Modeling and Simulation of a Salient-Pole Synchronous Generator With Dynamic Eccentricity Using Modified Winding Function Theory

Iman Tabatabaei, Jawad Faiz, *Senior Member, IEEE*, H. Lesani, and M. T. Nabavi-Razavi

***Abstract*—This paper models and simulates a salient-pole synchronous generator using a modified winding function theory and more precise stator and rotor winding distribution with dynamic eccentricity between the stator and rotor. Air-gap permeance is also computed more accurately compared to currently available methods. Inductances with this method are compared to those obtained from other methods and it is shown that the results are closer to those obtained from finite element computations. Finally, the calculated inductances are used in a coupled electromagnetic model for simulation and studying the frequency spectrum of the stator line current in the presence of dynamic eccentricity.**

***Index Terms*—Eccentricity, line current frequency spectrum, salient-pole synchronous generator, winding function.**

List of Symbols

L_{aa}, L_{bb}, L_{cc}	Self-inductance of stator phase windings.
L_{ab}, L_{ac}, L_{bc}	Mutual-inductance of stator phase windings.
L_f	Self-inductance of excitation winding (including leakage flux).
L_{af}, L_{bf}, L_{cf}	Mutual-inductances between stator phases and excitation winding.
V, I and λ	Voltage, current and flux-linkage of windings.
T_e	Developed electromagnetic torque.
W_{co}	Co-energy.
δ	Degree of dynamic eccentricity.

I. Introduction

SYNCHRONOUS machines are the most important and valuable devices in power systems. These generators are generally well constructed and robust, but the possibilities of incipient faults are inherent due to stresses involved in the electromechanical energy conversion process. Fault diagnosis can produce significant cost saving by allowing for the scheduling of preventive maintenance, thereby preventing extensive downtime periods caused by extensive failure [1]. In addition to bad performance, faults reduce the life span of synchronous generators. Fault diagnosis of large and costly generators in power stations, oil refineries and petrochemical industries is thus required for preventive maintenance of the machines.

Various fault diagnosis techniques have been proposed including expert system [2], neural networks (NN) [3], fuzzy logic approaches [2] and fuzzy neural networks [4]. The expert systems and fuzzy logic approaches have some intrinsic shortcomings, such as the difficulty of acquiring knowledge and maintaining fault databases. In NN approaches, the training data must be sufficient and compatible to ensure proper training.

A synchronous generator fault not only damages the machine itself but may also cause an interruption in power and hence loss of revenue. It is therefore of great importance to recognize imminent failures in the machines as early as possible, so that they can be corrected, thereby improving the reliability of power system. In various diagnostic techniques, monitoring and measuring electrical, magnetic, chemical, acoustic and thermodynamic quantities as well as measuring partial discharge are required.

About 60% of faults in electrical machines are caused by mechanical parts such as bearings, shaft and coupling. Nearly 80% of these faults result in the displacement of the axis of symmetry or the rotating axis of the rotor. Therefore, existing asymmetry between rotor and stator cause 50% of faults in motors [5]. Furthermore, if these faults have not been diagnosed and prevented, the rotor may touch the stator and result in irreparable damage of machines.

In the case of static eccentricity, the rotating axis of the rotor coincides with its axis of symmetry, but these are displaced with respect to stator axis of symmetry. In the dynamic eccentricity condition, the stator axis of symmetry coincides with the rotating axis of the rotor, but the rotor axis symmetry is displaced with respect to the two former axes. Finally, in the mixed eccentricity condition, all three axes are displaced with respect to each other.

This paper investigates the dynamic eccentricity condition in a salient-pole synchronous machine caused by geometrical asymmetry between its rotor and stator. The contributions of the present paper includes: 1). Accounting for linear rise of mmf across the slots, 2). Approximation of air-gap with three terms instead of two terms as reported in [11].

II. A Modified Winding Function Theory

Modeling faulty electrical machines requires long computation using techniques that are normally more complicated than

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that of healthy machines. To date, finite element (FE), equivalent magnetic circuit [7] and winding function [6] methods have been used for modeling, simulation and analysis of electrical machines with different types of fault. Other methods using some simplifying assumptions have been proposed to solve this problem for particular cases [8]. In this paper, a machine is modeled by using a modified winding function theory for computing magnetic inductances and electromagnetic coupling equations between the stator phase windings and the excitation winding.

