right] \text{ (\% of } \mathrm{V_d}\text{, Displaceable Volume)}$$

Equation (2-11) *Recovery Factor at Break Through, % of V_d*

$$E_{rBT} = S_{wAVBT} - S_{wi} \text{ (\% of } \mathrm{V_p}\text{, Pore Volume)}$$

Equation (2-12) *Recovery Factor at Break Through, % of V_p*

It is advantageous to have an estimate of oil production, recovery factor etc. as a function of time for economic considerations and/or modeling for incremental economics with respect to implementing a given water flood project. N_p as a function of average water saturation:

$$N_p = V_p \cdot \left(S_{wAV} - S_{wi}\right) \Big/ B_{oi}$$

Equation (2-13) *Cumulative Oil Production, N_p,*

Oil production rate as a function of fraction water flow:

$$Q_o = Q_i \cdot \left(1 - f_w\right) \Big/ B_{oi}$$

Equation (2-14) *Oil Production Rate, Q_o*

Time function can be estimated as:

$$T = V_i \Big/ Q_i$$

Equation (2-15) *Time Scale*

The total recovery factor then can be determined from:

$$E_r = N_p / N \quad (\text{ \% of OOIP })$$

Equation (2-16) *Recovery Factor, N_p/N*

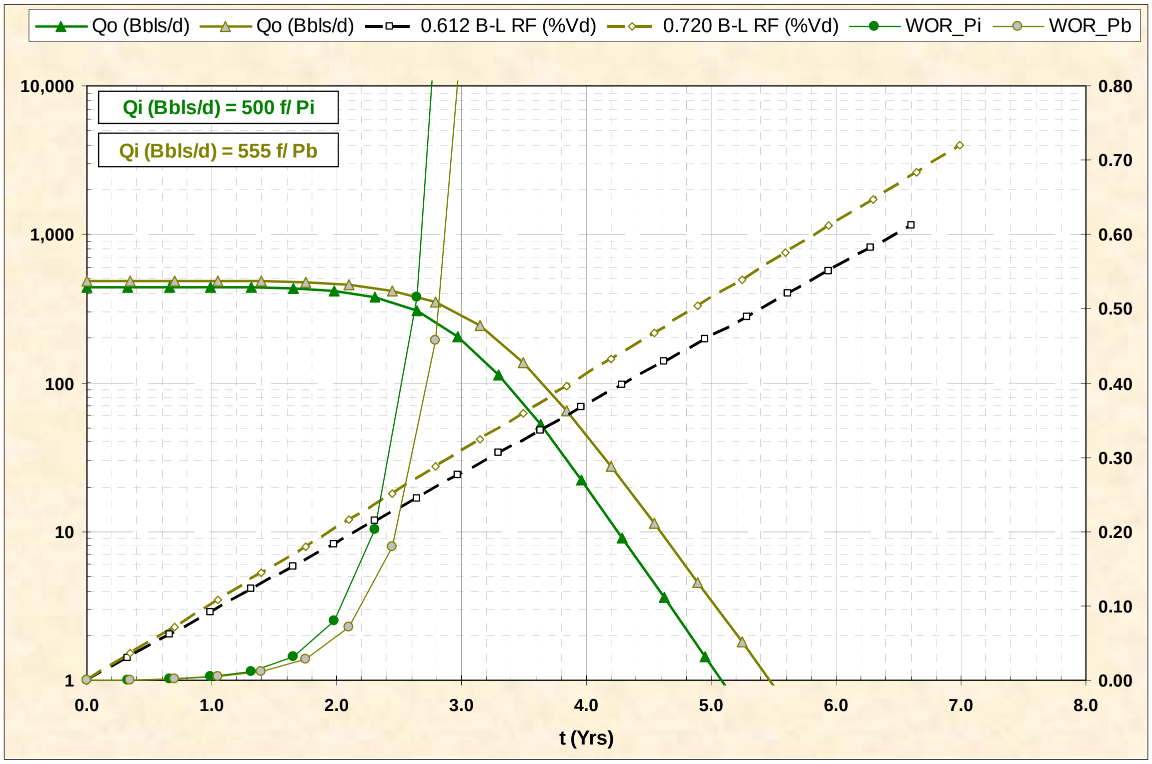

Figure 2 - B – WOR, Qo, Qi, B-L RF vs. Time (Yrs.)

Figure 2-B shows the results of applying the Buckely-Leverett Approach to the data previously shown in Figure 2-A with differing injection rates one starting from the initial reservoir pressure and the other start at bubble point pressure. The rapidly increasing WOR and diminishing oil flow rates are clearly identifiable. Note that the recovery efficiency is linear with respect to time using this approach.

Material Balance Approach

Using the material balance approach assumes that the flow with the reservoir is incompressible and that the reservoir volume is constant and the fluids entering the reservoirs volume are via injection and production wells. Start of the water flood is at x where x designates the corresponding values at that point in time and/or pressure. A tank model is assumed.

III. Water Flooding Above Bubble Point Pressure

The water flood is start at a pressure between initial and bubble point:

$$N_x = N - V_p \cdot S_{ox} \big/ B_{ox}$$

Equation (3-0) Cumulative Oil Production @ Start of Water Flood

Applying the assumption/approximation of $S_{ox} = S_{oi}$ we can derive the primary recovery efficiency at the start of the water flood:

$$E_{rp} = \left(1 - B_{oi} \big/ B_{ox}\right)$$

Equation (3-1) Primary Recovery Efficiency @ Start of Water Flood

$$E_D = \left(S_{oi} - S_{or}\right) \big/ S_{oi}$$

Equation (3-2) Maximum Displacement Efficiency

$$E_V = W_i \big/ V_d$$

Equation (3-3) Sweep Efficiency

$$E_r = 1 - \left(B_o \big/ B_{oi}\right) \cdot \left(1 - E_V \cdot E_D\right)$$

Equation (3-4) Total Recovery Efficiency

$$E_{r\_mx} = 1 - \left(B_{ox} \big/ B_{oi}\right) \cdot \left(1 - E_D\right)$$

Equation (3-5) Maximum Total Recovery Efficiency

$$N_p = N - \left[V_p \cdot E_v \cdot S_{or} \big/ B_o + \left(1 - E_v\right) \cdot S_{oi} \big/ B_{oi}\right]$$

Equation (3-6) Cumulative Oil Produced

With appropriate substitutions and assumptions N_p becomes:

$$N_p = N - \left[V_p \cdot S_{oi} / B_o + \left(1 - E_D \cdot E_v\right)\right]$$

