Limites quando x tende ao infinito

4 páginas com 18 questões

Considerações iniciais para se calcular limites quando x tende ao infinito:

$\lim_{x\to+\infty} \frac{1}{x} = 0$ (quando se divide 1 por um número muito grande, o resultado é tão pequeno que se aproxima de ZERO) à Limites laterais: $\begin{cases} \lim_{x\to+\infty} \frac{1}{x} = 0 \\ \lim_{x\to-\infty} \frac{1}{x} = 0 \end{cases}$ à como os limites laterais são iguais, então o limite da função é: $\lim_{x\to-\infty} \frac{1}{x^n} = 0$, para n ímpar.

Limites laterais: $\begin{cases} \lim_{x\to+\infty} \frac{1}{x^2} = 0 \\ \lim_{x\to-\infty} \frac{1}{x^2} = 0 \end{cases}$ à como os limites laterais são iguais, então o limite da função é:

$\lim_{x\to-\infty} \frac{1}{x^n} = 0$, para n par.

1. $\lim_{x\to-\infty} \left(-3x^3 + 5x^2 + x + 2\right) = ?$ Solução: colocando-se o termo de mais alto grau x^3 em evidência,

$\lim_{x\to-\infty} x^3.\left(-3 + \frac{5}{x} + \frac{1}{x^2} + \frac{2}{x^3}\right) = \lim_{x\to-\infty} x^3.\left(-3 + 5.\frac{1}{x} + 1.\frac{1}{x^2} + 2\frac{1}{x^3}\right)$, como os termos fracionários tendem a zero, temos: $\lim_{x\to-\infty} x^3.(-3) = +\infty$

2. $\lim_{x\to+\infty} \left(x^4 - 3x^2 + 5x + 1\right) = ?$ Solução: $\lim_{x\to+\infty} \left(x^4 - 3x^2 + 5x + 1\right)$

$= \lim_{x\to+\infty} x^4.\left(1 + \frac{-3}{x^2} + \frac{5}{x^3} + \frac{1}{x^4}\right) = \lim_{x\to+\infty} x^4.(1) = +\infty$

3. $\lim_{x\to-\infty} \frac{2x^3 + 4x^2 - 1}{x^3 + x^2 + x + 1} = ?$ à $\lim_{x\to-\infty} \frac{2x^3 + 4x^2 - 1}{x^3 + x^2 + x + 1} = \lim_{x\to-\infty} \frac{x^3.\left(2 + \frac{4}{x} - \frac{1}{x^3}\right)}{x^3.\left(1 + \frac{1}{x} + \frac{1}{x^2} - \frac{1}{x^3}\right)} = \lim_{x\to-\infty} \frac{x^3.2}{x^3.1} = 2$

4. $\lim_{x\to+\infty} \frac{\sqrt{x^2 + x + 3}}{2x + 1} = ?$ à $\lim_{x\to+\infty} \frac{\sqrt{x^2\left(1 + \frac{1}{x} + \frac{3}{x^2}\right)}}{x.\left(2 + \frac{1}{x}\right)} = \lim_{x\to+\infty} \frac{x.\sqrt{1 + \frac{1}{x} + \frac{3}{x^2}}}{x.\left(2 + \frac{1}{x}\right)} = \lim_{x\to+\infty} \frac{\sqrt{1 + \frac{1}{x} + \frac{3}{x^2}}}{2 + \frac{1}{x}} = \frac{1}{2}$

5. $\lim_{x\to-\infty} \frac{\sqrt{x^2 + x + 3}}{2x + 1} = ?$ à $\lim_{x\to-\infty} \frac{\sqrt{x^2\left(1 + \frac{1}{x} + \frac{3}{x^2}\right)}}{x.\left(2 + \frac{1}{x}\right)} = \lim_{x\to-\infty} \frac{-x.\sqrt{1 + \frac{1}{x} + \frac{3}{x^2}}}{x.\left(2 + \frac{1}{x}\right)} = \lim_{x\to-\infty} \frac{\sqrt{1 + \frac{1}{x} + \frac{3}{x^2}}}{2 + \frac{1}{x}} = -\frac{1}{2}$

