An Application of Simplex Linear Programming

TABLE of CONTENTS

An Application of Simplex Linear Programming

LIST of FIGURES

LIST of TABLES

LIST of EQUATIONS

An Application of Simplex Linear Programming

Introduction

The use of linear programming in the petroleum industry has many applications primarily to optimize the profitability of a particular project or projects and/or to enhance a company's bottom line. Some typical applications are:

1. To determine the optimal crude oil and/or gas production from various producing fields each of which produce different grades of petroleum and have differing operating cost structures.
2. To determine the amount of fuel supplies that a particular refinery should make available to its various marketing areas in order to minimize the overall costs of refinery runs and the distribution costs of the fuel.
3. To determine an optimal plan for exchanging petroleum producing properties
4. To determine the optimal schedule for refinery's operations and product blending subject to numerous constraints.

The application presented here is a product-blending problem. Appendix One is a brief synopsis of the basic principles and methodology of Simplex Linear Programming and should be referenced.

Fuel Blending - Statement of Problem

A refinery blends five different raw feedstocks to produce two grades of motor gasoline, Premium (a.k.a. Fuel A) and Regular (a.k.a. Fuel B). It also produces Aviation Gas (a.k.a. Fuel C) and Fuel Oil (a.k.a. Fuel D). The available quantity of each raw feedstock and its respective Octane number is given in Table. [1]

Table 1 –
Available Raw Feedstock and Octane Numbers

Feedstock #	Octane Number	Quantity (Bbl/d)
1	70	2000
2	80	4000
3	85	4000
4	90	5000
5	99	3000

Premium is required to have an Octane of at least 95 and Regular at least 85. The current contract requires at least 8,000 Bbl/d , $\mathbf{Q_R}$, of Regular. However, the refinery can sell its entire output of Premium and Regular. All raw feedstock not blended into motor gasoline with and

[1] Fortran and Computer Mathematics
Carlile, R. E. & Gillett, B. E.
Petroleum Publishing Co. Tulsa, OK, 1973
Chapter 21: Operations Research Techniques

Octane number of 90 or greater is sold as Aviation Gasoline. Raw feedstock with an Octane number of 85 or less not used to blend motor gasoline is sold as Fuel Oil.

The profit margin, $/Bbl, price symbols, fuel symbols, octane relationships and octane symbols for each of the four grades is as follows:

Table 2 – Profit Margins & Equation Symbols

Fuel	$/ Bbl	Price Symbol	Fuel Symbol	Octane #	Octane Symbol
Premium	3.75	P_A	A	≥ 95	δ
Regular	2.85	P_B	B	≥ 85	γ
Aviation	2.75	P_C	C	≥ 90	β
Fuel Oil	1.25	P_D	D	≤ 85	α

The objective of the refinery then becomes that of maximizing the profit margin given the conditions and contract requirements as stated above. The Octane Symbol given in Table 2 is the limiting value of the octane requirement for each fuel type. It will be convenient to re-state these known variable and conditions in a tabular form so has to better understand the application of Simplex Linear Programming in its solution.

Problem Data - Relationships

The following table is used to visualize the relationships of minimum and maximum Octane numbers used to blend each fuel type:

Table 3 – Feedstock Octanes Used to Blend Each Fuel Type

Fuel A	Fuel B	Fuel C	Fuel D	Octane	ω_j
≥	≥	N/A	≤	70	ω_1
≥	≥	N/A	≤	80	ω_2
≥	≥	N/A	≤	85	ω_3
≥	≤	≥	N/A	90	ω_4
≤	≤	≥	N/A	99	ω_5
≥ 95	≥ 85	≥ 90	≤ 85		

Feedstock Octane numbers, ω_j , marked “N/A” are not used to blend that particular fuel, e.g. Fuel C and Fuel D. The other inequality signs designate that that fuel may be used to blend the Fuel and the relationship with respect to the desired final Octane number for the blend. The bottom row in the table shows the relationship between the final blended fuel and it’s Octane number.