Equation (3-7) *Cumulative Oil Produced*

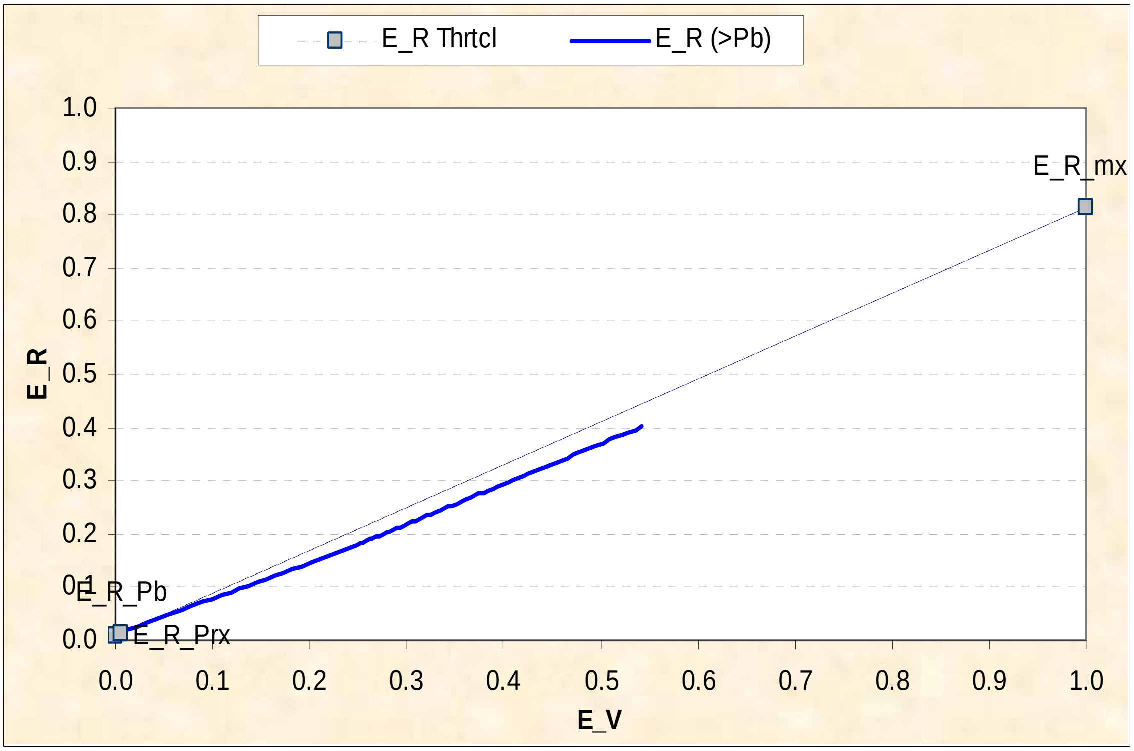

Figure 3 - A – Ev vs. Er – Water Flood Above Bubble Point Pressure

Figure 3-A shows the theoretical and the computed values for E_r and E_v for the data used in this document.

IV. Water Flooding Below Bubble Point Pressure

The water flood is started below the bubble point pressure. Gas saturation of a function of Oil Formation Volume Factor, B_{ox}, where x designates the corresponding values at the point in time and/or pressure below bubble point pressure at which water flood is started and the Primary Recovery, E_{rp}, a function of B_{ox}, N_{px} and S_{ox} is as follows:

$$E_{rp} = 1 - \left(\frac{B_{oi}}{B_{ox}} \cdot \frac{S_{ox}}{S_{oi}} \right)$$

Equation (4-0) *Total Primary Recovery Efficiency*

$$S_{gx} = S_{oi} \cdot \left[1 - \frac{B_{ox}}{B_{oi}} \cdot \left(1 - E_{rp} \right) \right]$$

Equation (4-1) *Gas Saturation Function of Primary Recovery*

$$N_{px} = N - V_p \cdot \left(\frac{S_{ox}}{B_{ox}} \right)$$

Equation (4-2) *Oil Produced From Primary Recovery*

At the end of the fill up period N_p and E_v are estimated as follows:

$$S_{g@fu} = S_{oi} \cdot \left[1 - \frac{B_{ox}}{B_{oi}} \cdot \left(1 - E_{r@fu} \right) \right]$$

Equation (4-3) *Gas Saturation at End of Fill Up*

$$E_{V@fu} = \left(S_{g@fu} \middle/ \left(S_{oi} - S_{or} \right) \right)$$

Equation (4-4) *Sweep Efficiency at End of Fill Up*

$$N_{p@fu} = \left(V_p \cdot E_{rp} \cdot S_{or} \right) \big/ B_{ox} + \left(V_p \cdot \left(1 - E_{rp} \right) \cdot S_{oi} \right) \big/ B_{ox}$$

Equation (4-5) *Oil Produced at End of Fill Up*

$$E_{r@fu} = N_{p@fu} \big/ N$$

Equation (4-6) *Total Recovery Efficiency at End of Fill Up*

During the water flooding after fill up:

$$E_D = \left(S_{oi} - S_{or}\right) / S_{oi}$$

Equation (4-7) *Maximum Displacement Efficiency*

$$E_V = \left(W_i - W_p\right) \cdot B_w / V_d$$

Equation (4-8) *Sweep Efficiency*

$$E_r = \left(1 - B_{oi} / B_o\right) \cdot \left(1 - E_V \cdot E_D\right)$$ [2]

Equation (4-9) *Total Recovery Efficiency*

$$N_p = N - \left(V_p \cdot E_V \cdot S_{or}\right) / B_o + \left(V_p \cdot \left(1 - E_V\right) \cdot S_{oi}\right) / B_o$$

