

CS623: CAD FOR VLSI DESIGN
Lecture 35

Unate Function

A logic function is **monotone increasing** in a variable x_j if changing x_j from 0 to 1 causes **all** the outputs of f that change, to increase also from 0 to 1.

A logic function is **monotone decreasing** in a variable x_j if changing x_j from 0 to 1 causes **all** the outputs of f that change, to decrease also from 1 to 0.

A function that is either monotone increasing or monotone decreasing in x_j is said to be **monotone** or **unate** in x_j .

A function is **unate (monotone)** if it is unate in all its variables.

For eg. $f = x_1\overline{x_2} + \overline{x_2}x_3$ is increasing in x_1 and x_3 and decreasing in x_2 and hence it is unate.

Function and its Cover

Modelling a function as a cover, the cover is monotone increasing in a variable x_j if all the cubes in the cover has either 1 or 2 in a position j of the input part.

The cover is monotone decreasing in a variable x_j if all the cubes in the cover has either 0 or 2 in a position j of the input part.

A cover is said to beunate if it is monotone in all the input variables.

Proposition: If a cover $C(f)$ is unate in x_j , then f is unate in x_j .

Proof: If you check all cubes in $C(f)$ they do not have in their position j both 0 and 1. This implies that there does not exist two product terms such that one contains x_j and another $\overline{x_j}$. The $C(f)$ implies an SOP. Hence the proof.

By definition, if a function f is unate, it is not necessary that its cover $C(f)$ is unate.

$$\left| \begin{array}{cccc} 1 & 1 & 0 & 4 \\ 2 & 0 & 2 & 4 \end{array} \right|$$

The above cover is unate in x_1 and x_3 but not in x_2 . The function is $\overline{x_2} + x_1\overline{x_3}$, which is unate in x_2 also.

We prove that if a cover of a unate function consists only of prime cubes, then it must be a unate cover.

Proposition: A logic function f is monotone increasing (decreasing) in x_j if and only if no prime implicant of f has a 0 (1) in the j^{th} position.

Proof: Let f be a single-output logic function w.l.g. and we prove the increasing part.

If part: If no cube has a 0 in j position then, the cover is unate and hence the function is unate.

Only if part: If a prime cube c contains a 0 in the j position, consider the product term implied by c' got by replacing the 0 with a 1 in the j position. Since c is prime, c' is a product term of $\overline{f}$. For the input combination for c' in which $x_j = 1$ $f = 0$. Changing x_j from 0 to 1, causes f to change from 1 to 0. Hence f is not unate, a contradiction.

This implies

Proposition: A prime cover of a unate function is unate.

In the previous example, the following was not a prime cover

$$\left| \begin{array}{cccc} 1 & 1 & 0 & 4 \\ 2 & 0 & 2 & 4 \end{array} \right|$$

The prime cover is

$$\left| \begin{array}{cccc} 1 & 2 & 0 & 4 \\ 2 & 0 & 2 & 4 \end{array} \right|$$

Proposition: A unate cover is a tautology iff it contains a row of all 2's.

Else minterm $(0,0,\dots,0)$ will not be covered.

Proposition: Let C be a unate cover and P be any subset of C , and c be any cube of C . Then $c \subseteq P$ iff $c \subseteq s$ for some s in P .

Proof: If $c \subseteq s$ for some $s \in P$, then $c \subseteq P$. If $c \subseteq P$ then, let C be positive unate and hence every vertex will contain $(1,1,1,\dots,1)$. Hence $\forall s \in P, c \cap s \neq \phi$. Now $P_c \equiv 1$. By previous proposition there should exist a row of $(2,2,\dots,2)$ in P_c . This should have been generated by some $s \in P$. Now $c \subseteq s$.

Proposition: Every prime of a unate function is essential.

Proof: Let F be the set of primes

Suppose $p \in F$ is not essential. This implies $\exists P \subseteq F, p \notin P$, such that $p \subseteq P$. This implies $\exists s \in P$ such that $p \subseteq s$, contradicts the fact that p is prime.

Proposition: Let C be aunate cover, and P be the set of all primes of $f(C)$, the function specified by C , then, $P \subseteq C$.

Proof: Let p be a prime of $f(C)$. Then, $p \subseteq C$, hence $p \subseteq c$, $c \in C$. Since p is prime $p = c$.

Proposition: In addition to above if C is minimal with respect to single cube containment, then $C = P$, and C is the unique minimum cover.

Proof: Single cube containment is $\forall a, b \in C$ $a \not\subseteq b$. Here for any cube $c \in C$ is contained in some prime and all primes are in C . Since C is minimal, all elements of C are primes. $P = C$. Note that all primes are essential and this implies the unique minimum cover.

The Procedure to find minimum cardinality prime cover of a unate function

1. Expand each cube to a prime
2. Remove any cube contained by any other cube in the cover.

Exercises to the Student :))

Proposition: If a logic function f is monotone increasing in x_j , then the complement of f , $\overline{f}$, is monotone increasing in $\overline{x_j}$.

Proposition: The complement of a unate function is unate.

Proposition: The cofactors of a unate function f with respect to x_j and $\overline{x_j}$ are unate.

Choice of the Splitting variable:

If a function is unate then do not split - a known algorithm to get the minimum prime cover.

If a function is not unate, chose the **unate** variable such that the total number of cubes which are part of **both** the cofactors are as small as possible.

Procedure BINATE_SELECT(cover C);

// n be the number of variables

Begin

1. $\forall j, 1 \leq j \leq n$

$p_0(j) \leftarrow$ number of cubes with 0 in j^{th} position.

$p_1(j) \leftarrow$ number of cubes with 1 in j^{th} position.

2. **if** ($\max_j \min(p_0(j), p_1(j)) == 0$) **return**(C is unate).

else

Find all binate variables $J = \{j | \min(p_0(j), p_1(j)) > 0\}$. Now, $\forall j \in J$ find that j with maximum $(p_0(j) + p_1(j))$ and **return**(x_j).

End