The available quantity of each feedstock is designated as $\mathbf{Q_i}$. The quantity of each feedstock used to blend each fuel type is designated as $\mathbf{X_{zi}}$ where Z=A, B, C or D and i=1,5. Feedstocks 1 through 3 are not used to blend Fuel C and feedstocks 4 and 5 are not used to blend Fuel D in order to satisfy the Octane requirements. Alternatively stated:

$$X_{C1} = X_{C2} = X_{C3} = 0$$

$$X_{D4} = X_{D5} = 0$$

Table 4 designates the quantity of each raw feedstock used to blend each of the fuel type along with the Octane numbers and available quantity of each feed stock:

Table 4 – Feedstock Quantities Available for Each Fuel Type

ω_j	Octane	Fuel A	Fuel B	Fuel C	Fuel D	Quantity	Q_j
ω_1	70	X_{A1}	X_{B1}	X_{C1}	X_{D1}	2000	Q_1
ω_2	80	X_{A2}	X_{B2}	X_{C2}	X_{D2}	4000	Q_2
ω_3	85	X_{A3}	X_{B3}	X_{C3}	X_{D3}	4000	Q_3
ω_4	90	X_{A4}	X_{B4}	X_{C4}	X_{D4}	5000	Q_4
ω_5	99	X_{A5}	X_{B5}	X_{C5}	X_{D5}	3000	Q_5

Objective Function

The objective function, i.e. the total profit to eventually realize, can now be formulated as:

Equation (0-1) *Objective Function*

$$Z = P_A * \sum_{j=1}^{5} X_{A \cdot j} + P_B * \sum_{j=1}^{5} X_{B \cdot j} + P_C * \sum_{j=1}^{5} X_{C \cdot j} + P_D * \sum_{j=1}^{5} X_{D \cdot j}$$

It is intuitive that we need to maximize the profitability as computed in the objective function.

Constraint Rationales

The Constraints are listed to fit on one page below. Constraints 1 through 5 are all related to the available quantity of each raw feedstock. The numerical 0 coefficients are displayed for clarity. For example, Constraint 1 implies that the quantity of raw feedstock #1 eventually used to blend Fuel A and/or Fuel B and/or Fuel C and/or Fuel D cannot be greater than the available quantity of feedstock # 1.

Constraints 6 through 9 are derived from the Octane requirements for Fuels A through D. Since we must combine raw feedstocks 1 thru 5 to achieve the Octane requirement for Fuel A thru D:

$$\frac{\omega_1 X_{A1} + \omega_2 X_{A2} + \omega_3 X_{A3} + \omega_4 X_{A4} + \omega_5 X_{A5}}{X_{A1} + X_{A2} + X_{A3} + X_{A4} + X_{A5}} \geq \delta \qquad \text{Fuel A}$$

$$\frac{\omega_1 X_{B1} + \omega_2 X_{B2} + \omega_3 X_{B3} + \omega_4 X_{B4} + \omega_5 X_{B5}}{X_{B1} + X_{B2} + X_{B3} + X_{B4} + X_{B5}} \geq \gamma \qquad \text{Fuel B}$$

$$\frac{\omega_1 X_{C1} + \omega_2 X_{C2} + \omega_3 X_{C3} + \omega_4 X_{C4} + \omega_5 X_{C5}}{X_{C1} + X_{C2} + X_{C3} + X_{C4} + X_{C5}} \geq \beta \qquad \text{Fuel C}$$

$$\frac{\omega_1 X_{D1} + \omega_2 X_{D2} + \omega_3 X_{D3} + \omega_4 X_{D4} + \omega_5 X_{D5}}{X_{D1} + X_{D2} + X_{D3} + X_{D4} + X_{D5}} \leq \alpha \qquad \text{Fuel D}$$

The numerical 0 factors shown in Constraints 8 and 9 are again displayed for clarity.

Constraint 10 results from the fact that the minimum contract quantity of Fuel B is 8000 Bbl/d, i.e. $\mathbf{Q_R}$.

Constraint 11 results from the sum of the quantity of Fuel B and the sum of the quantity of Fuels A, B, and C must not exceed the total available feedstock.

$$Q_S = \sum_{j=1}^{5} Q_j - Q_R = 10{,}000 \;\; Bbl/d$$

Constraint 12 is a non-negativity constrain since. feed stocks 1 thru 3 are not used to blend Fuel C and feed stocks 4 and 5 are not used to blend Fuel D because of the Octane requirements.