Equation (4-10) *Oil Produced During Water Flood*

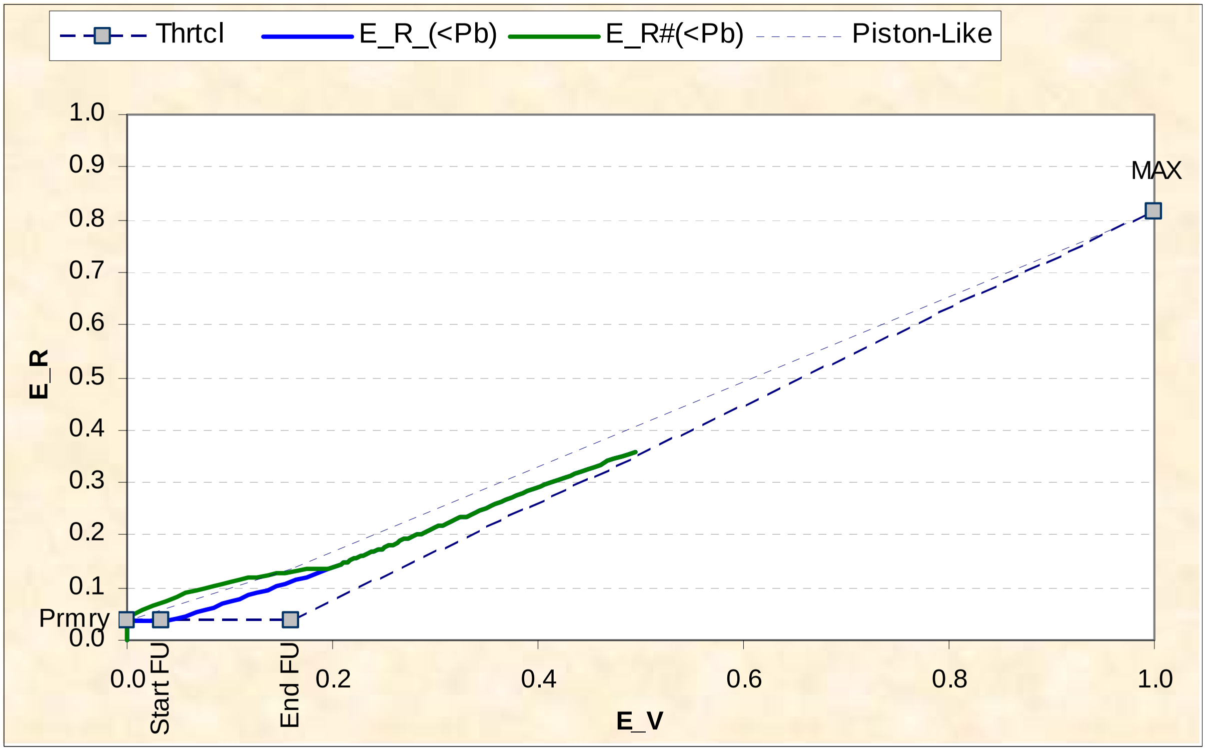

Figure 4 - A – Ev vs. Er – Water Flood Below Bubble Point Pressure

---

[2] Note that E_r is essentially a function of B_o and a Material Balance computation can be used to create B_o as a function of time and/or pressure.

V. Gas Reservoirs

The recovery for a depleted volumetric gas reservoir, E_p, is:

$$E_{rp} = 1 - \frac{B_{gi}}{B_{ga}}$$

Equation (5-1) Recovery Factor Depleted Gas Reservoir

where B_{gi} and B_ga are gas formation volume factors in ft$^{\wedge 3}$/scf.

Should the depleted gas reservoir be water flooded, then the ultimate recovery, E_{wf}, can be expressed as:

$$E_{wf} = 1 - \frac{B_{gi} \cdot S_{gr}}{B_{ga} \cdot S_{gi}}$$

Equation (5-2) Ultimate Recovery Factor Depleted Gas Reservoir

Where S_{gi} is the initial gas saturation and S_{gr} is the residual gas saturations. Incremental gas recovery resulting from the water flood is:

$$\Delta E = E_{wf} - E_p = \frac{B_{gi}}{B_{ga}} \left(1 - \frac{S_{gr}}{S_{gi}} \right)$$

Equation (5-3) Incremental Gas Recovery

VI. Recovery Efficiency Comparsions [3]

The following figure compares the total recovery efficiencies from Material Balance with out water flooding, the Buckley-Leverett approach, Material Balance with water flooding started above the bubble point pressure and via Material Balance with water flooding started below the bubble point pressure.

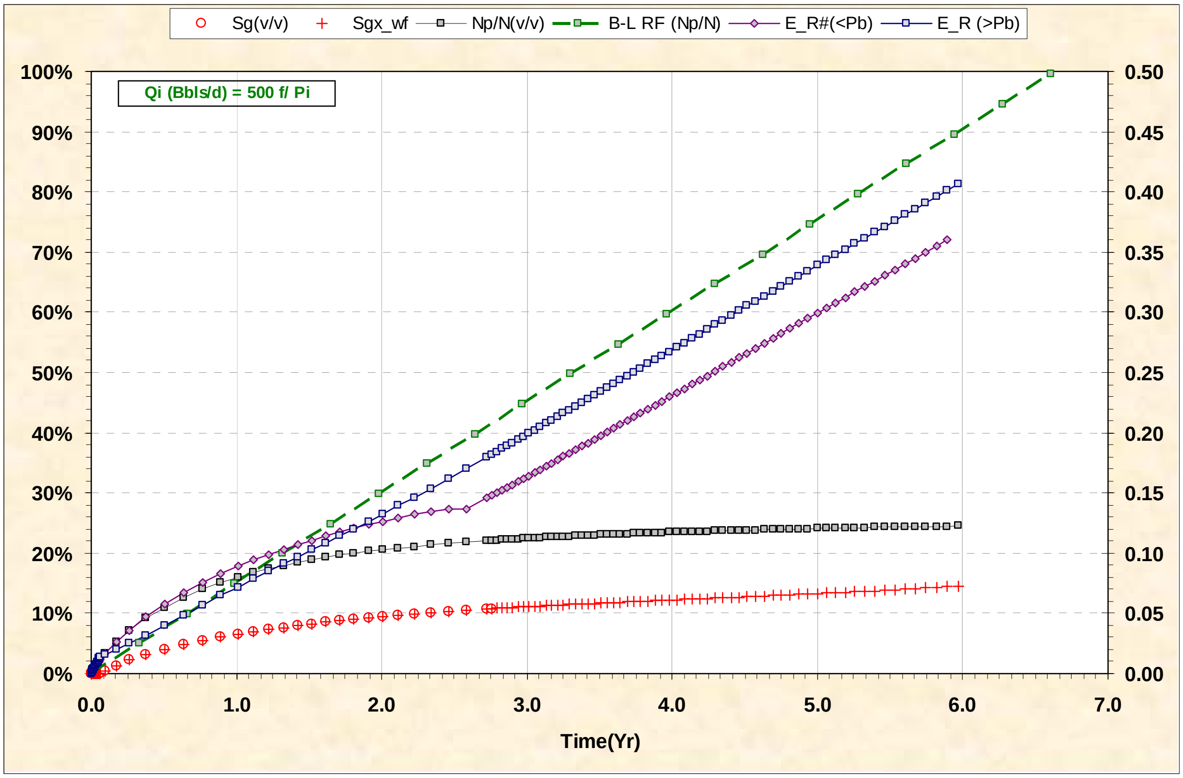

Figure 5 - A – Recovery Efficiency Comparison – Via Models

Note that the recovery efficiencies (except for the N_p/N derived from the Material Balance computations) are linear in later time and hence will yield recovery efficiencies greater than 1 which would not be exactly correct.

