Exercícios — Regra de L'Hospital

I. Limites que redundam em $\frac{0}{0}$ ou $\frac{\infty}{\infty}$

1. $\lim_{x\to 3}\frac{e^x - e^3}{x-3} = ?$ **à** fazendo $\frac{f(x)}{g(x)} = \frac{e^x - e^3}{x-3}$ e $\frac{f(3)}{g(3)} = \frac{e^3 - e^3}{3-3} = \frac{0}{0}$, derivando o numerador e o denominador, separadamente, temos: $\frac{f'(x)}{g'(x)} = \frac{e^x}{1}$ logo $\lim_{x\to 3}\frac{e^x - e^3}{x-3} =$

   $\lim_{x\to 3}\frac{e^x}{1} = e^3$

2. $\lim_{x\to 0}\frac{e^x - e^{-x} - x^2}{2x - \operatorname{sen} x} = ?$ **à** fazendo $\frac{f(x)}{g(x)} = \frac{e^x - e^{-x} - x^2}{2x - \operatorname{sen} x}$ e $\frac{f(0)}{g(0)} = \frac{0}{0}$; derivando o numerador e denominador, separadamente, temos: $\frac{f'(x)}{g'(x)} = \frac{e^x + e^{-x} - 2x}{-\cos x}$ e $\frac{f'(0)}{g'(0)} = -\frac{2}{1}$

   logo $\lim_{x\to 0}\frac{e^x - e^{-x} - x^2}{2x - \operatorname{sen} x} = \lim_{x\to 0}\frac{e^x + e^{-x} - 2x}{-\cos x} = -2$

3. Calcule o limite $\lim_{x\to 0}\frac{x - \operatorname{sen} x}{3x^2} = ?$ **à** $\frac{f(x)}{j(x)} = \frac{x - \operatorname{sen} x}{3x^2}$ e $\frac{f(0)}{j(0)} = \frac{0}{0}$ $\to$ derivando o numerador e o denominador, separadamente, temos: $\frac{f'(x)}{j'(x)} = \frac{1 - \cos x}{6x}$ e $\frac{f'(0)}{j'(0)} = \frac{0}{0}$;

   $\frac{f''(x)}{j''(x)} = \frac{\operatorname{sen} x}{6}$ e $\frac{f''(0)}{j''(0)} = \frac{0}{6} = 0$ $\to$ $\lim_{x\to 0}\frac{x - \operatorname{sen} x}{3x^2} = \lim_{x\to 0}\frac{\operatorname{sen} x}{6} = 0$

4. Calcule o limite $\lim_{x\to 1}\frac{Ln[\cos(x-1)]}{1 - \operatorname{sen}\frac{px}{2}} = ?$ $\frac{f(x)}{j(x)} = \frac{Ln[\cos(x-1)]}{1 - \operatorname{sen}\frac{px}{2}}$ e $\frac{f(1)}{j(1)} = \frac{0}{0}$ $\to$ derivando o numerador e o denominador, separadamente, temos: $\frac{f'(x)}{j'(x)} = \frac{\frac{-\operatorname{sen}(x-1)}{\cos(x-1)}}{-\frac{p}{2}.\cos\frac{px}{2}} = \frac{2.\operatorname{sen}(x-1)}{p.\cos\frac{px}{2}.\cos(x-1)}$

   e $\frac{f'(1)}{j'(1)} = \frac{0}{0}$ **à** $\frac{f''(x)}{j''(x)} = \frac{2.\cos(x-1)}{\frac{p^2}{2}.\operatorname{sen}\frac{px}{2}.\cos(x-1) - p.\cos(x-1).\operatorname{sen}(x-1)}$ e $\frac{f''(1)}{j''(1)} = -\frac{4}{p^2}$

$$\lim_{x\to 1}\frac{Ln[\cos(x-1)]}{1 - \operatorname{sen}\frac{px}{2}} = \lim_{x\to 1}\frac{2.\cos(x-1)}{\frac{p^2}{2}.\operatorname{sen}\frac{px}{2}.\cos(x-1) - p.\cos(x-1).\operatorname{sen}(x-1)} = -\frac{4}{p^2}$$

5. Calcule o limite $\lim_{x\to+\infty}\frac{1-x-e^{-x}}{2x^3}=?$ **à** $\frac{f(x)}{j(x)}=\frac{1-x-e^{-x}}{2x^3}$ e $\frac{f(+\infty)}{j(+\infty)}=-\frac{\infty}{\infty}\to$

derivando o numerador e o denominador, separadamente, temos: $\frac{f'(x)}{j'(x)}=\frac{-1+e^{-x}}{6x^2}$ e