Modified Objective Function

In order to solve the linear programming problem using the Simplex Method we must change the constraint inequalities to equalities using slack and surplus variables as noted in Appendix One. We must also be able to define an identity matrix within the constrain coefficients. In doing so we may need to arrive at a modified objective function:

Equation (0-2) *Modified Objective Function*

$$Z = P_A * \sum_{j=1}^{5} X_{A\cdot j} + P_B * \sum_{j=1}^{5} X_{B\cdot j} + P_C * \sum_{j=1}^{5} X_{C\cdot j} + P_D * \sum_{j=1}^{5} X_{D\cdot j} - M_{33} * X_{33} - M_{34} * X_{34}$$

Modified Constraint Rationales

The Modified Constraints are listed on one page below. See Appendix One for a description of slack, surplus and artificial variables. Figure A-2 has been constructed to help visualize how these variables have been added to arrive at the modified constraints. Figure A-3 has been constructed to the existence of an identity matrix.

In the modified constrains, X_{30} and X_{29} are slack variables; X_{33} and X_{34} are artificial variables and X_{21} through X_{28} and X_{31} and X_{32} are surplus variables. $-M_{33}$ and $-M_{34}$ are arbitrarily large negative coefficients of the artificial variables in the modified objective function.

Constraints Equations

Equation (1-1) *Constraint 1*

$$X_{A1} + X_{B1} + 0 * X_{C1} + X_{D1} \le Q_1$$

Equation (2-1) *Constraint 2*

$$X_{A2} + X_{B2} + 0 * X_{C2} + X_{D2} \le Q_2$$

Equation (3-1) *Constraint 3*

$$X_{A3} + X_{B3} + 0 * X_{C3} + X_{D3} \le Q_3$$

Equation (4-1) *Constraint 4*

$$X_{A4} + X_{B4} + X_{C4} + 0 * X_{D4} \le Q_4$$

Equation (5-1) *Constraint 5*

$$X_{A5} + X_{B5} + X_{C5} + 0 * X_{D5} \le Q_5$$

Equation (6-1) *Constraint 6*

$$(\delta - \omega_1)X_{A1} + (\delta - \omega_2)X_{A2} + (\delta - \omega_3)X_{A3} + (\delta - \omega_4)X_{A4} + (\delta - \omega_5)X_{A5} \le 0$$

Equation (7-1) *Constraint 7*

$$(\gamma - \omega_1)X_{B1} + (\gamma - \omega_2)X_{B2} + (\gamma - \omega_3)X_{B3} + (\gamma - \omega_4)X_{B4} + (\gamma - \omega_5)X_{B5} \le 0$$

Equation (8-1) *Constraint 8*

$$0 * (\beta - \omega_1)X_{C1} + 0 * (\beta - \omega_2)X_{C2} + 0 * (\beta - \omega_3)X_{C3} + (\beta - \omega_4)X_{C4} + (\beta - \omega_5)X_{C5} \le 0$$

Equation (9-1) *Constraint 9*

$$(\alpha - \omega_1)X_{D1} + (\alpha - \omega_2)X_{D2} + (\alpha - \omega_3)X_{D3} + 0 * (\alpha - \omega_4)X_{D4} + 0 * (\alpha - \omega_5)X_{D5} \ge 0$$