---

[3] Np/N is Primary Recovery from the Material Balance computations;
Q_i, Injection Rate is 500 (bbl/d)
B-L is Buckley Leverett Approach
(<Pb) is Material Balance Approach Below Bubble Point Pressure and
(>Pb) is Material Balance Approach Above Bubble Point Pressure

The author has adopted a modified asymptotic approach where the modeled water flood recovery efficiencies are adjusted with respect to the maximum displacement efficiency. In this regard, actual production data can be better matched for future production/injection projections.

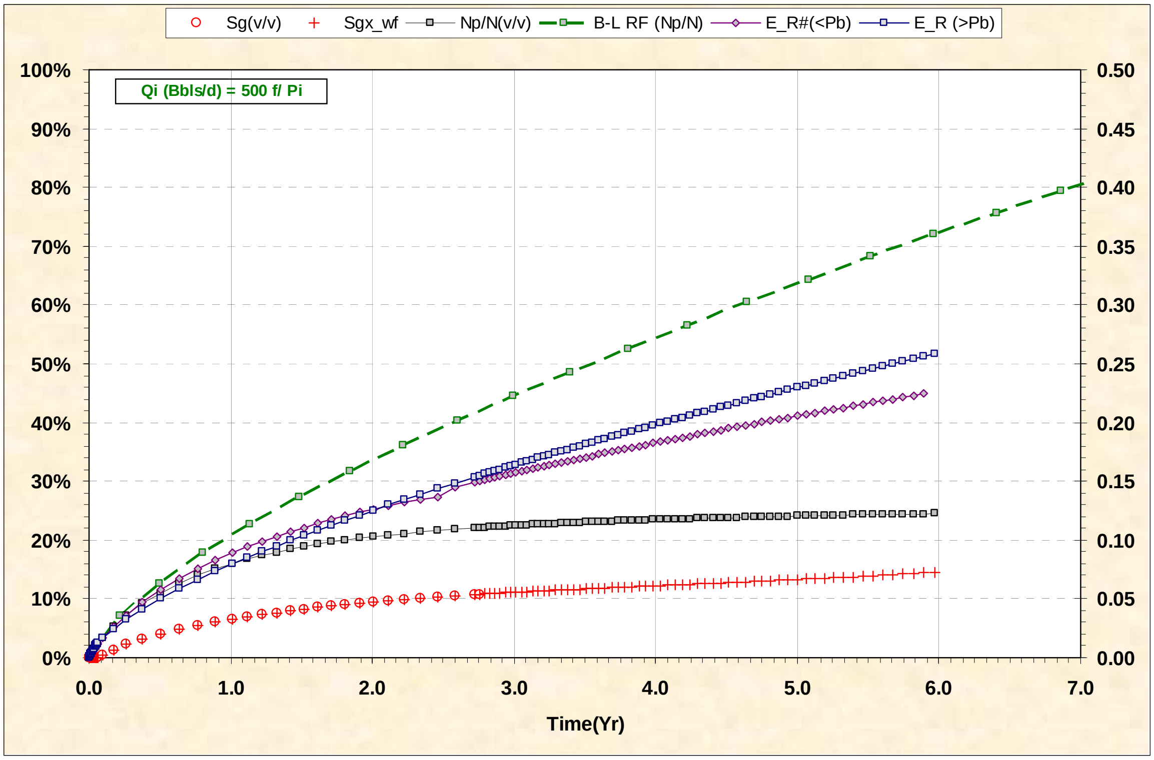

Figure 5 - B – Recovery Efficiency Comparison - Modified [4]

---

[4] Np/N is Primary Recovery from the Material Balance computations;
Q_i, Injection Rate is 500 (bbl/d)
B-L is Buckley Leverett Approach Modified Via E_D Asymptote
(<Pb) is Material Balance Approach Below Bubble Point Pressure Modified Via E_D Asymptote and
(>Pb) is Material Balance Approach Above Bubble Point Pressure Modified Via E_D Asymptote