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6. $\lim\limits_{x\to-\infty}\dfrac{\sqrt[3]{x^3+x-1}}{x+3}=?$ **à** $\lim\limits_{x\to-\infty}\dfrac{\sqrt[3]{x^3.\left(1+\frac{1}{x^2}-\frac{1}{x^3}\right)}}{x.\left(1+\frac{3}{x}\right)}=\lim\limits_{x\to-\infty}\dfrac{x.\left(\sqrt[3]{1+\frac{1}{x^2}-\frac{1}{x^3}}\right)}{x.\left(1+\frac{3}{x}\right)}=\lim\limits_{x\to-\infty}\dfrac{\sqrt[3]{1+\frac{1}{x^2}-\frac{1}{x^3}}}{1+\frac{3}{x}}$

$=1$

7. $\lim\limits_{x\to+\infty}\dfrac{3x^2+x+9}{\sqrt{x^4+2}}=?$ $\lim\limits_{x\to+\infty}\dfrac{x^2.\left(3+\frac{1}{x}+\frac{9}{x^2}\right)}{\sqrt{x^4.\left(1+\frac{2}{x^2}\right)}}=\lim\limits_{x\to+\infty}\dfrac{x^2.\left(3+\frac{1}{x}+\frac{9}{x^2}\right)}{x^2..\sqrt{1+\frac{2}{x^2}}}=\lim\limits_{x\to+\infty}\dfrac{\left(3+\frac{1}{x}+\frac{9}{x^2}\right)}{\sqrt{1+\frac{2}{x^2}}}=$

$\lim\limits_{x\to+\infty}\dfrac{3+\frac{1}{x}+\frac{9}{x^2}}{\sqrt{1+\frac{2}{x^2}}}=3$

8. $\lim\limits_{x\to-\infty}\dfrac{3x^2+x+1}{\sqrt{x^4+2}}=?$ **à** $\lim\limits_{x\to-\infty}\dfrac{x^2.\left(3+\frac{1}{x}+\frac{1}{x^2}\right)}{\sqrt{x^4.\left(1+\frac{2}{x^2}\right)}}=\lim\limits_{x\to-\infty}\dfrac{x^2.\left(3+\frac{1}{x}+\frac{1}{x^2}\right)}{x^2..\sqrt{1+\frac{2}{x^2}}}=3$

9. $\lim\limits_{x\to+\infty}\left(\sqrt{x-1}-\sqrt{x+3}\right)=?$ **à** $\lim\limits_{x\to+\infty}\dfrac{\left(\sqrt{x-1}-\sqrt{x+3}\right).\left(\sqrt{x-1}+\sqrt{x+3}\right)}{\sqrt{x-1}+\sqrt{x+3}}=$

$\lim\limits_{x\to+\infty}\dfrac{x-1-(x+3)}{\sqrt{x-1}+\sqrt{x+3}}=\lim\limits_{x\to+\infty}\dfrac{x-1-x-3}{\sqrt{x-1}+\sqrt{x+3}}=\lim\limits_{x\to+\infty}\dfrac{-4}{\sqrt{x-1}+\sqrt{x+3}}=0$

10. $\lim\limits_{x\to-\infty}\left(\sqrt{x^2+x+3}-x\right)=?$ **à** $\lim\limits_{x\to-\infty}\dfrac{\left(\sqrt{x^2+x+3}-x\right).\left(\sqrt{x^2+x+3}+x\right)}{\sqrt{x^2+x+3}+x}=$

$\lim\limits_{x\to-\infty}\dfrac{x^2+x+3-x^2}{\sqrt{x^2+x+3}+x}=\lim\limits_{x\to-\infty}\dfrac{x+3}{\sqrt{x^2+x+3}+x}=\lim\limits_{x\to-\infty}\dfrac{x.\left(1+\frac{3}{x}\right)}{\sqrt{x^2.\left(1+\frac{1}{x}+\frac{3}{x}\right)}+x}=$

$\lim\limits_{x\to-\infty}\dfrac{x.\left(1+\frac{3}{x}\right)}{x.\left(\sqrt{1+\frac{1}{x}+\frac{3}{x}}\right)+x}=\lim\limits_{x\to-\infty}\dfrac{x.\left(1+\frac{3}{x}\right)}{x.\left(\sqrt{1+\frac{1}{x}+\frac{3}{x}}+1\right)}=\lim\limits_{x\to-\infty}\dfrac{1+\frac{3}{x}}{\sqrt{1+\frac{1}{x}+\frac{3}{x}}+1}=\dfrac{1}{2}$

