Exercícios – Regra de L'Hospital

Oito Limites com Alguns Resolvidos (4,6 e 8)

1. $\lim\limits_{x\to\infty}\dfrac{(\ln x)^2+e^x}{x^2}$ ® ∞

2. $\lim\limits_{x\to 0}\left(x^2.\ln x\right)$ ® 0

3. $\lim\limits_{x\to 0}\left(\dfrac{1}{\cos x-1}-\dfrac{1}{\operatorname{sen} x}\right)$ ® ∞

4. $\lim\limits_{x\to\infty}\dfrac{e^x}{\ln x}$ =? Solução: $\lim\limits_{x\to\infty}\dfrac{e^x}{\ln x}$ **à** $\dfrac{\infty}{\infty}$. Fazendo: $\dfrac{f(x)}{j(x)}=\dfrac{e^x}{\ln x}$ e derivando o numerado e o denominador, separadamente, temos: $\dfrac{f'(x)}{j'(x)}=\dfrac{e^x}{\frac{1}{x}}=x.e^x$ e $\lim\limits_{x\to\infty}\dfrac{e^x}{\ln x}=\infty$.

5. $\lim\limits_{x\to\frac{p}{2}}(p-2).tgx$ ® ∞

6. $\lim\limits_{x\to 1}\left(\dfrac{1}{x}\right)^{\ln(1-x)}$ Solução: $\lim\limits_{x\to 1}\left(\dfrac{1}{x}\right)^{\ln(1-x)}$ **à** 1^∞

Fazendo: $y=\left(\dfrac{1}{x}\right)^{Ln(1-x)}$, aplicando logaritmo neperiano a ambos os membros, temos:

$\ln y=\ln(1-x).\ln\left(\dfrac{1}{x}\right)$ **à** $\lim\limits_{x\to 1}[\ln(1-x)].\ln\left(\dfrac{1}{x}\right)$ **à** $\infty.0$. Fazendo: $\dfrac{f(x)}{j(x)}==\dfrac{\ln\left(\frac{1}{x}\right)}{\frac{1}{\ln(1-x)}}$ **à**

$\dfrac{f(1)}{j(1)}=\dfrac{0}{0}$. Aplicando a Regra de L'Hôspital: $\dfrac{f'(x)}{j'(x)}=\dfrac{\frac{1}{x}}{\frac{-x}{1-x}}=-\dfrac{1-x}{x^2}$ **à**

$\dfrac{f'(1)}{j'(1)}=-\dfrac{1-1}{1^2}=0$ logo $\ln\lim\limits_{x\to 1}\left(\dfrac{1}{x}\right)^{\ln(1-x)}=0=\ln e^0$ **à** $\lim\limits_{x\to 1}\left(\dfrac{1}{x}\right)^{\ln(1-x)}=1$

7. $\lim\limits_{x\to 1}(x)^{\frac{1}{x-1}}=?$ Solução: $\lim\limits_{x\to 1}(x)^{\frac{1}{x-1}}$ **à** 1^∞. Fazendo: $y=(x)^{\frac{1}{x-1}}$ e aplicando logaritmo de Nepper a ambos os membros dessa expressão, temos:

$\ln y=\ln(x)^{\frac{1}{x-1}}$ **à** $\lim\limits_{x\to 1}\ln y=\lim\limits_{x\to 1}\left(\dfrac{1}{x-1}\right)\ln(x)$ **à** $\infty.0$. Fazendo: $\dfrac{f(x)}{j(x)}=\dfrac{\ln x}{x-1}$ e

$\dfrac{f'(x)}{j'(x)}=\dfrac{\frac{1}{x}}{1}=\dfrac{1}{x}$ **à** $\dfrac{f'(1)}{j'(1)}=\dfrac{1}{1}$. Logo: $\lim\limits_{x\to 1}\ln y=\lim\limits_{x\to 1}\ln(x)^{\left(\frac{1}{x-1}\right)}$ **à**

$\ln\lim\limits_{x\to 1}(x)^{\left(\frac{1}{x-1}\right)}=1=\ln e^1$ **à** $\lim\limits_{x\to 1}(x)^{\frac{1}{x-1}}=e^1$

8. $\lim_{x\to 0}\left(\frac{x}{1-x}\right)^{\ln(1+x)}$?　　Solução: $\lim_{x\to 0}\left(\frac{x}{1-x}\right)^{\ln(1+x)}$ à $-\infty^{0}$. Fazendo $y=\left(\frac{x}{1-x}\right)^{\ln(1+x)}$, aplicando logaritmo neperiano a ambos os membros, temos: $Lny = Ln(1+x).Ln\left(\frac{x}{1-x}\right)$ à $0.\infty$. Fazendo: $\frac{f(x)}{j(x)} = \frac{Ln(1+x)}{\frac{1}{Ln\left(\frac{x}{1-x}\right)}}$ à

$\frac{f'(x)}{j'(x)} = \frac{\frac{1}{1+x}}{\frac{1}{\frac{x}{(1-x)^2}}} = \frac{x}{(1+x).(1-x)^2}$ à $\frac{f'(0)}{j'(0)} = \frac{0}{(1+0).(1-0)^2} = 0$. Pela a Regra de L'Hôspital: . $\ln\lim_{x\to 0}\left(\frac{x}{1-x}\right)^{\ln(1+x)} = 0 = \ln e^{0}$. Logo: $\lim_{x\to 0}\left(\frac{x}{1-x}\right)^{\ln(1+x)} = 1$.