

## CSC 751

# Image Processing Notes

## Document II

October 7, 2003

"Anyone who cannot cope with mathematics is not fully human. At best he is a tolerable subhuman who has learned to wear shoes, bathe, and not make messes in the house."

Robert Anson Heinlein  
*Time Enough for Love*

To the reader: The author has **no interest** in anything "GUI" or the menial details of "making an image look "good"". What the author does enjoy doing is being able to programmatically decipher an image by segmentation and applying various algorithms for understanding and describing a region. Then, machine learning is used and this makes image processing interesting. I suppose, you could say my interest is not in Image Processing, but more of Computer Vision. I'm more interested in getting my hands dirty, than using a pre-built "System.Util.DoEverythingForMe" approach...no offense to the I.T. workers of this World.

## October 6, 2003, Monday

PA #1 due Wednesday October 8, 2003 by Midnight.

Histogram equalization and the CDF (cumulative Distribution Function)

**Separability:** Speeds up computation faster because the filter is decomposed from a  $n \times m$  to 2 1-D filters of size  $n \times 1$  and  $1 \times m$ .

Filters that are separable and not separable.

1	2	1
2	4	2
1	2	1

**Is this filter separable?** Yes, we could multiply the outside rows by 1 and the middle by 2 to get what we need.

0	1	0
1	2	1
0	1	0

**Is this filter separable?** No. There is no possible way to multiply by 0 and then by 1 to get 1, as the values will be zero.

**Is the median filter separable?** We will come back to this-> translation: "*It sounds like a good test question!*"

*A search on Google for keyword "separable median" presents the following results:*

**Efficient Serial And Parallel Algorithms For Median Filtering**

<http://citeseer.nj.nec.com/ranka91efficient.html>

Chapter 3 (Lab)

[http://poseidon.csd.auth.gr/LAB\\_PUBLICATIONS/Books/dip\\_material/chapter\\_3/](http://poseidon.csd.auth.gr/LAB_PUBLICATIONS/Books/dip_material/chapter_3/)

<http://course.ie.cuhk.edu.hk/~ieg7007/LectureNote/Lec5.pdf>

It turns out the median filter **is and is not separable**.

## Distance Metrics

1. Euclidean Distance

$$\text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

2. City block Distance

$$\text{Distance} = |x_1 - x_2| + |y_1 - y_2|$$

3. Chessboard Distance

$$\text{Distance} = \max(|x_1 - x_2|, |y_1 - y_2|)$$

### Chamfer 3-4 Distance transform

#### Connectivity Analysis

- 1) 4-Connectivity
- 2) 8-Connectivity

One approach is to use a Seed-Filled algorithm.

- Starting with a seed, pixel in a region, recursively add adjacent pixels.
- How do we get the seed?

## October 8, 2003, Wednesday

PA #1 due tonight, Midnight  
PA #2, on the web, due 10/24

- Finish chapter 3
- Read Chapter 4. We will probably not get into the frequency domain until after the test on Wednesday
- No class on Monday.
- Midterm 10/15
- New Version of ImageLib out (again).
- Also, a windows version of ImageLib is out. Use mingw, instead of cygwin. Less space used and less of a headache.
- <http://www.mcs.sdsmt.edu/csc464/>

In the Assignment (PA #2), use the noise generation functions given to us for #26 and 27.

For the magnitude, use the arc tangent function, atan2 (man atan2).

$$\tan^{-1} \frac{G_y}{G_x}$$

You may ignore image edges when performing spatial-domain filtering. (extra credit for handling them)

**Last Class:**  
Separability

## Image Arithmetic

### Distance Metrics

Euclidean

D4 (city block)

D8 (chessboard distance)

### Distance transforms

D34: Chamfer 3-4 Distance transform

- good approximation to the Euclidean distance, within 8 percent (without the sqrt and expensive math operations).

How do we compute the Chamfer 3-4 distance transform?