$\frac{f'(+\infty)}{j'(+\infty)}=-\frac{1}{\infty}$ $\lim_{x\to+\infty}\frac{1-x-e^{-x}}{2x^3}=\lim_{x\to+\infty}\frac{-1+e^{-x}}{6x^2}=-\frac{1}{\infty}=0$

6. $\lim_{x\to0}\frac{\ln x}{\ln(1-\cos x)}=?$ $\frac{f(x)}{j(x)}=\frac{\ln x}{\ln(1-\cos x)}=$ e $\frac{f(0)}{j(0)}=\frac{\infty}{\infty}$. Derivando o numerador e o denominador, separadamente, temos:

$$\frac{f'(x)}{j'(x)}=\frac{\frac{1}{x}}{\frac{\operatorname{sen}x}{1-\cos x}}=\frac{1-\cos x}{x.\operatorname{sen}x}=\frac{-2\operatorname{sen}\frac{x}{2}.\operatorname{sen}\frac{x}{2}}{x.2.\operatorname{sen}\frac{x}{2}.\cos\frac{x}{2}}=\frac{-\operatorname{sen}\frac{x}{2}}{x.\cos\frac{x}{2}}$$ e $\frac{f'(0)}{j'(0)}=\frac{0}{0}$. Derivando

novamente numerador e o denominador, vem: $$\frac{f''(x)}{j''(x)}=\frac{-\frac{1}{2}.\cos\frac{x}{2}}{1.\cos\frac{x}{2}+x.\left(-\frac{1}{2}.\operatorname{sen}\frac{x}{2}\right)}$$ e

$$\frac{f''(0)}{j''(0)}=\frac{\frac{1}{2}.\cos\frac{0}{2}}{1.\cos\frac{0}{2}+0.\left(-\frac{1}{2}.\operatorname{sen}\frac{0}{2}\right)}=\frac{\frac{1}{2}}{1}$$ logo $\lim_{x\to0}\frac{\ln x}{\ln(1-\cos x)}=\frac{1}{2}$

II. Limites que redundam em $0.\infty$

7. Calcue limite $\lim_{x\to0}\left(x.e^{\frac{1}{x}}\right)=?$ **à** $\lim_{x\to0}\left(x.e^{\frac{1}{x}}\right)=0.\infty$ **à** Fazeno $\frac{f(x)}{j(x)}=\frac{e^{\frac{1}{x}}}{\frac{1}{x}}$ e

$\frac{f(0)}{j(0)}=\frac{\infty}{\infty}$ $\to$ derivando o numerador e o denominador, separadamente, temos:

$$\frac{f'(x)}{j'(x)}=\frac{-\frac{1}{x^2}.e^{\frac{1}{x}}}{-\frac{1}{x^2}.}=e^{\frac{1}{x}}$$ e $\frac{f'(0)}{j'(0)}=\infty$ $\lim_{x\to0}\left(x.e^{\frac{1}{x}}\right)=\lim_{x\to0}\left(e^{\frac{1}{x}}\right)=\infty$

8. $\lim_{x\to0}\cot g2x.\cot g\left(\frac{p}{2}-x\right)=?$ **à** $\lim_{x\to0}\cot g2x.\cot g\left(\frac{p}{2}-x\right)=\infty.0$

$\frac{f(x)}{g(x)} = \frac{\cot g\left(\frac{p}{2} - x\right)}{\frac{1}{\cot g 2x}}$ e $\frac{f(0)}{g(0)} = \frac{\cot\ g\left(\frac{p}{2} - 0\right)}{\frac{1}{\cot\ g 0}} = \frac{0}{0}$ **à** $\frac{f(x)}{g(x)} = \frac{tgx}{tg 2x}$ **à** derivando o numerador e o denominador, separadamente, temos: $\frac{f'(x)}{g'(x)} = \frac{\sec^2 x}{2.\sec^2 2x}$ **à** $\frac{f'(0)}{g'(0)} = \frac{\sec^2 0}{2.\sec^2 2.0} = \frac{1}{2}$ Logo

$$\lim_{x \to 0} \cot g 2x . \cot g\left(\frac{p}{2} - x\right) = \lim_{x \to 0} \frac{\sec^2 x}{2.\sec^2 2x} = \frac{1}{2}$$

III. Limites que redundam em $\infty - \infty$

9. Calcule o limite $\lim_{x \to \frac{p}{2}} (\sec x - tgx) = ?$ **à** $\lim_{x \to \frac{p}{2}} (\sec x - tgx) = \infty - \infty$; Fazendo $\frac{f(x)}{j(x)} =$