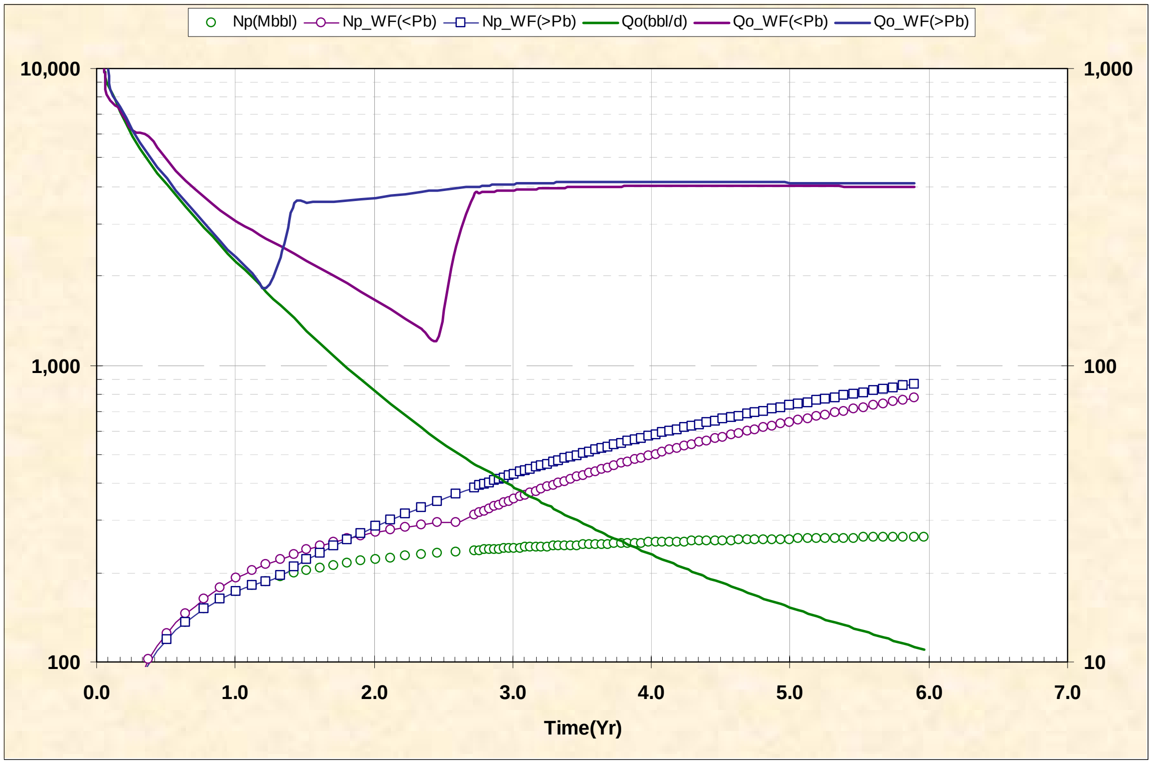

Figure 5 - C – Production Profile Comparison – Via Models

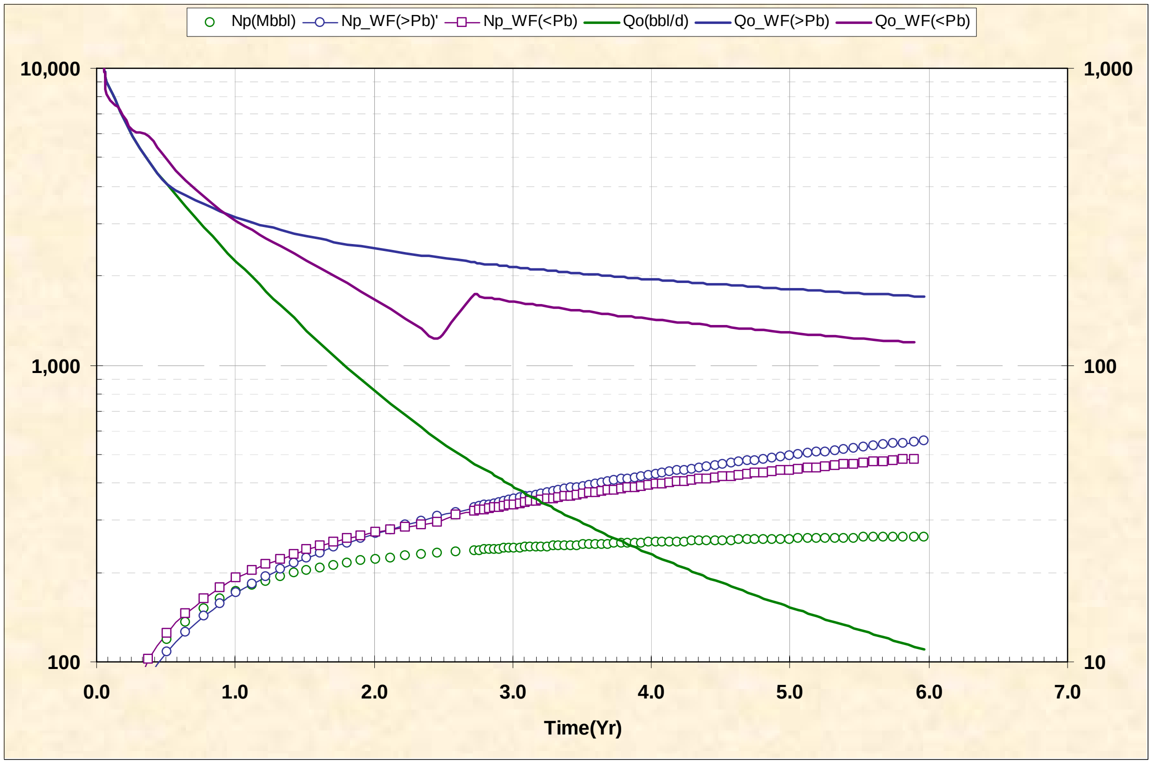

Figure 5 - D – Production Profile Comparison - Modified

VII. Dykstra-Parsons' Method

The reservoir is characterized by stratified non-uniform layers a varying permeability. In this reservoir the injected water will advance more rapidly in the higher permeability layers. The volumetric sweep efficiency, E_V, then becomes a measure of the three-dimensional effect of the reservoir layering, the invasion sweep efficiency, E_i, and the macroscopic sweep efficiency, E_s.

$$E_V = E_i \cdot E_s$$

Equation (7-1) *Volumetric Sweep Efficiency*

The Dykstra-Parson's Method is used to approximate the water flood performance for stratified non-uniform layered reservoirs. The Dykstra-Parsons' Method incorporates the following assumptions in order to make the associated computations practical:

1. The reservoir layers are of equal thickness with each layer having uniform horizontal and vertical perm abilities and that no cross flow exists between layers.
2. Piston like displacement occurs such that only one fluid phase is flowing in any given volume within the reservoir layer.
3. Steady state flow in a linear direction occurs.
4. The flowing and in-situ fluids are immiscible and incompressible.
5. The pressure profile within each layer is the same.
6. Fill-up occurs in each layer prior to incremental production as a result of water flooding. Fill-up times should be considered with respect to the total water flooding application.
7. Each reservoir layer has the same rock and fluid properties with only the layer's absolute permeability differentiating them.

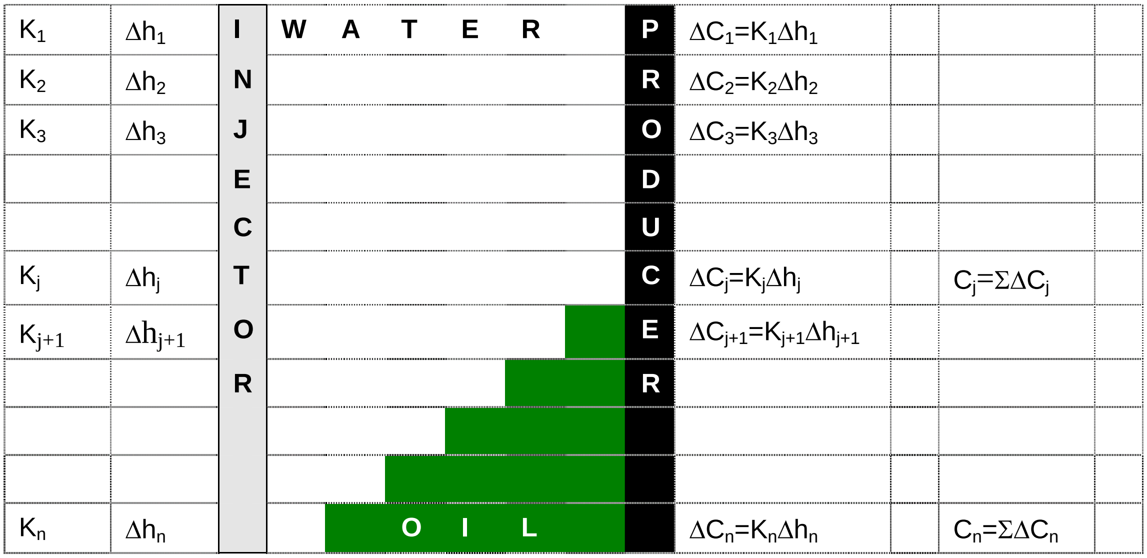

Figure 7 - A – Displacement of Oil by Water in a Stratified Reservoir

ΔC is incremental capacity ($k \cdot h$ (md-ft)) and C is cumulative capacity and h is reservoir interval height (ft).