8	7	6	7	8
7	4	3	4	7
6	3	0	3	6
7	4	3	4	7
8	7	6	7	8

Two-pass recursive transform

#### Pass 1 (forward Pass)

```
For (I=0; I<nrows; I++)
  For (j=0; j<ncols; j++)
  {
      Dij=MIN
  }
```

#### Pass 2 (backward pass)

```
For (I==nrows-2; I>=0; I--)
{
  For (j=ncols-2; j>=0; j--)
  {
      d[I][j]=min
```

Finish up by dividing result by 3

### **11:37 am: Demo of chamfer 3-4 distance transform.**

#### **Steps:**

- 1) Loaded brain image
- 2) Performed Binary Threshold
- 3) Do the distance transform
- 4) Contrast stretch
- 5) Pseudocolor

### **Connectivity Analysis (“Blob Labeling”)**

- discovering adjacency relationships

#### **Approaches:**

- 1) Seed fill (recursive)
  - a. Adjacency
    - i. 4-Connected
    - ii. 8-Connected

What distance metric would be used for these adjacencies?

Speedup and optimizations

- (1) Can create our own stack
- (2)**L-shaped template

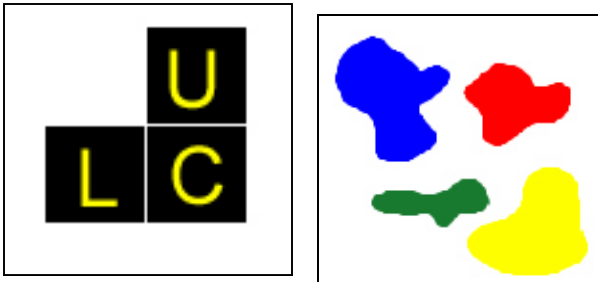
## October 10, 2003, Friday

Size of the Gaussian Smoothing kernel: (look at October 26, 2003)

$$6\sqrt{\sigma}$$

**Size filtering:** Gets rid of small points

Look



### Blob Coloring Algorithm

C is  $f(x,y)$   
U is  $f(x,y-1)$   
L is  $f(x-1,y)$

1. We have a special matrix with the assignments of each blob stored in it.
2. Moving from left to right in the image, we do the following:
  - a. If we encounter a pixel where L and U are unassigned, make a new #
  - b. If L is set, U is not or vice-versa, then set  $C=L$  or  $U$  (respectively)
  - c. If both are set and equal, use that value to set C.
  - d. If U and L differ, use the value of U.
3. Was there a point here where values could be combined? What did it have to do with the Union Find algorithm?

### Union Find Algorithm

See your data structures book on this algorithm:

The **union-find algorithm** gives a nice method for keeping track of disjoint sets (<<this relates to blob coloring, as each is a disjoint set>>). It allows performing set-union operations on sets which are in some way equivalent, while ensuring that the end product of such a union is disjoint from any other set. (<http://www.cs.rug.nl/~michael/ismm2000.pdf>).

### Quantization Issues

Printing grayscale images on B&W printers.

- (1) Binary thresholding
- (2) Intermediate shades of B&W to represent different intensities
- (3) Dithering
- (4) Error Diffusion
- (5)

**This works only for 4-connectivity.**

**How would this mask would change if we had 8-connectivity?**

## October 13, 2003, Monday

Holiday-no class.