$\sec x - tgx = \frac{1}{\cos x} - \frac{\text{sen}\, x}{\cos x} = \frac{1 - \text{sen}\, x}{\cos x}$ e $\frac{f\left(\frac{p}{2}\right)}{j\left(\frac{p}{2}\right)} = \frac{0}{0}$; derivando o numerador e o denominador, separadamente, temos: $\frac{f'(x)}{j'(x)} = \frac{-\cos x}{-\text{sen}\, x} = \frac{\cos x}{\text{sen}\, x}$ e $\frac{f'\left(\frac{p}{2}\right)}{j'\left(\frac{p}{2}\right)} = \frac{0}{1} = 0$

$$\lim_{x \to \frac{p}{2}} (\sec x - tgx) = \lim_{x \to \frac{p}{2}} \frac{\cos x}{\text{sen}\, x} = 0$$

IV. Limites que redundam em 0^0 *,* 1^∞ *,* ∞^0

10. Calcular o limite $\lim_{x \to 0} (x^x) = ?$ **à** $\lim_{x \to 0} (x^x) = 0^0$

$y = (x^x) \Rightarrow \ln y = x . \ln x$, fazendo $\frac{f(x)}{j(x)} = \frac{\ln x}{\frac{1}{x}}$ e $\frac{f(0)}{j(0)} = \frac{\infty}{\infty}$ **à** derivando o numerador e o denominador, separadamente, temos: $\frac{f'(x)}{j'(x)} = \frac{1/x}{-1/x^2} = -x$ e $\frac{f'(0)}{j'(0)} = 0$ Logo

$\lim_{x \to 0} \ln(x^x) = \ln \lim_{x \to 0} (x^x) = 0 = \ln e^0 = \ln 1$ **à** $\lim_{x \to 0} (x^x) = 1$

11. Calcule o limite $\lim_{x \to 0} (e^x + x)^{\frac{1}{x}} = ?$ **à** $\left[(e^x + x)^{\frac{1}{x}}\right]_{X \to 0} \to 1^\infty$

$\ln\left(e^x+x\right)^{\frac{1}{x}} = \frac{1}{x}.\ln\left(e^x+x\right)$ e $\left[\frac{1}{x}.\ln\left(e^x+x\right)\right]_{x\to 0} \to 0.\infty$; Fazendo: $\frac{f(x)}{j(x)} = \frac{\ln\left(e^x+x\right)}{x}$ e

$\frac{f(0)}{j(0)} = \frac{0}{0}$ $\Rightarrow$ derivando o numerador e o denominador, separadamente, temos: $\frac{f'(x)}{j'(x)} = \frac{\frac{e^x+1}{e^x+x}}{1} =$

$\frac{e^x+1}{e^x+x}$ e $\frac{f'(0)}{j'(0)} = \frac{2}{1} = 2$ **à** $\lim_{x\to 0}\ln\left[\left(e^x+x\right)^{\frac{1}{x}}\right] = 2 = \ln e^2$ ou $\ln\left[\lim_{x\to 0}\left(e^x+x\right)^{\frac{1}{x}}\right] = \ln e^2$

$\lim_{x\to 0}\left(e^x+x\right)^{\frac{1}{x}} = e^2$

12. Calcule o limite $\lim_{x\to 0}\left(\cot gx\right)^{\operatorname{sen} x} = ?$

$\left[\left(\cot gx\right)^{\operatorname{sen} x}\right]_{x\to 0} \to \infty^0$ que é uma indeterminação.

$\ln\left(\cot gx\right)^{\operatorname{sen} x} = \operatorname{sen} x.\ln\left(\cot gx\right)$ e $\left[\operatorname{sen} x.\ln\left(\cot gx\right)\right]_{x\to 0} \to 0.\infty$

$\frac{f(x)}{j(x)} = \frac{\ln\left(\cot gx\right)}{1/\operatorname{sen} x}$ e derivando o numerador e o denominador, separadamente, temos:

$\frac{f'(x)}{j'(x)} = \frac{\frac{-\operatorname{cos}ec^2 x}{\cot gx}}{\frac{-\cos x}{\operatorname{sen}^2 x}} = \frac{1}{\operatorname{sen} x.\cos x}.\frac{\operatorname{sen}^2 x}{\cos x} = \frac{\operatorname{sen} x}{\cos^2 x}$ e $\frac{f'(0)}{j'(0)} = \frac{0}{1} = 0$ $\lim_{x\to 0}\ln\left[\left(\cot gx\right)^{\operatorname{sen} x}\right] = 0 = \ln e^0 = \ln$

1 ou $\ln\left[\lim_{x\to 0}\left(\cot gx\right)^{\operatorname{sen} x}\right] = \ln 1$ logo $\lim_{x\to 0}\left(\cot gx\right)^{\operatorname{sen} x} = 1$