The oil and water invasion profiles for the system are diagrammed in Figure 7-A. For the i^{th} layer where the flood front is located at x_i and ΔP_i is the pressure differential between the front and the producing point of the i^{th} layer are:

$$Q_w = A \cdot k_i \, \lambda_w \cdot \frac{\Delta P}{x_i}$$

Equation (7-2) *D-P i^{th} Layer Water Flow Rate*

$$Q_o = A \cdot k_i \cdot \lambda_o \cdot \frac{\Delta P - \Delta P_i}{L - x_i}$$

Equation (7-3) *D-P i^{th} Layer Oil Flow Rate*

Where ΔP is the pressure differential between the efflux end of the layer and the influx end at a distance L apart. Therefore the pressure differential can be expressed as:

$$\Delta P = \frac{Q_w \cdot x_i}{A \cdot k_i \cdot \lambda_w} + \frac{Q_o \cdot \left(L - x_i\right)}{A \cdot k_i \cdot \lambda_o}$$

Equation (7-4) *Pressure Differential i^{th} Layer*

Assuming that the j^{th} layer has been broken through by the water flood then all layers with permeability greater than that of the j^{th} layer will also have been broken through. Therefore the fraction of the reservoir for which the layers have been completely flooded out is (j/n). Those layers having permeability less than the j^{th} layer will only be partially swept. The (fractional) total recovery efficiency, E_i, which is defined as the fraction of the reservoir which has been invaded by water can be derived and the relationship is as follows: [5] {*The reader is referred to the various references for the derivations which will not be presented here.*}

$$E_i = \frac{1}{n}\left[j + \frac{(n-j)\cdot M}{M-1} - \frac{1}{M-1} \cdot \sum_{i=i>j}^{n} \left(M^2 + \frac{k_i}{k_j}\left(1 - M^2\right) \right)^{0.5} \right]$$

Equation (7-5) *Fractional Total Recovery Efficiency i^{th} Layer*

When the j^{th} layer has broken through, only water is flowing in the layers with permeability greater than that of the j^{th} layer. Hence, the water flow rate is:

---

[5] Dykstra, H., and Parsons, R.L., “The Prediction of Oil Recovery by Waterflood”, Chapter 12, Secondary Recovery of Oil in the United States, 2nd Edition, API, New York, NY, 1950.

$$Q_w = \sum_{i=i<j}^{n} \left[\frac{k_i \cdot k_{rw}}{\mu_w} \cdot \frac{A \cdot \Delta P}{L} \right]$$

Equation (7-6) *D-P Water Flow Rate*

Only oil is flowing within the layers with permeability less than that in the jth layer:

$$Q_o = \sum_{i=i>j}^{n} \left\{ \frac{A \cdot k_i \cdot \frac{k_{rw}}{\mu_w} \cdot \frac{\Delta P}{L}}{\left[M^2 + \frac{k_i}{k_j} \cdot \left(1 - M^2\right) \right]^{0.5}} \right\}$$

Equation (7-7) *D-P Oil Flow Rate*

And the water oil ratio, WOR = Q_w/Q_o, when the jth layer has broken through is:

$$WOR = \frac{\sum_{i=i-j}^{n} k_i}{\sum_{i=i>j}^{n} k_i \Big/ \left[M^2 + \frac{k_i}{k_j} \cdot \left(1 - M^2\right) \right]^{0.5}}$$

Equation (7-8) *Volumetric Sweep Efficiency*

VIII. Modified Dykstra-Parsons' Method

The "Permeability Variation", V_k, is defined as the median permeability minus the permeability at 84.1 cumulative percent divided by the median permeability, k_{50}. In the Modified Dykstra-Parsons' Method, only the Permeability Variation is necessary to characterize the distribution in so far has the computations are concerned since it is the magnitudes of the permeability are not so important in as much as only the ratios of permeability that occur in the calculations.

$$V_k = \frac{k_{50} - k_{84.1}}{k_{50}}$$

Equation (8-1) *D-P Permeability Variation*

Vertical Sweep Efficiency is also known as and/or referred to as Coverage. Dykstra and Parsons constructed Coverage versus Permeability Variation which gives WOR as a function of V_k and Mobility, M. Hence for any coverage computation a set of curves at WOR's of 0.1, 0.2, 0.5, 1, 2, 5, 10, 25, 50 and 100 are needed. Fassihi et. al. improved the coverage curves of Dykstra and Parsons with empirical correlations and are shown in Appendix Two. The Y correlation parameter was correlated in the form:

$$Y = \frac{(WOR + 0.4) \cdot (18.948 - 2.499 \cdot V_k)}{(M + 1.137 - 0.8094 \cdot V_k) \cdot 10^{f(V_k)}}$$ [6]

Equation (8-2) *Y-Correlation Parameter*

$$f(V_k) = -0.6891 + 0.9735 \cdot V_k + 1.6453 \cdot V_k^2$$

Equation (8-3) *Y-Correlation Exponent Function*

The Y-Correlation factor and Vertical Sweep Efficiency can also be expressed as:

$$Y = a_1 \cdot C^{a_2} \cdot (1 - C)^{a_3}$$

Equation (8-4) *Y-Correlation and Coverage*

Table 1 – Y-Correlation and Coverage Constants

a1	3.3340888568	
a2	0.7737348199	
a3	-1.225859406	

---

[6] Fassihi, M. R., New Correlations for Calculation of Vertical Coverage and Areal Sweep Efficiency, SPE Reservoir Engineering, November 1986.

Areal Sweep Efficiency, E_A, is the fraction of the water flood pattern area actually contacted by water. E_A is a function of the pattern geometry, the mobility ratio, M, and the volume of water injected, W_p. Fassihi et. al. correlated daya from Dyes et. al. for a correlation for E_A:with correlation constants listed in Table 3.

$$\frac{1-E_A}{E_A} = \left[a_1 \cdot \ln\left(M + a_2\right) + a_3\right] \cdot f_w + a_4 \cdot \ln\left(M + a_5\right) + a_6$$

Equation (8-5) *Areal Sweep Efficiency Correlation*

Table 2 – Coefficients in Areal Sweep Efficiency Correlations [7]

Coefficient	5-Spot	Direct Line	Staggered Line			
a_1	-0.2062	-0.3014	-0.2077			
a_2	-0.0712	-0.1568	-0.1059			
a_3	-0.5110	-0.9402	-0.3526			
a_4	0.3048	0.3714	0.2608			
a_5	0.1230	-0.0865	0.2444			
a_6	0.4349	-0.8805	0.3158			

And the Volumetric Sweep Efficiency is:

$$E_V = \frac{E_A / E_D}{\left\{M^{0.5} - \left[\left(M-1\right)\cdot\left(1-E_A / E_D\right)\right]^{0.5}\right\}^2}$$

Equation (8-6) *Volumetric Sweep Efficiency*

---

[7] Ibib.