## October 15, 2003, Wednesday

**MIDTERM**

**Started: 11:52 am**

**Ended: 1:00 pm**

Theme: Histogram Equalization

## October 17, 2003, Friday

Went over test from Wednesday

## October 20, 2003, Monday

Read Chapter 4

PA #2

PA #3, you select the project

CSC751: Select an article for review by the 29<sup>th</sup>

### Frequency Domain

Fourier Transform

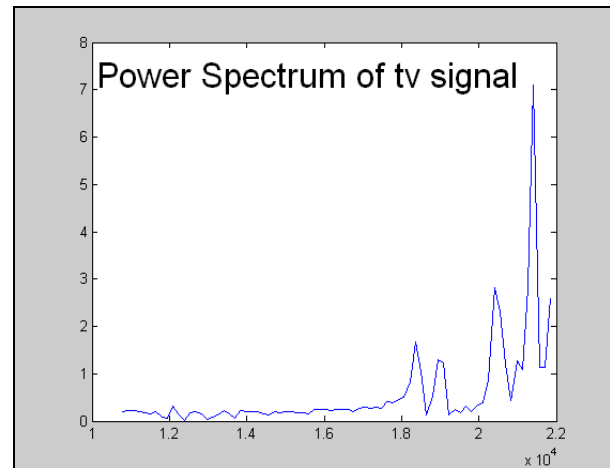
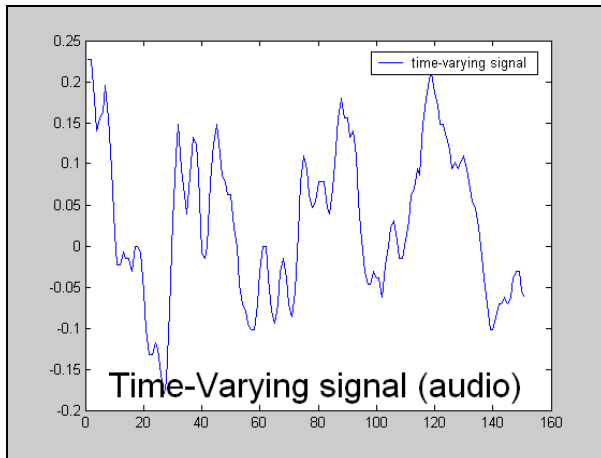
Function  $f(x)$  maybe represented with a series expansion , e.g. power series

$$f(x) = \sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots + a_n x^n$$

Or use the Fourier Series

$$f(x) = \sum_{-\infty}^{\infty} C_u e^{i2\pi ux} \quad \text{where } i = \sqrt{-1}$$

written as a function of  $u$ .



## Continuous Case

### Inverse

### Forward

Fourier transform yields frequency components in a time or space-varying signal.

### Euler's formula

In general, the FT of a signal is complex.

Complex  $z = x + iy$  (see figure 10.20.1)

for  $i = \sqrt{-1}$

$x, y \in \text{Re}$

**Mag**  $|z| = \sqrt{x^2 + y^2}$

**Phase**  $\theta = \tan^{-1}\left(\frac{y}{x}\right)$

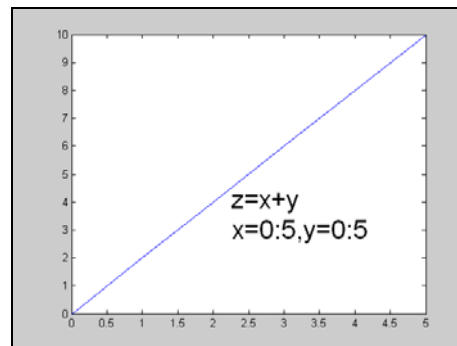


Figure 10-20.1

## Complex conjugate

What is the complex conjugate and what does it mean to us?

In general, you can always obtain the complex conjugate of any expression by simply replacing  $j$  with  $-j$ . In the complex plane, this is a *vertical flip* about the real axis; *i.e.*, complex conjugation replaces each point in the complex plane by its *mirror image* on the other side of the  $x$ -axis.

## DFT Properties

**Periodicity:** DFT and its inverse are periodic with a period of  $N$  for an  $N$ -point DFT.

In General, continuous signals are band limited.

Periodicity implies:

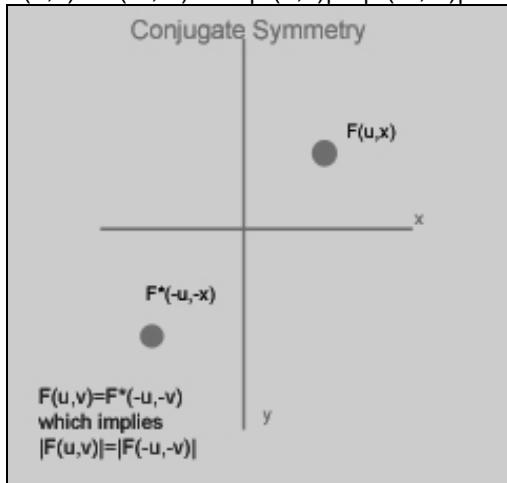
$$F(u) = F(u+N)$$

$$\text{And } f(x) = f(x+N)$$

The inverse is also periodic

## Conjugate Symmetry

$$F(u,v) = F^*(-u,-v) \text{ and } |F(u,v)| = |F(-u,-v)|$$



**October 22, 2003, Wednesday**

Topics for final group project

Heckbert Median cut: For 24-bit color to 8-bit color conversions.

Why do we want to re-compute the histogram in our code?

Indent.pro -> indent looks for the file automatically. Rename to .indent.pro for hiding it.

Magnitude: used the log to compress the data (for an FFT demonstration the prof. Is doing).