One of the advantages of this method is that it is possible to predict transient and steady-state performance of any machine with any type of winding distribution and air-gap length, while taking into account the effect of all spatial and time harmonics. This means that all faults occurring in the stator windings, rotor turns and air-gap eccentricity can be included in the model obtained using this theory.

This theory was initially presented for the computation of single-phase induction motors [7]. The theory was then extended for the steady-state equivalent circuits of two-phase asymmetrical induction motors, including odd- and even-harmonics of the magnetic field. This method was also presented to analyze linear induction motors [8]. By extension of the theory and combining it with concepts of coupled circuits, equivalent d-q-o models for synchronous and induction machines were proposed and used in the analysis of induction machines with concentrated windings [10]. In recent years, this theory has been widely used to study the transient behavior of induction motor under inner faults such as rotor broken bars, stator turns short-circuit and different types of the air-gap eccentricity.

Although this theory has been considerably used in the analysis of induction motors [10], it is considered to be less attractive for synchronous machines. The most important work on this machine is modeling a salient-pole synchronous machine under dynamic eccentricity [9].

The principal equation of the theory which presents the mutual inductance of two arbitrary windings x and y in respect to the winding distribution $\mathrm{n_x}$ and $\mathrm{n_y}$ is as follows [12]:

$$L_{yx} = 2\pi\langle n_x n_y\rangle - 2\pi\frac{\langle Pn_x\rangle\langle Pn_y\rangle}{\langle P\rangle} \tag{1}$$

where operator $\langle \mathrm{f}\rangle$ is defined as the mean of function f over [0, 2π] and P is the permeance distribution of the air-gap. If α is an arbitrary angle in the stator reference frame, it follows that:

$$\langle f\rangle = \frac{1}{2\pi}\int_0^{2\pi} f(\alpha)d\alpha. \tag{2}$$

The derivation of the equations (1) and (2) is discussed in [12]. These equations have been developed by taking into account a more precise distribution of stator phases and rotor excitation windings and also a more precise computation of the air-gap permeance. Its application results are compared with that obtained by the winding function and FE method [9]. This comparison shows that the result obtained is closer to that of the FE computation than that of the normal winding function.

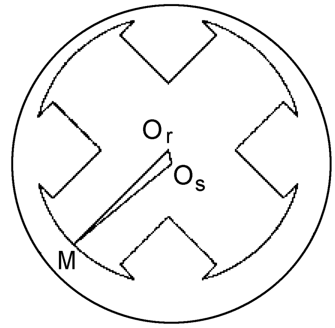

Fig. 1. Positions of rotor and stator air-gap dynamic eccentricity.

III. Air-Gap Permeance Computation

Air-gap permeance is proportional to the inverse of the air-gap length. This quantity is thus neglected for the points far from the poles shoes air-gap, and taken into account only for the air-gap between the salient-poles and the stator. Therefore, the air-gap permeance distribution, between the salient pole of the rotor and stator, is as follows:

$$P(\alpha) = \mu_0\frac{r_{av}(\alpha)}{g(\alpha)} \tag{3}$$

where r_{av} and g are the mean radius of air-gap and air-gap distribution, respectively. These two quantities are constant for the all points between the salient-pole and stator, in the symmetrical case. These two geometrical quantities of the machine are calculated in the dynamic air-gap eccentricity presented in Fig. 1. It is noted that $\mathrm{O_r}$ and $\mathrm{O_s}$ are the centerline of rotor and stator respectively.