APPENDIX ONE

Nomenclature

Symbol	**Definition**	**Units**
A	Reservoir or Contour Area	acres
B_o	Oil Formation Volume Factor	rb/bbl
B_{oi}	Initial Oil Formation Volume Factor	rb/bbl
E_A	Areal Sweep Efficiency	v/v
E_D	Displacement Efficiency (Microscopic)	v/v
E_i	Invasion Sweep Efficiency	v/v
E_r	Total Recovery Efficiency	v/v
E_{rp}	Total Primary Recovery Factor	v/v
E_V	Sweep Efficiency (Macroscopic)	v/v
f_w	Fractional Flow	v/v
h	Reservoir or Contour Height	ft
k_o	Oil Permeability	md
k_{ro}	Relative Oil Permeability	v/v
k_{rw}	Relative Water Permeability	v/v
k_w	Water Permeability	md
λ_o	Oil Mobility	darcy/cp
λ_w	Water Mobility	darcy/cp
M	Mobility Ratio (M_Displacing/M_Displaced)	
μ_o	Oil Viscosity	cp
MOS	Movable Oil Saturation	v/v
μ_w	Water Viscosity	cp
N	OOIP	rbbl
N_p	Oil Produced (at time and/or pressure)	bbl
Q_i	Water or Fluid Injection Rate	bbl/d
S_g	Gas Saturation	v/v
S_{oi}	Initial Oil Saturation	v/v
S_{or}	Residual Oil Saturation	v/v
S_{wAV}	Average Water Saturation	v/v
S_{wi}	Initial Water Saturation	v/v
S_{wir}	Irreducible Water Saturation	v/v
T	Time	days or yrs
T_{BT}	Time to Water Break Through	days or yrs
V_d	Displaceable Volume	rbbl

V_i	Volume of Injected Water or Fluid	bbl
V_k	Permeability Variation	v/v
V_p	Pore Volume	rbbl
WOR	Water Oil Ratio	v/v

APPENDIX TWO

Vertical Coverage and Areal Sweep Efficiency

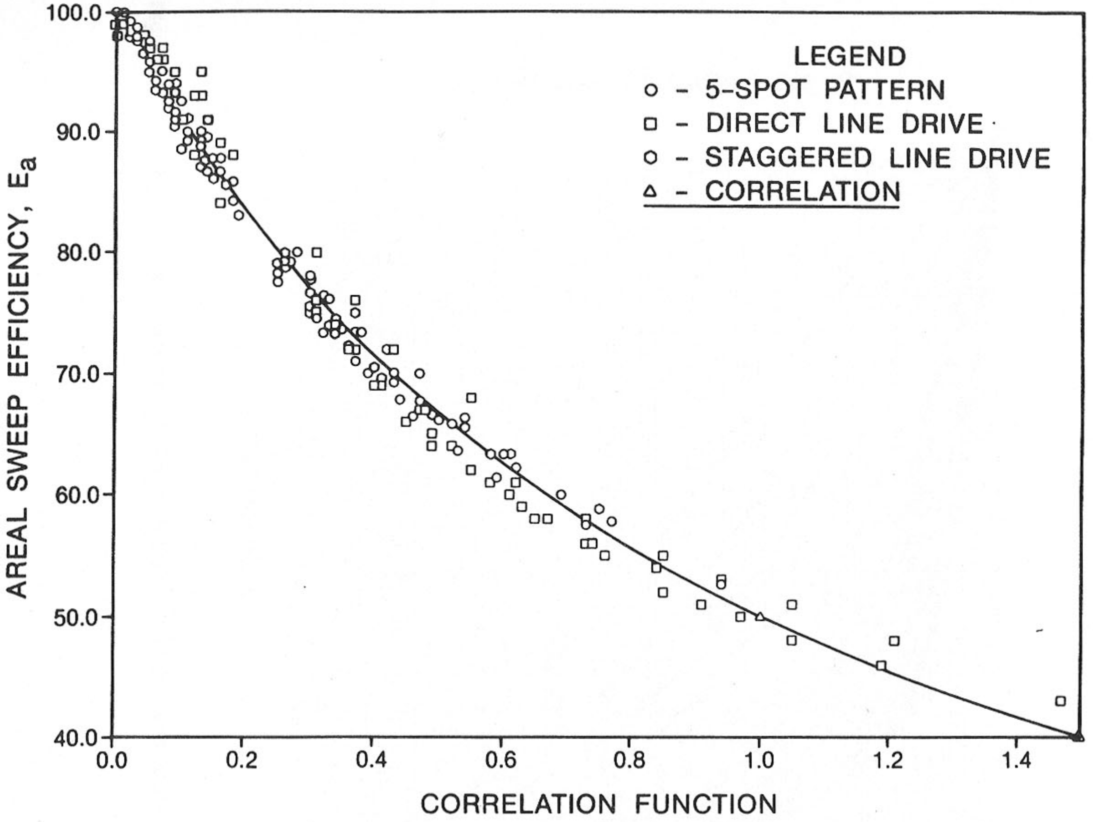

Figure A-2-1 – The Actual and The Correlated Areal Sweep Efficiencies [8]

---

[8] Fassihi, M. R., O'Brien, W. J., A Predictive Model for Water Flood Performance Using a Handheld Calculator, ARCO Oil and Gas Co., Plano, Texas, July 1984.