Areas highly correlated in an image will be low-frequency components.

Edges are responsible for high-frequency components.

## October 24, 2003, Friday

PA#2

PA#3

### *Project requirements*

- project proposal (1 page) - due Wednesday 10/29/03
- C/C++ implementation - due Monday 12/01/03
- technical paper (~10 pages) - due Monday 12/01/03
- 15 minute oral presentations - 12/01/03 through 12/12/03

### **Possible topics**

<b>Project Topic</b>	<b>Group Size</b>
Histogram specification / adaptive histogram equalization	(2)
Image transforms (2-D FFT, Walsh, Hadamard, Hartley)	(2-4)
Frequency domain filtering (ideal, Butterworth, homomorphic)	(2-4)
Morphological filtering (binary, grayscale)	(2-4)
Color models and quantization (HSI, Heckbert median cut, dithering)	(2-3)
Image restoration (inverse filter, Wiener filter, interactive restoration)	(2-3)
Geometric transformations (warping, morphing)	(2-4)
Image encoding (runlength, contour, quadtree, Huffman)	(2-4)
Connectivity analysis (seed fill, blob labeling)	(2)
Region-based segmentation (thresholding, pixel aggregation, split-and-merge)	(2-3)
Boundary-based segmentation (edge linking, boundary following)	(2-3)
Hough transform (parametric, general)	(2)
Texture analysis (moments, co-occurrence matrices)	(2-4)
Representation / description schemes (Fourier descriptors, polygonal approximations, signatures)	(2-4)
Pattern recognition (correlation, minimum distance classifiers, artificial neural network classifiers)	(2-4)
Other topics of interest	?

**Review Paper:** 4 pages (its okay if its over 4).

## Conjugate Symmetry

### October 26, 2003, Sunday

Notes about Gaussian Smoothing:

#### How do we define the size of the kernel?

I did not write down the formula because I didn't remember it exactly. I believe Schalkoff recommends using a filter width  $w=6*\sqrt{2}*\sigma$ , where the filter size  $m=2w+1$  for an  $m \times m$  filter. In practice, this resulted in long computation times. I believe I am using  $w=3*\sigma$  in my code, which seemed to give similar results at significantly less cost.

### October 27, 2003, Monday

Finish Chapter 4

Start Chapter 9 (Morphological filters)

Pa#3, proposal due at start of class

-Consult with me first

#### Laplacian

There are two in this assignment; one takes the absolute value of

$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$

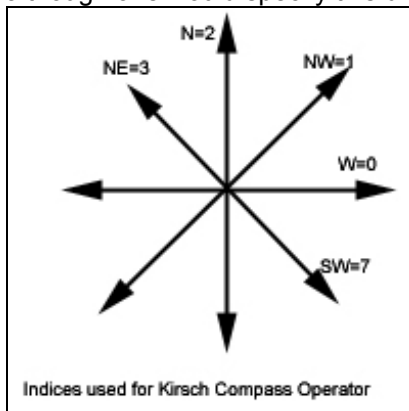
$\begin{bmatrix} -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$

