

CS623: CAD FOR VLSI DESIGN

Lecture 32

Shannon Expansion

Any logical function **g** on variables $\{x_1, \dots, x_n\}$ can be expressed as

$x_i g_{x_i} + \overline{x_i} g_{\overline{x_i}}$, where g_a is the **cofactor** of g with respect to the variable/cube a .

The proof is straightforward.

Given a set of cubes $\mathcal{C} = \{c^1, \dots, c^t\}$ and a cube p , the cofactor of $\mathcal{C}$ with respect to p is denoted by $\mathcal{C}_p$ and is computed by evaluating the cofactor of each of the cubes in the cover $\mathcal{C}$ with respect to p .

Given p , the cofactor of c^i w.r.t p is another cube in which each of its elements are defined by $(c_i^p)_k =$

ϕ , if $c^j \cap p = \phi$

2, if $p_k = 0$ or 1

4, if $p_k = 3$

c_k^i , otherwise.

Given $\mathcal{C} =$

$$\begin{vmatrix} 1 & 1 & 0 & 2 & 4 & 4 \\ 0 & 1 & 2 & 0 & 4 & 4 \\ 1 & 1 & 1 & 1 & 4 & 3 \end{vmatrix}$$

This corresponds to the two output functions

$$g_1 = \overline{x_1}x_2\overline{x_4} + x_1x_2x_3x_4 + x_1x_2\overline{x_3}$$

$$g_2 = \overline{x_1}x_2\overline{x_4} + x_1x_2\overline{x_3}$$

The cube corresponding to x_4 is [222144] and the cofactor g_{x_4} of x_4 with respect to $\mathcal{C}$ is

$$\begin{vmatrix} 1 & 1 & 0 & 2 & 4 & 4 \\ 1 & 1 & 1 & 2 & 4 & 3 \end{vmatrix}$$

This corresponds to $x_1x_2\overline{x_3} + x_1x_2x_3$ of g_1 and

$x_1x_2\overline{x_3}$ of g_2

Similarly,

The cube corresponding to $\overline{x_4}$ is [222044] and the cofactor $g_{\overline{x_4}}$ of $\overline{x_4}$ with respect to $\mathcal{C}$ is

$$\begin{vmatrix} 1 & 1 & 0 & 2 & 4 & 4 \\ 0 & 1 & 2 & 2 & 4 & 4 \end{vmatrix}$$

This corresponds to $x_1x_2\overline{x_3} + \overline{x_1}x_2$ of g_1 and

$x_1x_2\overline{x_3} + \overline{x_1}x_2$ of g_2

Note that $g = x_4g_{x_4} + \overline{x_4}g_{\overline{x_4}}$

Some Propositions

$(fg)_{x_j} = f_{x_j}g_{x_j}$. This is proved by Shannon's expansion.

$(\overline{f})_{x_j} = \overline{(f_{x_j})}$. Prove by Shannon's expansion and the fact that f_{x_j} is independent of x_j .

Let $\mathcal{C} = \{c^i\}$ be a cover of cubes and let p be a cube. Then, $p \cap \mathcal{C} \equiv p\mathcal{C}$. $\checkmark$

Given a cube c , then the cofactor of c w.r.t c is a tautology.

A set of cubes $\mathcal{C}$ covers a cube p if and only if $\mathcal{C}_p$ is a tautology.

Proof: Clearly, $\mathcal{C}$ covers p iff $\mathcal{C} \cap p = p$. $(\mathcal{C}p)_p = \mathcal{C}_p p_p = p_p = 1$ implies $\mathcal{C}_p = 1$.

Let $f = x_j f_{x_j} + \overline{x_j} f_{\overline{x_j}}$ be the Shannon expansion of a completely specified logical function f . f is a tautology iff f_{x_j} and $f_{\overline{x_j}}$ are both tautologies.

Summary: Operations are Intersection, Complementation and Tautology.

$$\text{operate}(f,g) = \text{merge} (p \text{ operate}(f_p, g_p), \bar{p} \text{ operate}(f_{\bar{p}}, g_{\bar{p}}))$$

The algorithm is a recursive process that creates a binary tree and the results at each node will be a merging of the results obtained from the two subtrees below it.