Vector $\mathrm{O_sO_r}$ is called the dynamic eccentricity vector. The eccentricity factor is the ratio of the length of this vector, to the symmetrical air-gap length over the pole shoes (g_0). This vector rotates around stator symmetrical axes with angular speed equal to the mechanical speed of the rotor. Therefore, once the motor starts up, it is assumed that this vector coincides with the reference axes of the mechanical angle, and moreover, is always equal to the mechanical angle of the rotor. Fig. 2 shows the position of an arbitrary point M on the rotor pole shoes in the dynamic eccentricity condition. The distance of this point from the rotor center is equal to $\mathrm{R_r}$ (rotor radius), then

$$O_sM = \delta g\cos(\alpha-\theta) + \sqrt{R_r^2 - \delta^2 g^2\sin^2(\alpha-\theta)}. \tag{4}$$

For the inner radius of stator, $\mathrm{R_s}$, the mean length and radius of the air-gap above the poles shoes are as follows:

$$g_e(\alpha) = R_s - O_sM \tag{5}$$

$$r_{av}(\alpha) = \frac{1}{2}(R_s + O_sM). \tag{6}$$

Since the air-gap length above the poles shoes is much smaller than the rotor radius, the second term in (4) may be approximated with $\mathrm{R_r}$. Equations (5)–(6) can therefore be rewritten as

$$g_e(\alpha) = g\,(1-\delta\ \cos(\alpha-\theta)) \tag{7}$$

$$r_{av}(\alpha) = r_0 + \frac{1}{2}\delta g\ \cos(\alpha-\theta) \approx r_0 \tag{8}$$

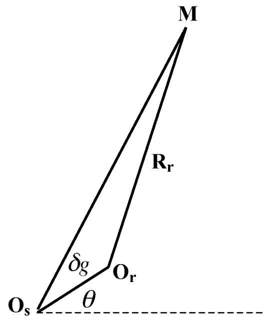

Fig. 2. Position of a point on the rotor pole shoes during air-gap dynamic eccentricity.

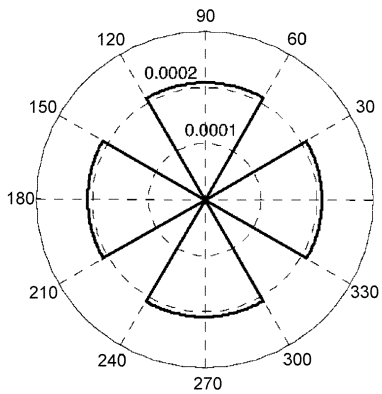

Fig. 3. Polar distribution of air-gap magnetic permeance of a healthy synchronous machine.

where the term $(1/2)\delta g\ cos(\alpha - \theta)$ has been ignored due to the small air-gap length compared to the rotor radius. The mean radius of the air-gap poles is therefore almost equal to this radius in the healthy machine. Moreover, the magnetic permeance distribution of the air-gap above the pole shoes is obtained by substituting (7), (8) into (3) as follows:

$$P(\varphi) = \frac{\mu_0 r_0}{g_0\,(1 - \delta\ \cos(\alpha - \theta))}. \tag{9}$$

Figs. 3 and 4 present the polar distribution of air-gap magnetic permeance of a synchronous machine in the healthy condition and 25% dynamic eccentricity. In such eccentricity, these distributions rotate with mechanical speed of the rotor. In [6], this distribution has been approximated by the first ten terms of its Fourier series, leading to a reduction in the accuracy of the computations.

IV. Inductance Computations

All synchronous machine inductances can be computed by substituting the magnetic permeance distribution of the air-gap and the excitation of the rotor into (1). In [11], a linear rise of mmf of the air-gap across the stator slots has been considered and the first three terms of its Fourier series has been used. In this paper, the mmf rise is taken as shown in Fig. 5. The stator winding of a typical synchronous machine is shown in Fig. 6, with its specifications given in Table I [9].

Fig. 7 shows the turn functions of stator phases winding. Fig. 8 presents a similar function for the excitation winding.

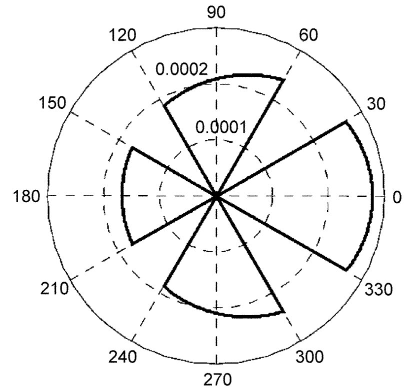

Fig. 4. Polar distribution of air-gap magnetic permeance of a synchronous machine with 25% dynamic eccentricity.