$\begin{bmatrix} 0 & -1 & 0 \end{bmatrix}$

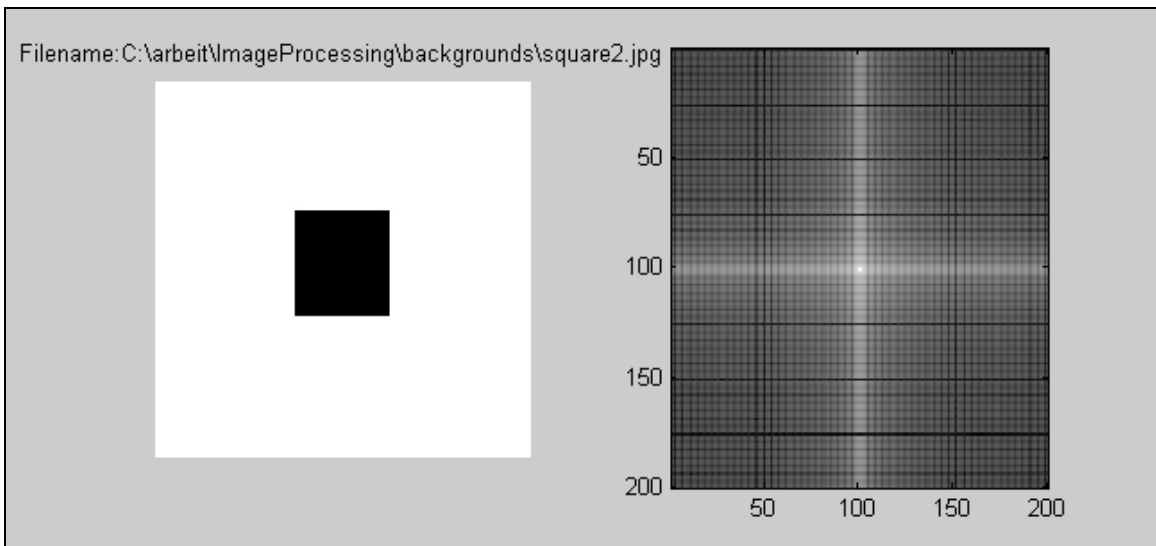
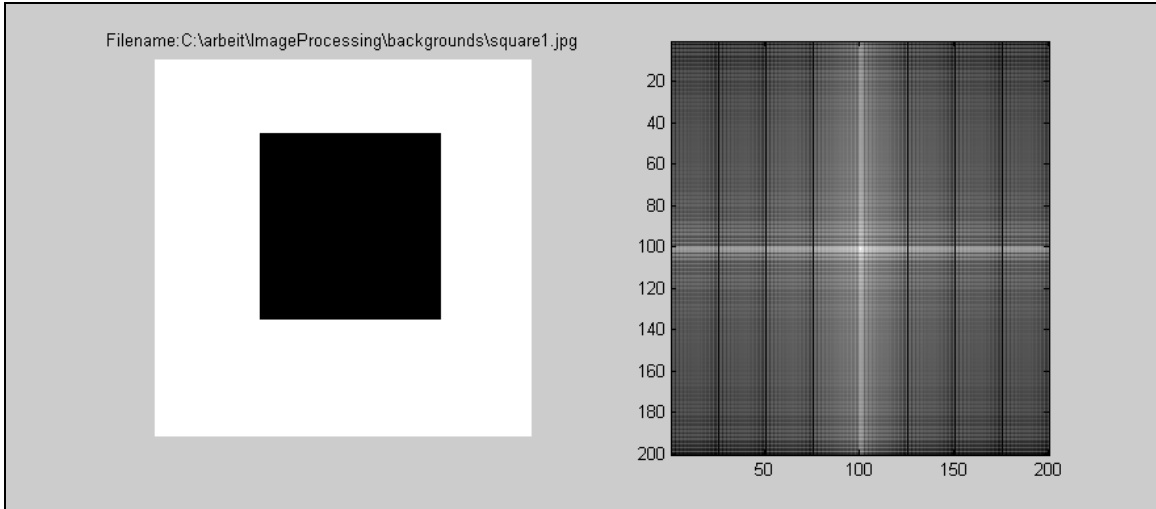
One of them takes the absolute value of that mask.

Embossing:  $F(x+1,y+1) \leftarrow$  subtract these

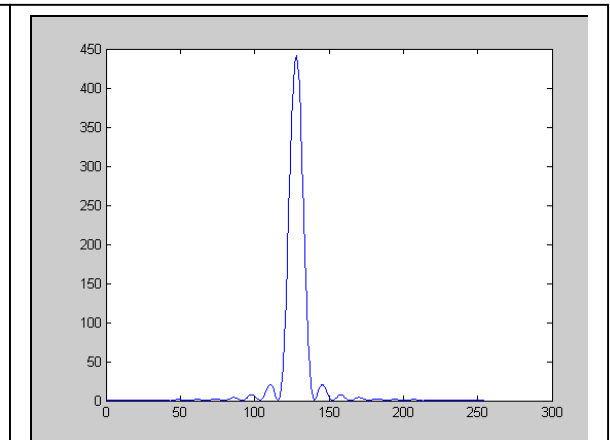
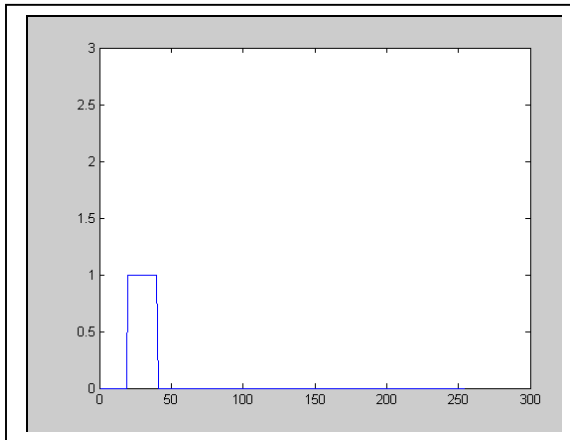
Kirsch Edge Gradient. The directions start at 0 degrees, with index 0, called "West" in class, although one would specify this direction as East, normally.



