Limites Trigonométricos

Regra de L'Hospital

I. Limites que redundam em $\frac{0}{0}$ ou $\frac{\infty}{\infty}$

1. Calcule o limite $\lim\limits_{x \to 0} \frac{x - \text{sen}\, x}{3x^2} = ?$

$$\frac{f(x)}{j(x)} = \frac{x - \text{sen}\, x}{3x^2} \quad \text{e} \quad \frac{f(0)}{g(0)} = \frac{0}{0} \quad \to \quad \frac{f'(x)}{j'(x)} = \frac{1 - \cos x}{6x} \quad \text{e} \quad \frac{f'(0)}{g'(0)} = \frac{0}{0}$$

$$\frac{f''(x)}{j''(x)} = \frac{\text{sen}\, x}{6} \quad \text{e} \quad \frac{f''(0)}{g''(0)} = \frac{0}{6} = 0 \quad \to \quad \lim_{x \to 0} \frac{x - \text{sen}\, x}{3x^2} = \lim_{x \to 0} \frac{\text{sen}\, x}{6} = 0$$

2. Calcule o limite $\lim\limits_{x \to 1} \frac{Ln[\cos(x-1)]}{1 - \text{sen}\frac{px}{2}} = ?$

$$\frac{f(x)}{j(x)} = \frac{Ln[\cos(x-1)]}{1 - \text{sen}\frac{px}{2}} \quad \text{e} \quad \frac{f(1)}{g(1)} = \frac{0}{0} \quad \to \quad \frac{f'(x)}{j'(x)} = \frac{\frac{-\text{sen}(x-1)}{\cos(x-1)}}{-\frac{p}{2}.\cos\frac{px}{2}} = \frac{2.\text{sen}(x-1)}{p.\cos\frac{px}{2}.\cos(x-1)} \quad \text{e} \quad \frac{f'(1)}{g'(1)} = \frac{0}{0}$$

$$\frac{f''(x)}{j''(x)} = = \frac{2.\cos(x-1)}{\frac{p^2}{2}.\text{sen}\frac{px}{2}.\cos(x-1) - p.\cos(x-1).\text{sen}(x-1)} \quad \text{e} \quad \frac{f''(1)}{g''(1)} = -\frac{4}{p^2}$$

$$\lim_{x \to 1} \frac{Ln[\cos(x-1)]}{1 - \text{sen}\frac{px}{2}} = \lim_{x \to 1} \frac{2.\cos(x-1)}{\frac{p^2}{2}.\text{sen}\frac{px}{2}.\cos(x-1) - p.\cos(x-1).\text{sen}(x-1)} = -\frac{4}{p^2}$$

3. Calcule o limite $\lim\limits_{x \to +\infty} \frac{1 - x - e^{-x}}{2x^3} = ?$

$$\frac{f(x)}{j(x)} = \frac{1 - x - e^{-x}}{2x^3} \quad \text{e} \quad \frac{f(+\infty)}{g(+\infty)} = -\frac{\infty}{\infty} \quad \to \quad \frac{f'(x)}{j'(x)} = \frac{-1 + e^{-x}}{6x^2} \quad \text{e} \quad \frac{f'(+\infty)}{g'(+\infty)} = -\frac{1}{\infty}$$

$$\lim_{x\to+\infty}\frac{1-x-e^{-x}}{2x^3}=\lim_{x\to+\infty}\frac{-1+e^{-x}}{6x^2}=-\frac{1}{\infty}=0$$

II. Limites que redundam em $0.\infty$

1. Calcule limite $\lim_{x\to 0}\left(x.e^{\frac{1}{x}}\right) = ?$ **à** $\lim_{x\to 0}\left(x.e^{\frac{1}{x}}\right) = 0.\infty$ **à**

$$\frac{f(x)}{j(x)} = \frac{e^{\frac{1}{x}}}{\frac{1}{x}} \text{ e } \frac{f(0)}{g(0)} = \frac{\infty}{\infty} \rightarrow \frac{f'(x)}{j'(x)} = \frac{-\frac{1}{x^2}.e^{\frac{1}{x}}}{-\frac{1}{x^2}.} = e^{\frac{1}{x}} \text{ e } \frac{f'(0)}{g'(0)} = \infty$$

$$\lim_{x\to 0}\left(x.e^{\frac{1}{x}}\right) = \lim_{x\to 0}\left(e^{\frac{1}{x}}\right) = \infty$$

2. $\lim_{x\to 0} x.\text{sen}(\frac{1}{x}) = ?$ **à** $\lim_{x\to 0} x.\text{sen}(\frac{1}{x}) = \lim_{t\to\infty}\frac{1}{t}.\text{sen}\, t = \lim_{t\to\infty}\frac{\text{sen}\, t}{t} = 1$

Fazendo $t = \frac{1}{x}\begin{cases} x \rightarrow 0 \\ t \rightarrow \infty \end{cases}$