Equation (10-1) *Constraint 10*

$$X_{B1} + X_{B2} + X_{B3} + X_{B4} + X_{B5} \ge Q_R$$

Equation (11-1) *Constraint 11*

$$X_{A1} + X_{A2} + X_{A3} + X_{A4} + X_{A5} + X_{C4} + X_{C5} + X_{D1} + X_{D2} + X_{D3} \le Q_S$$

Equation (12-1) *Constraint 12*

$$X_{C1} + X_{C2} + X_{C3} + X_{D4} + X_{D5} \le 0$$

Modified Constraints Equations

Equation (1-2) *Modified Constraint 1*

$$X_{A1} + X_{B1} + 0 * X_{C1} + X_{D1} \le Q_1 - X_{21}$$

Equation (2-2) *Modified Constraint 2*

$$X_{A2} + X_{B2} + 0 * X_{C2} + X_{D2} \le Q_2 - X_{22}$$

Equation (3-2) *Modified Constraint 3*

$$X_{A3} + X_{B3} + 0 * X_{C3} + X_{D3} \le Q_3 - X_{23}$$

Equation (4-2) *Modified Constraint 4*

$$X_{A4} + X_{B4} + X_{C4} + 0 * X_{D4} \le Q_4 - X_{24}$$

Equation (5-2) *Modified Constraint 5*

$$X_{A5} + X_{B5} + X_{C5} + 0 * X_{D5} \le Q_5 - X_{25}$$

Equation (6-2) *Modified Constraint 6*

$$(\delta - \omega_1)X_{A1} + (\delta - \omega_2)X_{A2} + (\delta - \omega_3)X_{A3} + (\delta - \omega_4)X_{A4} + (\delta - \omega_5)X_{A5} \le 0 - X_{26}$$

Equation (7-2) *Modified Constraint 7*

$$(\gamma - \omega_1)X_{B1} + (\gamma - \omega_2)X_{B2} + (\gamma - \omega_3)X_{B3} + (\gamma - \omega_4)X_{B4} + (\gamma - \omega_5)X_{B5} \le 0 - X_{27}$$

Equation (8-2) *Modified Constraint 8*

$$0 * (\beta - \omega_1)X_{C1} + 0 * (\beta - \omega_2)X_{C2} + 0 * (\beta - \omega_3)X_{C3} + (\beta - \omega_4)X_{C4} + (\beta - \omega_5)X_{C5} \le 0 - X_{28}$$

Equation (9-2) *Modified Constraint 9*

$$(\alpha - \omega_1)X_{D1} + (\alpha - \omega_2)X_{D2} + (\alpha - \omega_3)X_{D3} + 0 * (\alpha - \omega_4)X_{D4} + 0 * (\alpha - \omega_5)X_{D5} \ge 0 + X_{29} - X_{33}$$