DFT: Demos (11:15 am)



Box Function



FFT (magnitude) of box function (sinc function)

**Sinc function:** A damped sinusoidal function.

### Butterfly Method for FFT

Complexity for FFT (1-D):  $N \log N$

Complexity for FFT (2-D):  $N^2 \log N$

Band-limited signals?

Dirac Delta Comb filter

## Convolution

### Discrete Convolution

### Convolution Theorem

## Mathematical model of Image Filtering

### Why Add Zeros?

When a signal overlaps with another signal we cannot get back what we started with.

This is called “aliasing”

### Shannon-Whittaker Sampling Theorem

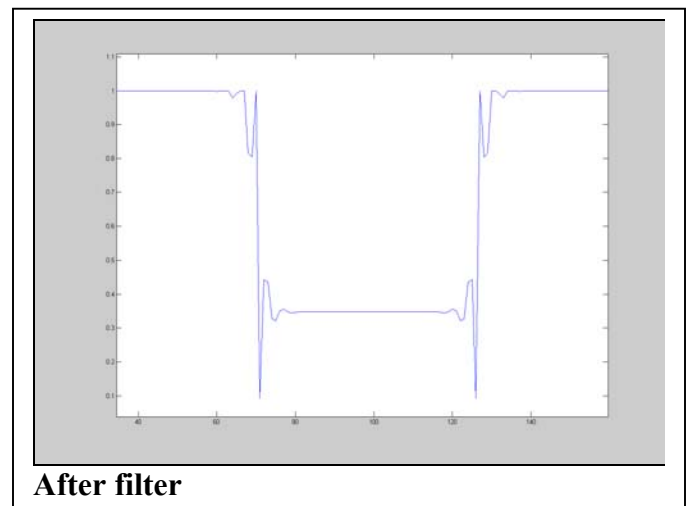
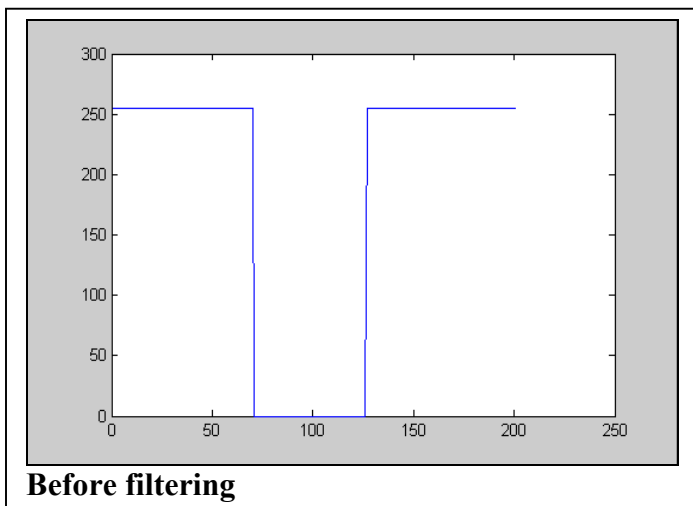
You must select  $\Delta x$  such that  $\Delta x$

How to handle **aliasing**?

- (1) Sampled at higher rate
- (2) Reduce the highest frequency (blurring-> or low-pass filter).

What is **homomorphic** filtering?

<http://www.cs.uwa.edu.au/~pk/Research/MatlabFns/>



**October 29, 2003, Wednesday**

**October 31, 2003, Friday**

Morphology

Set Theory

**November 5, 2003, Wednesday**

Thinning, Thickening, Skeletonization

"Grassfire algorithm"

"Hit or Miss transformation"

**Grayscale Morphology**

Erosion:

Subtract the value from the SE and then take the minimum.