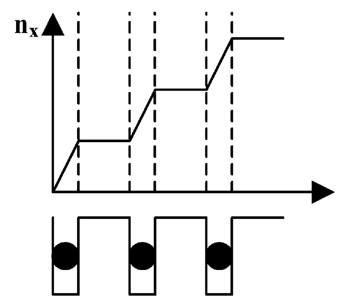

Fig. 5. Turn function of a winding with mmf rise across the slots.

Fig. 9 shows the self-inductance of the stator phase winding with 25% dynamic eccentricity.

V. Comparison With Finite Elements Computation Results

The FE method was used to determine the stator self-inductances of a faulty synchronous machine with 25% dynamic eccentricity The results from the FE method, shown in Fig. 10, can be compared with those in Fig. 9 obtained by the modified winding function method. This comparison indicates a good agreement between these two results. A comparison of self-inductance of phase A of the stator winding of a machine with 25% dynamic eccentricity is shown in Fig. 11. The self-inductances were obtained using the FE method and the unmodified winding function theory. This comparison indicates that the peak inductance values from the FE method are slightly distorted, while the unmodified winding function method results are relatively smooth. Fig. 11 depicts that the proposed modification on the winding function theory results in slightly distortion in the peak inductance values. The reasons for good agreement between the proposed method results and the FE results are as follows.

1) A precise distribution of air-gap magnetic permeance was included for the dynamic eccentricity condition.
2) A more precise distribution of the turn function of windings and linear mmf rise across the slots was considered.

Although it has not been proved here, the main reason for the difference between the results from the proposed method and FE method is probably due to neglecting saturation in the winding function. This claim needs more detailed study.

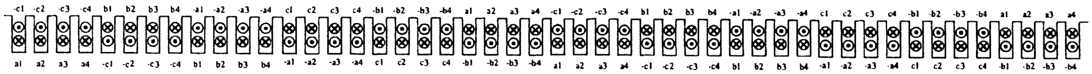

Fig. 6. Distribution of stator phases windings of a synchronous machine.

TABLE I
SPECIFICATIONS OF A 480 V, 60 Hz, 475 kW SYNCHRONOUS GENERATOR MACHINE [9]

Excitation voltage	60	V
Rotor length	273.05	mm
Airgap length	2.54	mm
Rotor radius	422.656	mm
Excitation winding resistance	0.01592	Ω
Stator phase winding resistance	0.3632	Ω
No. of stator phase winding turns	24	
No. of excitation winding turns	216	

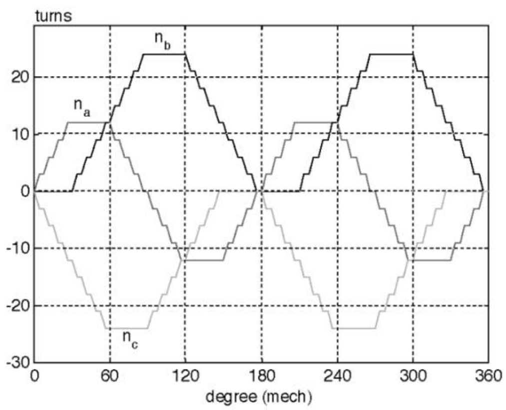

Fig. 7. Turn functions of stator phase windings taking into account the mmf rise across the slots.

VI. SIMULATION RESULTS

In order to simulate performance of a synchronous machine with a geometrical asymmetry condition, the electromagnetic coupling model of the machine circuits is solved using a 4th and 5th order Runge-Kutta method. One of the important and basic stages of this modeling technique is the calculation of inductances of the machine. All the inductances are computed at several rotor angular positions and stored within a computer file. The matrix form of the fundamental equations of this model is as follows:

$$\begin{bmatrix} V_a \\ V_b \\ V_c \\ V_f \end{bmatrix} = \begin{bmatrix} r_s & 0 & 0 & 0 \\ 0 & r_s & 0 & 0 \\ 0 & 0 & r_s & 0 \\ 0 & 0 & 0 & r_f \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \\ \lambda_f \end{bmatrix} \tag{10}$$