Equation (10-2) *Modified Constraint 10*

$$X_{B1} + X_{B2} + X_{B3} + X_{B4} + X_{B5} \ge Q_R + X_{30} - X_{34}$$

Equation (11-2) *Modified Constraint 11*

$$X_{A1} + X_{A2} + X_{A3} + X_{A4} + X_{A5} + X_{C4} + X_{C5} + X_{D1} + X_{D2} + X_{D3} \le Q_S - X_{31}$$

Equation (12-2) *Modified Constraint 12*

$$X_{C1} + X_{C2} + X_{C3} + X_{D4} + X_{D5} \le 0 - X_{32}$$

Solution Using Simplex Linear Program

The coefficients for the modified objective function, Z, the A-matrix and b-matrix are all displayed in the resulting output of the FORTRAN run: Note that for incorporation into the FORTRAN program inputs the b-matrix has been incorporated as A(x, 35).

```
Z=  3.75*X( 1)+    3.75*X( 2)+    3.75*X( 3)+    3.75*X( 4)+    3.75*X( 5)+
    2.85*X( 6)+    2.85*X( 7)+    2.85*X( 8)+    2.85*X( 9)+    2.85*X(10)+
    2.75*X(11)+    2.75*X(12)+    2.75*X(13)+    2.75*X(14)+    2.75*X(15)+
    1.25*X(16)+    1.25*X(17)+    1.25*X(18)+    1.25*X(19)+    1.25*X(20)+
     .00*X(21)+     .00*X(22)+     .00*X(23)+     .00*X(24)+     .00*X(25)+
     .00*X(26)+     .00*X(27)+     .00*X(28)+     .00*X(29)+     .00*X(30)+
     .00*X(31)+     .00*X(32)+ -999.00*X(33)+ -999.00*X(34)+

 Total Number Variables Objective Function =  34

  A( 1  1)  A( 1  2)  A( 1  3)  A( 1  4)  A( 1  5)  A( 1  6)  A( 1  7)
  A( 1  8)  A( 1  9)  A( 1 10)  A( 1 11)  A( 1 12)  A( 1 13)  A( 1 14)
  A( 1 15)  A( 1 16)  A( 1 17)  A( 1 18)  A( 1 19)  A( 1 20)  A( 1 21)
  A( 1 22)  A( 1 23)  A( 1 24)  A( 1 25)  A( 1 26)  A( 1 27)  A( 1 28)
  A( 1 29)  A( 1 30)  A( 1 31)  A( 1 32)  A( 1 33)  A( 1 34)  A( 1 35)
-------------------------------------------------------------------------
     1.000      .000      .000      .000      .000     1.000      .000
      .000      .000      .000     1.000      .000      .000      .000
      .000     1.000      .000      .000      .000      .000     1.000
      .000      .000      .000      .000      .000      .000      .000
      .000      .000      .000      .000      .000      .000  2000.000

  A( 2  1)  A( 2  2)  A( 2  3)  A( 2  4)  A( 2  5)  A( 2  6)  A( 2  7)
  A( 2  8)  A( 2  9)  A( 2 10)  A( 2 11)  A( 2 12)  A( 2 13)  A( 2 14)
  A( 2 15)  A( 2 16)  A( 2 17)  A( 2 18)  A( 2 19)  A( 2 20)  A( 2 21)
  A( 2 22)  A( 2 23)  A( 2 24)  A( 2 25)  A( 2 26)  A( 2 27)  A( 2 28)
  A( 2 29)  A( 2 30)  A( 2 31)  A( 2 32)  A( 2 33)  A( 2 34)  A( 2 35)
-------------------------------------------------------------------------
      .000     1.000      .000      .000      .000      .000     1.000
      .000      .000      .000      .000     1.000      .000      .000
      .000      .000     1.000      .000      .000      .000      .000
     1.000      .000      .000      .000      .000      .000      .000
      .000      .000      .000      .000      .000      .000  4000.000

  A( 3  1)  A( 3  2)  A( 3  3)  A( 3  4)  A( 3  5)  A( 3  6)  A( 3  7)
  A( 3  8)  A( 3  9)  A( 3 10)  A( 3 11)  A( 3 12)  A( 3 13)  A( 3 14)
  A( 3 15)  A( 3 16)  A( 3 17)  A( 3 18)  A( 3 19)  A( 3 20)  A( 3 21)
  A( 3 22)  A( 3 23)  A( 3 24)  A( 3 25)  A( 3 26)  A( 3 27)  A( 3 28)
  A( 3 29)  A( 3 30)  A( 3 31)  A( 3 32)  A( 3 33)  A( 3 34)  A( 3 35)
-------------------------------------------------------------------------
      .000      .000     1.000      .000      .000      .000      .000
     1.000      .000      .000      .000      .000     1.000      .000
      .000      .000      .000     1.000      .000      .000      .000
      .000     1.000      .000      .000      .000      .000      .000
      .000      .000      .000      .000      .000      .000  4000.000

  A( 4  1)  A( 4  2)  A( 4  3)  A( 4  4)  A( 4  5)  A( 4  6)  A( 4  7)
  A( 4  8)  A( 4  9)  A( 4 10)  A( 4 11)  A( 4 12)  A( 4 13)  A( 4 14)
  A( 4 15)  A( 4 16)  A( 4 17)  A( 4 18)  A( 4 19)  A( 4 20)  A( 4 21)
  A( 4 22)  A( 4 23)  A( 4 24)  A( 4 25)  A( 4 26)  A( 4 27)  A( 4 28)
  A( 4 29)  A( 4 30)  A( 4 31)  A( 4 32)  A( 4 33)  A( 4 34)  A( 4 35)
-------------------------------------------------------------------------
      .000      .000      .000     1.000      .000      .000      .000
      .000     1.000      .000      .000      .000      .000     1.000
      .000      .000      .000      .000     1.000      .000      .000
      .000      .000     1.000      .000      .000      .000      .000
      .000      .000      .000      .000      .000      .000  5000.000
```

A(5 1)	A(5 2)	A(5 3)	A(5 4)	A(5 5)	A(5 6)	A(5 7)
A(5 8)	A(5 9)	A(5 10)	A(5 11)	A(5 12)	A(5 13)	A(5 14)
A(5 15)	A(5 16)	A(5 17)	A(5 18)	A(5 19)	A(5 20)	A(5 21)
A(5 22)	A(5 23)	A(5 24)	A(5 25)	A(5 26)	A(5 27)	A(5 28)
A(5 29)	A(5 30)	A(5 31)	A(5 32)	A(5 33)	A(5 34)	A(5 35)

.000	.000	.000	.000	1.000	.000	.000
.000	.000	1.000	.000	.000	.000	.000
1.000	.000	.000	.000	.000	1.000	.000
.000	.000	.000	1.000	.000	.000	.000
.000	.000	.000	.000	.000	.000	3000.000