Dilation:

Add the value from the SE and take the maximum.

**November 7, 2003, Wednesday**

Image Restoration

Inverse Filtering

## November 9, 2003, Monday

**Finish Chapter 9, Morphology**

**Read Chapter 5, Restoration**

- Grayscale morphology
- intro to restoration
- math model of restoration
- Inverse Filtering
- Math. Model of Degradation
- Granulometry: counting objects in a view using morphology.

### Restoration

- Estimation of the our Error

(1) Mean square Error

$$MSE = \frac{1}{N^2} \sum_x \sum_y (f(x,y) - \hat{f}(x,y))^2$$

(2) RMSE

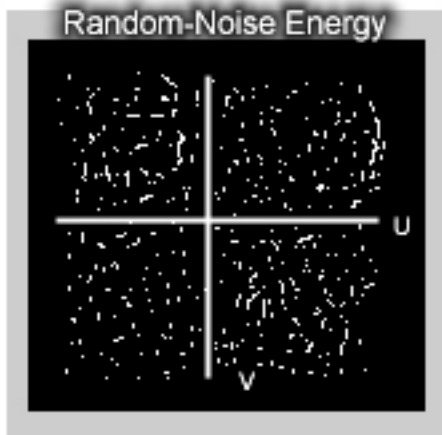
$$RMSE = \sqrt{MSE}$$

### Mathematical Model of Degradation

$$g(x,y) = f(x,y) * h(x,y) + \eta(x,y) \Rightarrow \mathfrak{F} \Rightarrow g(u,v) = f(u,v) * h(u,v) + \eta(u,v)$$

(With noise)

$$g(x,y) = f(x,y) * h(x,y) \Rightarrow \mathfrak{F} \Rightarrow g(u,v) = f(u,v) * h(u,v) \text{ (Without noise)}$$



## Inverse Filtering

$$F(u, v) = G(u, v) / H(u, v)$$

What happens with **inverse filtering** in the presence of noise?

## Wiener Filtering (Variation of Inverse Filtering)

$$\frac{|H(u, v)|^2}{|H(u, v)|^2 + \varphi \frac{|N(u, v)|^2}{|F(u, v)|^2}} \cdot \frac{1}{H(u, v)}$$

$$\frac{|N(u, v)|^2}{|F(u, v)|^2} \Rightarrow \text{is noise-to-signal ratio}$$

$\varphi$   $\Rightarrow$  Parameter for parametric Wiener Filter

### What if ...?

$N(u, v) \rightarrow 0 \rightarrow$  we have the inverse filter

$N(u, v) \rightarrow \infty \rightarrow$  filter tends to 0

Why is the inverse filter bad?

- (1) Noise- cannot handle noise.
- (2)  $H(u, v)$ -Can have 0's, which are not defined for 1.

### Types of noise

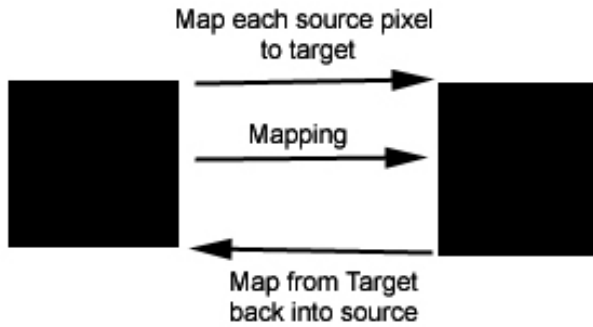
- (1) Impulse
- (2) Gaussian
- (3) Rayleigh
- (4) Gamma
- (5) Exponential
- (6) Uniform

### Noise Reduction

- (1) Image Averaging
  - a. Can reduce noise by  $\sqrt{n}$  if average N frames.
- (2) Neighborhood Operations
  - a. Mean
  - b. Median
- (3) Use band reject filter for points of a sine wave
  - a. Notch filters

### Geometric transforms

- Translation, rotation, and scaling: Affine transforms
- Geometric "warping"



- (1) Map all points in source image to corresponding points in target image
- (2) Interpolate between nearby pixels to determine output intensities.