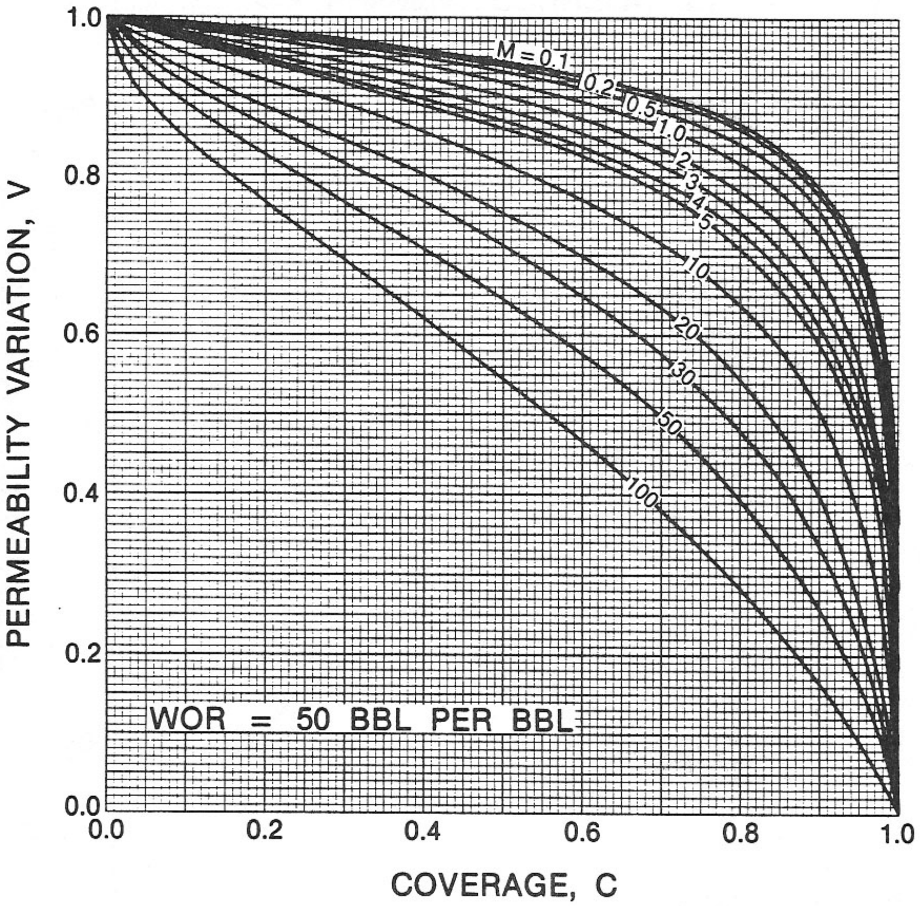

Figure A-2-2 – Coverage as a Function of Permeability Variation and Mobility Ratio [9]

---

[9] Fassihi, M. R., O'Brien, W. J., A Predictive Model for Water Flood Performance Using a Handheld Calculator, ARCO Oil and Gas Co., Plano, Texas, July 1984.

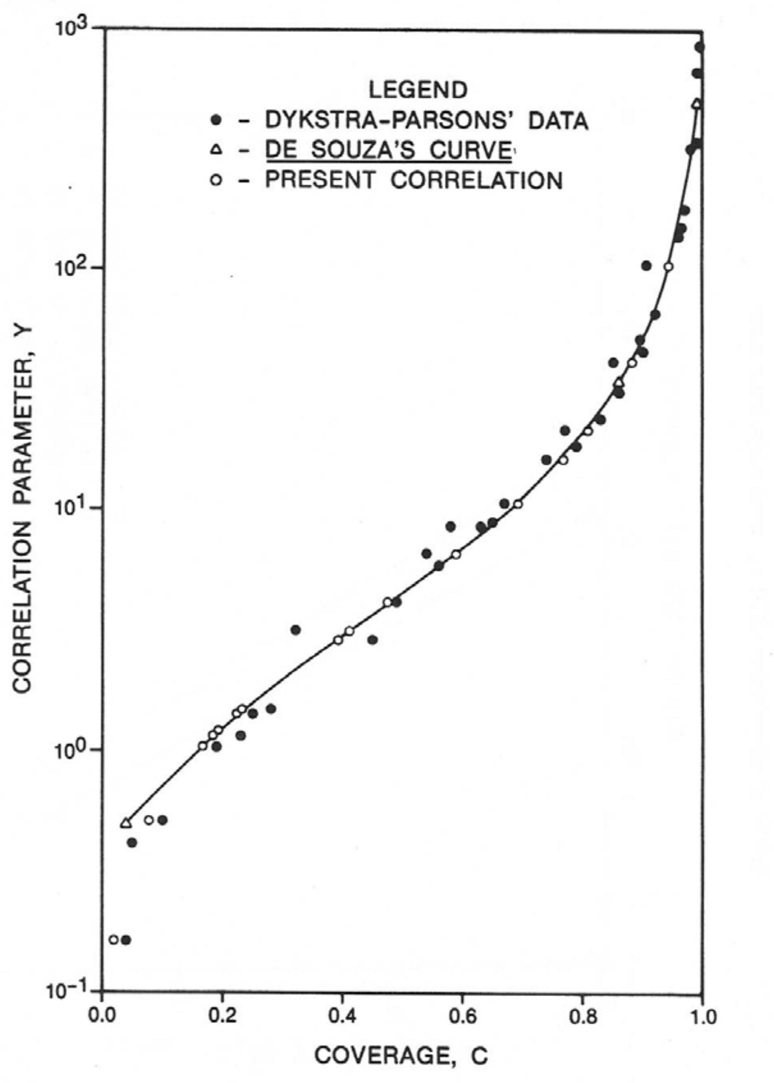

Figure A-2-3 – Coverage Correlation [10]

---

[10] Fassihi, M. R., O'Brien, W. J., A Predictive Model for Water Flood Performance Using a Handheld Calculator, ARCO Oil and Gas Co., Plano, Texas, July 1984.

REFERENCES

1. Bassiouni, Zaki, IHRDC Waterflooding, Saudi Aramco, September 2-6, 2000, Dhahran, Saudi Arabia.
2. Dykstra, H., and Parsons, R.L.: "The Prediction of Oil Recovery by Waterflood," Chapter 12, Secondary Recovery of Oil in the United States, 2nd Edition, API, New York, NY, 1950.
3. Fassihi, M. R., O'Biren, W. J., A Predictive Model For Waterflood Performance Using A Hand Held Calculators, ARCO Oil and Gas Company, Plano, Texas, July 1984.
4. Craig, Forrest F. Jr., The Reservoir Engineering Aspects of Waterflooding, SPE Monograph Volume 3, 1971.
5. Fassihi, M.R., New Correlations for Calculation of Vertical Coverage and Areal Sweep Efficiency, SPE Reservoir Engineering, November 1986.
6. Slider, H. C., "Worldwide Practical Petroleum Reservoir Engineering Methods,", Chapter 9.Water Flooding and its Variations, Pennwell Books, Tulsa, Oklahoma, 1983.
7. Willhite, Paul G. Waterflooding - SPE Textbook Series Vol. 3, 7th Printing 2001, Society of Petroleum Engineers, Richardson, TX 1986.

INDEX