$$\begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \\ \lambda_f \end{bmatrix} = \begin{bmatrix} L_{aa} & L_{ab} & L_{ac} & L_{af} \\ L_{ab} & L_{bb} & L_{bc} & L_{bf} \\ L_{ac} & L_{bc} & L_{cc} & L_{cf} \\ L_{af} & L_{bf} & L_{cf} & L_{ff} \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \\ I_f \end{bmatrix} \tag{11}$$

$$T_e = \left. \frac{\partial W_{co}}{\partial \theta} \right|_{I_f, I_a, I_b, I_c = constants} \tag{12}$$

$$W_{co} = \frac{1}{2} \begin{bmatrix} I_a \\ I_b \\ I_c \\ I_f \end{bmatrix}^T \begin{bmatrix} L_{aa} & L_{ab} & L_{ac} & L_{af} \\ L_{ab} & L_{bb} & L_{bc} & L_{bf} \\ L_{ac} & L_{bc} & L_{cc} & L_{cf} \\ L_{af} & L_{bf} & L_{cf} & L_{ff} \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \\ I_f \end{bmatrix}. \tag{13}$$

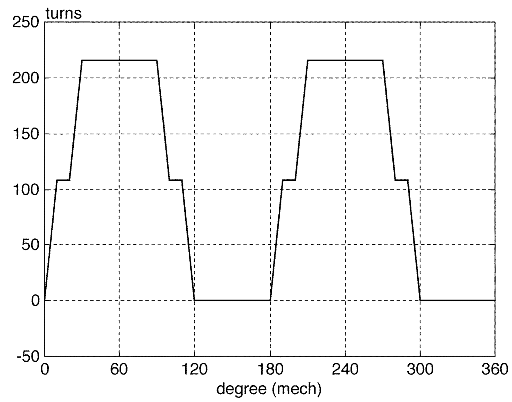

Fig. 8. Turn functions of excitation winding with 25% dynamic eccentricity.

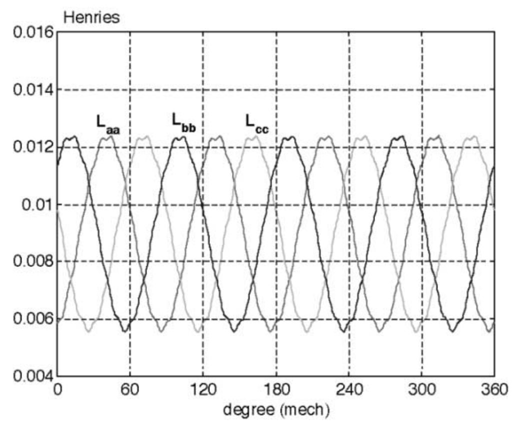

Fig. 9. Calculated self-inductance of stator phases winding with 25% dynamic eccentricity using modified winding function.

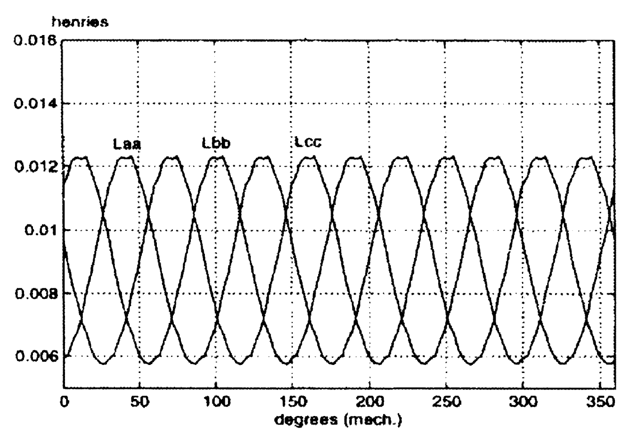

Fig. 10. Self-inductance of stator phases winding with 25% dynamic eccentricity using FE method [9].

By solving the above equations, the frequency spectrum of the stator phase current is estimated. Fig. 12 shows the frequency spectrum of the stator phase current for a healthy machine, as well as, 15% and 25% dynamic eccentricity conditions. Fig. 13 presents the variation of the amplitude versus eccentricity level.