**“Forward mapping”**

$$u = P_u(x, y) = ax + by + c$$

$$v = P_v(x, y) = dx + ey + f$$

**“Inverse Mapping”**

$$u = P_u(x, y) = a'x + b'y + c$$

$$v = P_v(x, y) = d'x + e'y + f$$

## November 12, 2003, Wednesday

Image Restoration:

- Geometric Model
- Inverse Filtering
- Wiener Filtering
- Noise Model
- Error measure
- Interactive restoration (BRFs for periodic interference)

### Intro to Geometric transformations (P. 271)



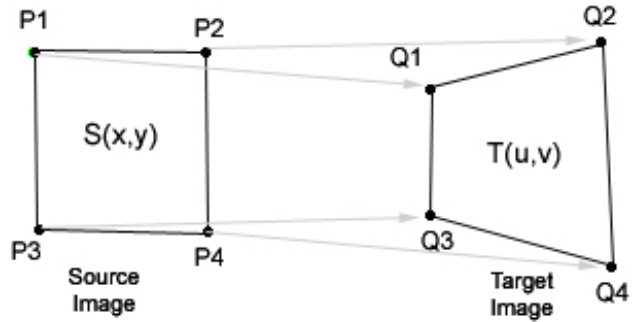
What are **affine transformations**?

- Restoration of geometric defects in image
- Translation, scaling, rotation (affine transforms)
- Warping transformations (rubber-sheet transformations)
- Morphing

### Process

- (1) Mapping from Source image to Target Image
  - a. Forward Mapping
  - b. Inverse Mapping.

Tie Points



How to use tie points to determine mapping

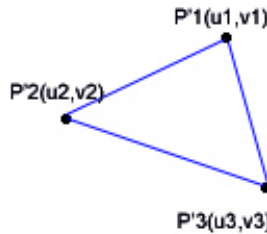
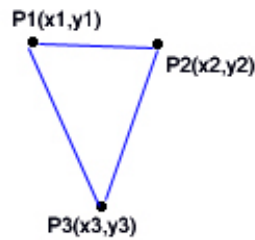
**G1 affine transforms**

$$u = ax + by + c$$

$$v = dx + ey + f$$

How do we solve for 3 equations and 3 unknowns?

**Gaussian Elimination**



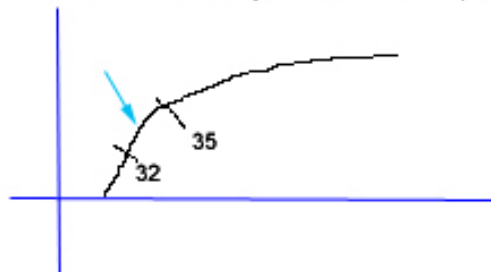
$$\begin{aligned} U1 &= ax1 + by1 + c \\ U2 &= ax2 + by2 + c \\ U3 &= ax3 + by3 + c \end{aligned}$$

$$\begin{aligned} V1 &= dx1 + ey1 + f \\ V2 &= dx2 + ey2 + f \\ V3 &= dx3 + ey3 + f \end{aligned}$$

- (2) Interpolation: to determine resulting intensity value.
- Nearest Neighbor: *Round off coordinates to nearest neighbor.*
  - Bilinear Interpolation: *take 4 nearest points or intensities.*

Linear transformation is demonstrated below

If we know the value of  $y$  at  $x=32$ , and we know  $x=35$ , what is the value of  $y$  at  $x=33$ ? Use interpolation!



Bilinear transformation is using 4 points.

- Bicubic interpolation: interpolate between 16 points

$$u = P_u(x, y) = ax + by + c$$

$$v = P_v(x, y) = dx + ey + f$$

**12:40 pm:** Did demons on cat face- scale, rotate, distortions

**12:45 pm:** "Sometimes using 4 ties points is not enough"

Morphing is the process of converting one image into another.

- display mapping as animated sequence.

How to determine mapping: Interpolate between source and target intensity  
Input Intensity--→ output intensity

**November 14, 2003, Friday**

## *References*

Pratt, William K., "*Digital Image Processing*", John Wiley and Sons, Inc, 2<sup>nd</sup> Edition, 1991

Gonzalez, Woods, "*Digital Image Processing*", 2<sup>nd</sup> Edition, Prentice Hall, 2002

<http://www-sop.inria.fr/epidaure/personnel/Celine.Fouard/ChamferMasks/discreteGeom.html>

Chamfer Masks and Discrete Geometry