III. Limites que redundam em $\infty - \infty$

Calcule o limite $\lim_{x\to\frac{p}{2}}(\sec x - tgx) = \infty - \infty$

$$\frac{f(x)}{j(x)} = \frac{1}{\cos x} - \frac{\text{sen}\, x}{\cos x} = \frac{1 - \text{sen}\, x}{\cos x} \text{ e } \frac{f\left(\frac{p}{2}\right)}{g\left(\frac{p}{2}\right)} = \frac{0}{0}$$

$$\frac{f'(x)}{j'(x)} = \frac{-\cos x}{-\text{sen}\, x} = \frac{\cos x}{\text{sen}\, x} \text{ e } \frac{f'\left(\frac{p}{2}\right)}{g'\left(\frac{p}{2}\right)} = \frac{0}{1} = 0$$

$$\lim_{x\to\frac{p}{2}}(\sec x - tgx) = \lim_{x\to\frac{p}{2}}\frac{\cos x}{\text{sen}\, x} = 0$$

IV. Limites que redundam em 0^0, 1^∞, ∞^0

1. Calcular o limite $\lim_{x\to 0}\left(x^x\right) = ?$ **à** $\lim_{x\to 0}\left(x^x\right) = 0^0$

$$y = \left(x^x\right) \Rightarrow \ln y = x.\ln x, \text{ fazendo } \frac{f(x)}{j(x)} = \frac{\ln x}{\frac{1}{x}} \text{ e } \frac{f(0)}{g(0)} = \frac{\infty}{\infty}$$

$$\frac{f'(x)}{j'(x)} = \frac{1/x}{-1/x^2} = -x \text{ e } \frac{f'(0)}{g'(0)} = 0$$

$$\lim_{x\to 0}(x^x) = \lim_{x\to 0}(-x) = 0$$

2. Calcule o limite $\lim_{x\to 0}(e^x + x)^{\frac{1}{x}} = ?$

$$\left[(e^x + x)^{\frac{1}{x}}\right]_{x\to 0} \to 1^\infty$$

$$\ln(e^x + x)^{\frac{1}{x}} = \frac{1}{x}.\ln(e^x + x) \text{ e } \left[\frac{1}{x}.\ln(e^x + x)\right]_{x\to 0} \to 0.\infty$$

$$\frac{f(x)}{j(x)} = \frac{\ln(e^x + x)}{x} \text{ e } \frac{f(0)}{j(0)} = \frac{0}{0} \Rightarrow \frac{f'(x)}{j'(x)} = \frac{\frac{e^x+1}{e^x+x}}{1} = \frac{e^x+1}{e^x+x} \text{ e } \frac{f'(0)}{j'(0)} = \frac{2}{1} = 2$$

$$\lim_{x\to 0}\ln\left[(e^x + x)^{\frac{1}{x}}\right] = 2 = \ln e^2 \text{ ou } \ln\left[\lim_{x\to 0}(e^x + x)^{\frac{1}{x}}\right] = \ln e^2$$

$$\lim_{x\to 0}(e^x + x)^{\frac{1}{x}} = e^2$$

3. Calcule o limite $\lim_{x\to 0}(\cot gx)^{\operatorname{sen} x} = ?$

$$\left[(\cot gx)^{\operatorname{sen} x}\right]_{x\to 0} \to \infty^0 \quad \text{que é uma indeterminação.}$$

$$\ln(\cot gx)^{\operatorname{sen} x} = \operatorname{sen} x.\ln(\cot gx) \text{ e } \left[\ln(\cot gx)^{\operatorname{sen} x}\right]_{x\to 0} \to 0.\infty$$

$$\frac{f(x)}{j(x)} = \frac{\ln(\cot gx)}{\operatorname{sen} x} \text{ e } \frac{f'(x)}{j'(x)} = \frac{\frac{-\cos ec^2 x}{\cot gx}}{\frac{-\cos x}{\operatorname{sen}^2 x}} = \frac{1}{\operatorname{sen} x.\cos x}.\frac{\operatorname{sen}^2 x}{\cos x} = \frac{\operatorname{sen} x}{\cos^2 x} \text{ e } \frac{f'(0)}{j'(0)} = \frac{0}{1} = 0$$

$$\lim_{x\to 0}\ln\left[(\cot gx)^{\operatorname{sen} x}\right] = 0 = \ln e^0 = \ln 1 \text{ ou } \ln\left[\lim_{x\to 0}(\cot gx)^{\operatorname{sen} x}\right] = \ln 1 \text{ logo } \lim_{x\to 0}(\cot gx)^{\operatorname{sen} x} = 1$$

Exercícios – Regra de L'Hospital

1. $\lim_{x\to 0}\frac{e^x - e^{-x} - x^2}{2x - \operatorname{sen} x}$ ® 2
2. $\lim_{x\to 3}\frac{e^x - e^3}{x - 3}$ ® e^3
3. $\lim_{x\to\infty}\frac{(\ln x)^2 + e^x}{x^2}$ ® ∞
4. $\lim_{x\to 0}\frac{\ln x}{\ln(1-\cos x)}$ ® $1/2$
5. $\lim_{x\to 0}(x^2.\ln x)$ ® 0
6. $\lim_{x\to 0}\left(\frac{1}{\cos x - 1} - \frac{1}{\operatorname{sen} x}\right)$ ® ∞
7. $\lim_{x\to\infty}\frac{e^x}{\ln x}$ ® ∞ ® 2
8. $\lim_{x\to\frac{p}{2}}(p - 2).tgx$ ® ∞
9. $\lim_{x\to 1}\left(\frac{1}{x}\right)^{\ln(1-x)}$ ® 2
10. $\lim_{x\to 1}(x)^{\frac{1}{x-1}}$ ® e^{-1}

11. $\lim_{x\to 0}\left(\frac{x}{1-x}\right)^{\ln(1+x)}$ ® 1