11. $\lim\limits_{x\to-\infty}\dfrac{2x+1}{\sqrt[3]{1-x}+\sqrt{1-x}}=?$ **à** $\lim\limits_{t\to+\infty}\dfrac{2\left(1-t^6\right)+1}{t^2+t^3}=\lim\limits_{t\to+\infty}\dfrac{-2.t^6+2+1}{t^3+t^2}=\lim\limits_{t\to+\infty}\dfrac{-2.t^6}{t^3}=\lim\limits_{t\to+\infty}-2.t^3=-\infty$

12. $\lim\limits_{x\to+\infty}\left(x.\sqrt{\frac{x-1}{x+1}}-x\right)=?$ **à** $\lim\limits_{x\to+\infty}x.\left(\sqrt{\dfrac{x\left(1-\frac{1}{x}\right)}{x\left(1+\frac{1}{x}\right)}}-1\right)=\lim\limits_{x\to+\infty}x.\left(\sqrt{\dfrac{1-\frac{1}{x}}{1+\frac{1}{x}}}-1\right)=0$

13. $\lim_{x\to+\infty}\left(\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x}\right)$=? **à** Fazendo $t=\sqrt{x}$, t^2=x **à** $\lim_{t\to+\infty}\left(\sqrt{t^2+\sqrt{t^2+t}}-t\right)$=

$\lim_{t\to+\infty}\frac{\left(\sqrt{t^2+\sqrt{t^2+t}}-t\right).\left(\sqrt{t^2+\sqrt{t^2+t}}+t\right)}{\sqrt{t^2+\sqrt{t^2+t}}+t}=\lim_{t\to+\infty}\frac{\sqrt{t^2+t}}{\sqrt{t^2+\sqrt{t^2+t}}+t}$ = Colocando-se t^2 em

evidência, vem: $\lim_{t\to+\infty}\frac{\sqrt{t^2.\left(1+\frac{1}{t}\right)}}{\sqrt{t^2+\sqrt{t^2.\left(1+\frac{1}{t}\right)}}+t}=\lim_{t\to+\infty}\frac{\sqrt{t^2.\left(1+\frac{1}{t}\right)}}{\sqrt{t^2+\sqrt{t^2.\left(1+\frac{1}{t}\right)}}+t}=\lim_{t\to+\infty}\frac{t.\left(\sqrt{1+\frac{1}{t}}\right)}{\sqrt{t^2+t.\left(\sqrt{1+\frac{1}{t}}\right)}+t}$

$=\lim_{t\to+\infty}\frac{t.\left(\sqrt{1+\frac{1}{t}}\right)}{\sqrt{t^2\left[1+\frac{1}{t}.\left(\sqrt{1+\frac{1}{t}}\right)\right]}+t}=\lim_{t\to+\infty}\frac{t.\left(\sqrt{1+\frac{1}{t}}\right)}{t\left[\sqrt{1+\frac{1}{t}.\left(\sqrt{1+\frac{1}{t}}\right)}\right]+t}=\lim_{t\to+\infty}\frac{t.\left(\sqrt{1+\frac{1}{t}}\right)}{t\left[\sqrt{1+\frac{1}{t}.\left(\sqrt{1+\frac{1}{t}}\right)}+1\right]}=$

$\lim_{t\to+\infty}\frac{\sqrt{1+\frac{1}{t}}}{\sqrt{1+\frac{1}{t}.\left(\sqrt{1+\frac{1}{t}}\right)}+1}=\frac{1}{2}$

14. $\lim_{x\to+\infty}\frac{x-\sqrt{x^2+x+1}}{\sqrt{x^2+3}-x}$ = ? **à** Mutiplicando-se o numerador e denominador pelos fatores racionalizantes $\left(x-\sqrt{x^2+x+1}\right)$ e $\sqrt{x^2+3}+x$, respectivamente, temos:

$\lim_{x\to+\infty}\frac{\left(x-\sqrt{x^2+x+1}\right).\left(x+\sqrt{x^2+x+1}\right).\left(\sqrt{x^2+3}+x\right)}{.\left(x+\sqrt{x^2+x+1}\right).\left(\sqrt{x^2+3}-x\right).\left(\sqrt{x^2+3}+x\right)}=\lim_{x\to+\infty}\frac{\left[x^2-\left(x^2+x+1\right)\right].\left(\sqrt{x^2+3}+x\right)}{\left[\left(x^2+3\right)-x^2\right].\left(x+\sqrt{x^2+x+1}\right)}=$

$\lim_{x\to+\infty}\frac{\left[x^2-x^2-x-1\right].\left(\sqrt{x^2+3}+x\right)}{\left[x^2+3-x^2\right].\left(x+\sqrt{x^2+x+1}\right)}=\lim_{x\to+\infty}\frac{[-x-1].\left(\sqrt{x^2+3}+x\right)}{[3].\left(x+\sqrt{x^2+x+1}\right)}=$

$\lim_{x\to+\infty}\frac{[-x-1].\left(\sqrt{x^2.\left(1+\frac{3}{x^2}\right)}+x\right)}{[3].\left(x+\sqrt{x^2.\left(1+\frac{1}{x}+\frac{1}{x^2}\right)}\right)}=\lim_{x\to+\infty}\frac{[-x-1].\left(x.\sqrt{1+\frac{3}{x^2}}+x\right)}{[3].\left(x+x\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}\right)}=\lim_{x\to+\infty}\frac{[-x-1].x\left(\sqrt{1+\frac{3}{x^2}}+1\right)}{[3].x\left(1+\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}\right)}=$

$\lim_{x\to+\infty}\frac{[-x-1].\left(\sqrt{1+\frac{3}{x^2}}+1\right)}{[3].\left(1+\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}\right)}=\lim_{x\to+\infty}\frac{[-x-1].2}{[3].2}=-\infty$

15. $\lim_{n\to+\infty}\frac{1+2+3+...+n}{n^2}$ = ? **à** $\lim_{n\to+\infty}\frac{\frac{n.(n+1)}{2}}{n^2}=\lim_{n\to+\infty}\frac{n^2+n}{2.n^2}=\lim_{n\to+\infty}\frac{n^2}{2.n^2}=1/2$

16. $\lim\limits_{n\to+\infty} \dfrac{1^2+2^2+3^2+\ldots+n^2}{n^3}$ =? **à** $\lim\limits_{n\to+\infty} \dfrac{\frac{n.(n+1).(2n+1)}{6}}{n^3} = \lim\limits_{n\to+\infty} \dfrac{n.(2n^2+3n+1)}{6.n^3} = \lim\limits_{n\to+\infty} \dfrac{2.n^3}{6.n^3} = \dfrac{2}{6} = \dfrac{1}{3}$

17. $\lim\limits_{n\to+\infty} \dfrac{1+4+7+\ldots+(3n-2)}{n^2}$ =? **à** $\lim\limits_{n\to+\infty} \dfrac{\frac{3n^2-n}{2}}{n^2} = \lim\limits_{n\to+\infty} \dfrac{3.n^2-n}{2.n^2} = \lim\limits_{n\to+\infty} \dfrac{3.n^2}{2.n^2} = \dfrac{3}{2}$

18. $\lim\limits_{n\to+\infty} \sqrt{2}.\sqrt[4]{2}.\sqrt[8]{2}.....\sqrt[2.n]{2}$ =? **à** $\lim\limits_{n\to+\infty} 2^{\frac{1}{2}}.2^{\frac{1}{4}}.2^{\frac{1}{8}}.....2^{\frac{1}{2n}} = \lim\limits_{n\to+\infty} 2^{\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+....+\frac{1}{2n}} =$

$2^{\lim\limits_{n\to+\infty}\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+....+\frac{1}{2n}} = 2^1 = 2$, **à** O expoente é uma Progressão Geométrica, PG: $\dfrac{1}{2}+\dfrac{1}{4}+\dfrac{1}{8}+\ldots$ de

soma: $S = \dfrac{a_1}{1-q} = \dfrac{1/2}{1-1/2} = 1$