A(6 1)	A(6 2)	A(6 3)	A(6 4)	A(6 5)	A(6 6)	A(6 7)
A(6 8)	A(6 9)	A(6 10)	A(6 11)	A(6 12)	A(6 13)	A(6 14)
A(6 15)	A(6 16)	A(6 17)	A(6 18)	A(6 19)	A(6 20)	A(6 21)
A(6 22)	A(6 23)	A(6 24)	A(6 25)	A(6 26)	A(6 27)	A(6 28)
A(6 29)	A(6 30)	A(6 31)	A(6 32)	A(6 33)	A(6 34)	A(6 35)

25.000	15.000	10.000	5.000	-4.000	.000	.000
.000	.000	.000	.000	.000	.000	.000
.000	.000	.000	.000	.000	.000	.000
.000	.000	.000	.000	1.000	.000	.000
.000	.000	.000	.000	.000	.000	.000

A(7 1)	A(7 2)	A(7 3)	A(7 4)	A(7 5)	A(7 6)	A(7 7)
A(7 8)	A(7 9)	A(7 10)	A(7 11)	A(7 12)	A(7 13)	A(7 14)
A(7 15)	A(7 16)	A(7 17)	A(7 18)	A(7 19)	A(7 20)	A(7 21)
A(7 22)	A(7 23)	A(7 24)	A(7 25)	A(7 26)	A(7 27)	A(7 28)
A(7 29)	A(7 30)	A(7 31)	A(7 32)	A(7 33)	A(7 34)	A(7 35)

.000	.000	.000	.000	.000	15.000	5.000
.000	-5.000	-14.000	.000	.000	.000	.000
.000	.000	.000	.000	.000	.000	.000
.000	.000	.000	.000	.000	1.000	.000
.000	.000	.000	.000	.000	.000	.000

A(8 1)	A(8 2)	A(8 3)	A(8 4)	A(8 5)	A(8 6)	A(8 7)
A(8 8)	A(8 9)	A(8 10)	A(8 11)	A(8 12)	A(8 13)	A(8 14)
A(8 15)	A(8 16)	A(8 17)	A(8 18)	A(8 19)	A(8 20)	A(8 21)
A(8 22)	A(8 23)	A(8 24)	A(8 25)	A(8 26)	A(8 27)	A(8 28)
A(8 29)	A(8 30)	A(8 31)	A(8 32)	A(8 33)	A(8 34)	A(8 35)

.000	.000	.000	.000	.000	.000	.000
.000	.000	.000	.000	.000	.000	.000
-9.000	.000	.000	.000	.000	.000	.000
.000	.000	.000	.000	.000	.000	1.000
.000	.000	.000	.000	.000	.000	.000

A(9 1)	A(9 2)	A(9 3)	A(9 4)	A(9 5)	A(9 6)	A(9 7)
A(9 8)	A(9 9)	A(9 10)	A(9 11)	A(9 12)	A(9 13)	A(9 14)
A(9 15)	A(9 16)	A(9 17)	A(9 18)	A(9 19)	A(9 20)	A(9 21)
A(9 22)	A(9 23)	A(9 24)	A(9 25)	A(9 26)	A(9 27)	A(9 28)
A(9 29)	A(9 30)	A(9 31)	A(9 32)	A(9 33)	A(9 34)	A(9 35)

.000	.000	.000	.000	.000	.000	.000
.000	.000	.000	.000	.000	.000	.000
.000	15.000	5.000	.000	.000	.000	.000
.000	.000	.000	.000	.000	.000	.000
-1.000	.000	.000	.000	1.000	.000	.000

A(10 1)	A(10 2)	A(10 3)	A(10 4)	A(10 5)	A(10 6)	A(10 7)
A(10 8)	A(10 9)	A(10 10)	A(10 11)	A(10 12)	A(10 13)	A(10 14)
A(10 15)	A(10 16)	A(10 17)	A(10 18)	A(10 19)	A(10 20)	A(10 21)
A(10 22)	A(10 23)	A(10 24)	A(10 25)	A(10 26)	A(10 27)	A(10 28)
A(10 29)	A(10 30)	A(10 31)	A(10 32)	A(10 33)	A(10 34)	A(10 35)

