Time-Varying Signal Processing: EEE598, Fall 2002

Assignment 5 Due date: Thursday, October 31, 2002

Use the MATLAB *Time-Frequency Toolbox* in the following. For full credit, you **must** provide all the plots and code, label all axes in Hz and seconds, and provide titles for each plot.

In sonar applications, linear frequency-modulated (LFM) chirps are used for transmission as chirps provide good range and Doppler resolution, both necessary to detect a target. In particular, an LFM chirp is first transmitted, and, in the presence of a target, an echo or target return is received. The received echo is detected and analyzed in order to estimate the target range and the target speed between the transmitted and received signals from the time delay and Doppler, respectively. Consider a transmitted LFM chirp $x(t)$ with sampling frequency $f_s = 102.4$ kHz and duration 10 ms. The chirp rate is $\alpha = 2\,(\text{kHz})^2$, and the initial frequency is 20 kHz (that is, at $t = 0$ the frequency is 20 kHz).

1. (i) Compute the WD of the LFM chirp and provide its image plot. When using `imagesc`, you can force the frequencies to increase in value along the frequency axis using `imagesc(t,f,WDx); axis xy`. (ii) Is the WD aliased? Explain why or why not.

2. (i) Demonstrate that the WD preserves the marginals and the energy of the LFM chirp. (ii) What can you do to your data to improve your results?

3. Let $w(t)$ be additive white Gaussian noise (AWGN) with zero mean. (i) Create L_x samples of the noise where L_x is the number of samples of the signal $x(t)$. (ii) Compute the WD of the noise and provide its image plot. (iii) Comment on the WD of the noise. Does it agree with its power spectral density?

4. (i) Create a new signal $y(t) = x(t) + w(t)$. Adjust the noise such that the signal-to-noise ratio (SNR) is 0 dB. (ii) Compute the WD of $y(t)$ and provide its image plot. Can you still distinguish the LFM?

5. We will investigate the unitarity property of the WD using the signals $x(t)$ and $y(t)$. This is an important property for the detection of signals in noise. In particular, one decides that a known LFM chirp is present in some noisy data by computing the inner product of $x(t)$ with the received signal $y(t)$. If the inner product exceeds a certain threshold, then one can conclude that the LFM chirp is present. This detector is called matched filter, and it is the optimal detector for the known deterministic signal case in AWGN. This same detection problem can also be also solved using the WD. (i) Using MATLAB, demonstrate that $|\langle y, x \rangle|^2 = |\langle\langle \text{WD}_y, \text{WD}_x \rangle\rangle|$. (ii) Provide the mesh plots of the WD of $y_0(t) = w(t)$ (H0 hypothesis, only noise) and $y_1(t) = x(t) + w(t)$ (H1 hypothesis, signal plus noise). (iii) Using the WD (and not the actual data), apply the matched filter to both $y_0(t)$ and $y_1(t)$ and remark on the results. (iv) Do you find that the highest signal correlation is obtained with $y_1(t)$ and should you expect this?

6. Use the function `chirp.m` from the signal processing toolbox. This function produces a real LFM. (i) Using the same specifications as in the main description of this problem, create a new signal $g(t)$, compute its WD and provide its image plot. (ii) Why do we observe 2 lines in the TF plane, and how can you explain the cross term geometry?

7. (i) Create a new signal $h(t)$ using `chirp.m` with the same sampling frequency $f_s = 102.4$ kHz and chirp rate $\alpha = 2\,(\text{kHz})^2$. The duration of the signal is now 20 ms. The signal's initial frequency is again 20 kHz. (ii) Compute the WD of this signal and provide its image plot. (iii) What can you say about the TF structure and CTs? (iv) Take the analytic part of the signal $h(t)$, repeat the process, and discuss the new TF structure.