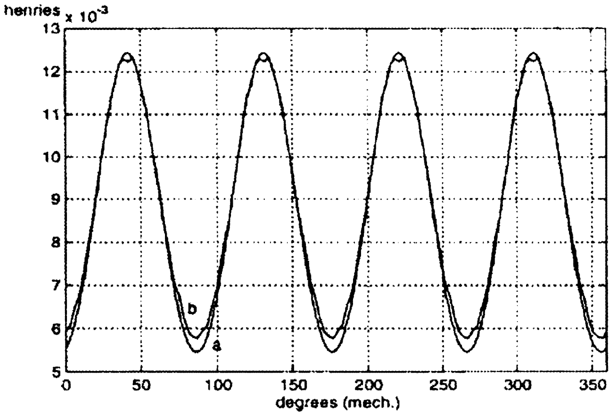

Fig. 11. Calculated self-inductance of phase a of stator with 25% dynamic eccentricity: (a) Unmodified and (b) modified winding function.

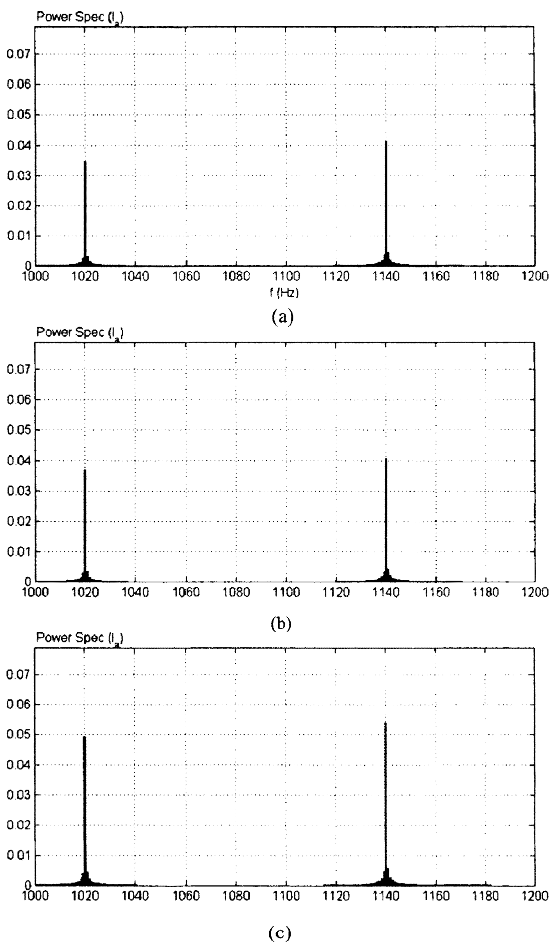

Fig. 12. Frequency spectrum of stator phase current (17th and 19th harmonics) for: (a) healthy machine, (b) 15% dynamic eccentricity, (c) 25% dynamic eccentricity.

VII. Conclusion

Modeling and simulation of a salient-pole synchronous generator has been carried out using a precise distribution of air-gap magnetic permeance present in the dynamic eccentricity condition. A more precise winding function and linear rise of mmf across the slots has also been taken into account. Probably the discrepancy between the winding function and FE method results is due to neglecting the saturation effect. However, the

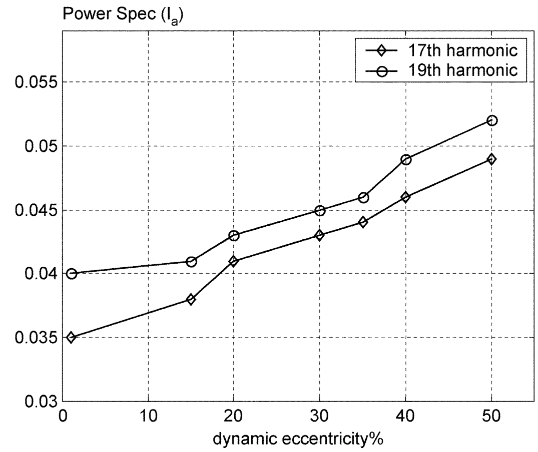

Fig. 13. Amplitude versus eccentricity level for the 17th and 19th harmonics.

results presented in this paper are closer to the FE method results than that of other available methods. The simulation results show that the 17th and 19th harmonics can be employed to diagnose the dynamic eccentricity of the machine.

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