.000	.000	.000	.000	.000	1.000	1.000
1.000	1.000	1.000	.000	.000	.000	.000
.000	.000	.000	.000	.000	.000	.000
.000	.000	.000	.000	.000	.000	.000
.000	-1.000	.000	.000	.000	1.000	8000.000

```
  A(11  1)  A(11  2)  A(11  3)  A(11  4)  A(11  5)  A(11  6)  A(11  7)
  A(11  8)  A(11  9)  A(11 10)  A(11 11)  A(11 12)  A(11 13)  A(11 14)
  A(11 15)  A(11 16)  A(11 17)  A(11 18)  A(11 19)  A(11 20)  A(11 21)
  A(11 22)  A(11 23)  A(11 24)  A(11 25)  A(11 26)  A(11 27)  A(11 28)
  A(11 29)  A(11 30)  A(11 31)  A(11 32)  A(11 33)  A(11 34)  A(11 35)
-----------------------------------------------------------------------------
     1.000     1.000     1.000     1.000     1.000      .000      .000
      .000      .000      .000      .000      .000      .000     1.000
     1.000     1.000     1.000     1.000      .000      .000      .000
      .000      .000      .000      .000      .000      .000      .000
      .000      .000     1.000      .000      .000      .000 10000.000

  A(12  1)  A(12  2)  A(12  3)  A(12  4)  A(12  5)  A(12  6)  A(12  7)
  A(12  8)  A(12  9)  A(12 10)  A(12 11)  A(12 12)  A(12 13)  A(12 14)
  A(12 15)  A(12 16)  A(12 17)  A(12 18)  A(12 19)  A(12 20)  A(12 21)
  A(12 22)  A(12 23)  A(12 24)  A(12 25)  A(12 26)  A(12 27)  A(12 28)
  A(12 29)  A(12 30)  A(12 31)  A(12 32)  A(12 33)  A(12 34)  A(12 35)
-----------------------------------------------------------------------------
      .000      .000      .000      .000      .000      .000      .000
      .000      .000      .000     1.000     1.000     1.000      .000
      .000      .000      .000      .000     1.000     1.000      .000
      .000      .000      .000      .000      .000      .000      .000
      .000      .000      .000     1.000      .000      .000      .000

 12 x 35  A-Matrix & B-Vector

 The Maximum Value of Z =  52829.900

 Optimal Solution :
 Variable    Value
 --------    -----
    X( 9)  4244.444
    X(10)  2055.556
    X( 8)  4000.000
    X( 4)   755.556
    X( 5)   944.444
    X( 6)  2000.000
    X( 7)  4000.000
```

Figure A-1 – FORTRAN Output / Solution

Equating the optimal solution's variable and values to the original linear programming problem we have a solution that suggest that we only produce Fuel A and B and no Fuel C or D:

Table 5 – Feedstock Quantities Solution for Each Fuel Type

Fuel A	Bbl/d	Fuel B	Bbl/d	Fuel C	Bbl/d	Fuel D	Bbl/d
X_{A1}	0.000	X_{B1}	2000.000	X_{C1}	0.000	X_{D1}	0.000
X_{A2}	0.000	X_{B2}	4000.000	X_{C2}	0.000	X_{D2}	0.000
X_{A3}	0.000	X_{B3}	4000.00	X_{C3}	0.000	X_{D3}	0.000
X_{A4}	755.556	X_{B4}	4244.444	X_{C4}	0.000	X_{D4}	0.000
X_{A5}	944.444	X_{B5}	2055.556	X_{C5}	0.000	X_{D5}	0.000

The maximum (optimized) profit is $52,830 per day for the refinery runs under these conditions.

APPENDIX ONE

Basic Principles – Methodology – Simplex Linear Programming

A linear programming model is composed of a linear function with several variables that can be optimized (maximized or minimized) subject to a number of linear constraints involving the variables in the linear function. The linear function is refereed to as the objective function. The coefficients of the variables in the objective function are referred to as cost coefficients. For a typical linear function with constraints, numerous feasible solutions exists such that every feasible solution will satisfy all of the constraints. The maximum value for the objective function will be satisfied by at least one of the feasible solutions. [2]

A set of algorithms incorporated into a FORTRAN program called the Simplex Method solves linear programming problems with any number of constraints and variables within reason. The most difficult job in applying linear programming is the formulation of the problem into a format that can be solved by the FORTRAN program. This involves essentially a reformulation of the original problem. The optimal solution for the reformulated problem will also be the optimal solution to the original problem.

One or more basic solutions can be ascertained which are subsets of the feasible solutions and are essentially solutions that can be thought of in a graphical sense as solutions at the "corners of the region of feasible solutions". The Simplex Method optimizes the objective function by examining a subset of the basic feasible solutions by moving from an initial basic feasible solution to an optimal basic feasible solution that does not decrease the value of the objective function.

In matrix notation, this concept can be expressed as: maximize the objective function, Z=C*X subject to the constraints, A*X=b. The constraints are analogous to a matrix of m equations and n unknowns. In matrix notation this notation may be expressed for a 2x4 matrix for example as follows:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \end{bmatrix} \qquad b = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$X = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} \qquad C = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix}$$

[2] For a more detailed explanation, the reader is refereed to the mathematical literature on the subject matter.

For this analogy the linear programming problem is to optimize (i.e. maximize or minimize) the objective function:

$$Z = c_1 * X_1 + c_2 * X_2 + c_3 * X_3 + c_4 * X_4$$

Subject to the constraints:

$$a_{11} * X_1 + a_{12} * X_2 + a_{13} * X_3 + a_{14} * X_4 = b_1$$
$$a_{21} * X_1 + a_{22} * X_2 + a_{23} * X_3 + a_{24} * X_4 = b_2$$

(It should be noted that maximizing a linear function is equivalent to minimizing the negative of the linear function.)

To apply the Simplex Method we must have $b_i \geq 0$ for all i=1,m , i.e. non-negativity restrictions and we must have an m x m identity sub matrix within matrix A.

The constraints are most often expressed in terms of inequalities, hence it is judicious to convert the inequalities with respect to the b-matrix to equalities by adding slack variables to a less than inequality and subtracting surplus variables to a greater than inequality. Correspondingly, zero cost coefficients are assigned for each slack and surplus variable in the objective function.

Should an m x m identity sub matrix not be realized, then additionally, artificial variables can be added to one or more constraints involving inequalities and each artificial variable added to the objective function with an arbitrarily large negative cost coefficient, e.g. $-M_{ij}$ to facilitate the optimal solution.

This appendix is a very brief description but serves to illustrate they methodology and other considerations that must be taken into account when applying the Simplex Method of Linear Programming.

APPENDIX TWO [3]

Matrix Subscripts & Values & Identity Matrix

A1					B1					C1					D1					21														<	Q1	A(1,Y)
	A2					B2					C2					D2					22													<	Q2	A(2,Y)
		A3					B3					C3					D3					23												<	Q3	A(3,Y)
			A4					B4					C4					D4					24											<	Q4	A(4,Y)
				A5					B5					C5					D5					25										<	Q5	A(5,Y)
A1	A2	A3	A4	A5																					26									<	0	A(6,Y)
					B1	B2	B3	B4	B5																	27								<	0	A(7,Y)
													C4	C5													28							<	0	A(8,Y)
															D1	D2	D3											+29				-33		>	0	A(9,Y)
					B1	B2	B3	B4	B5																				+30				-34	>	QR	A(10,Y)
A1	A2	A3	A4	A5									C4	C5	D1	D2	D3													31				<	QS	A(11,Y)
										C1	C2	C3						D4	D5												32			<	0	A(12,Y)
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34		B(i)	

Figure A-2 – Matrix Subscripts

1					1					1					1					1														2000
	1					1					1					1					1													4000
		1					1					1					1					1												4000
			1					1					1					1					1											5000
				1					1					1					1					1										3000
25	15	10	5	-4																					1									0
					15	5		-5	-14																	1								0
														-9													1							0
															15	5												-1				1		0
					1	1	1	1	1																				-1				1	8000
1	1	1	1	1									1	1	1	1	1													1				10000
										1	1	1						1	1												1			0
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	

Figure A-3 – Identity Matrix

[3] The subscripts are a mixture of letters and numbers that may appear confusing save the fact that the numbers are used to designate the addition of slack, surplus and artificial variables.