

CS623: CAD FOR VLSI DESIGN
Lecture 29

Topics for the Next two classes

- Basic Definitions
- A Minimization Algorithm based on Decomposition and Unate Functions

Let $B = \{0, 1\}$ and $Y = \{0, 1, 2\}$, where 2 stands for don't cares.

A Logic function on n input variables $x_1, x_2, \dots, x_n$ and m output variables $y_1, y_2, \dots, y_m$ is

$$F : B^n \rightarrow Y^m$$

This is a multiple output function, we have m outputs and it is *incompletely specified*.

Restricting $Y = \{0, 1\}$, we get *Completely Specified* functions.

Example: $n = 3$ and $m = 2$, two outputs and both incompletely specified

x_1	x_2	x_3	y_1	y_2
0	0	0	1	1
0	0	1	1	0
0	1	0	0	1
0	1	1	0	1
1	0	0	1	0
1	0	1	1	2
1	1	0	1	1
1	1	1	2	1

For each output $i, 1 \leq i \leq m$,

- X_i^{ON} is the set of inputs for which y_i is 1
- X_i^{OFF} is the set of inputs for which y_i is 0
- X_i^{DC} is the set of inputs for which y_i is 2

$$X_1^{ON} = \{000, 001, 100, 101, 110\}$$

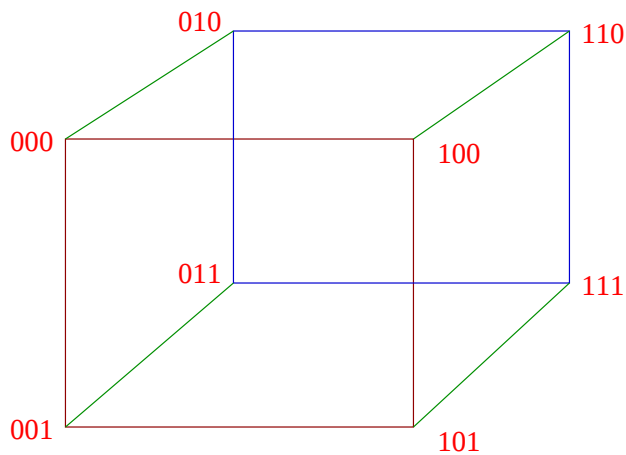
$$X_1^{OFF} = \{010, 011\}$$

$$X_1^{DC} = \{111\}$$

Boolean n -spaces.

An n input function can be represented as a n -dimensional cube.

A 3-input function is shown below.



Operations on *Completely Specified* Logic Functions

Complement - Swap the *ON* and *OFF* sets.

Intersection or *Product* of two functions f and g

- $h = f * g$: *ON*-set of h is the intersection of the *ON*-sets of f and g .

Difference of two functions f and g

- $h = f - g$: *ON*-set of h is the elements in the *ON*-set of f not belonging to the *ON*-set of g .

Union or *Sum* of two functions f and g

- $h = f + g$: *ON*-set of h is the union of the *ON*-sets of f and g .

Incompletely Specified Function F can be represented as *three* completely specified functions (f, d, r) .

- The ON set of f is the ON set of F and the OFF set of f is the union of the DC and OFF sets of F .

- The ON set of d is the DC set of F and the OFF set of d is the union of the ON and OFF sets of F .

- The ON set of r is the OFF set of F and the OFF set of r is the union of the ON and DC sets of F .

$f + d + r$ is a *tautology* - OFF set is $NULL$.

Consider a two-output and three-input Boolean function $F = \{f_1, f_2\}$.

Here $m = 2$ and $n = 3$.

$$f_1 = \overline{x_2} + x_1 \overline{x_3}$$

$$f_2 = x_2 + \overline{x_1} \overline{x_3}$$

Let p be a product term in F , say $\overline{x_2}$.

A *cube* p is specified by the row vector $c = [c_1, c_2, \dots, c_n, c_{n+1}, \dots, c_{n+m}]$

Where $c_i =$

- 0 if x_i appears complemented in p , $1 \leq i \leq n$
- 1 if x_i appears in p , $1 \leq i \leq n$
- 2 if x_i does not appear in p , $1 \leq i \leq n$

- 3 if p is not present in f_{i-n} , $n+1 \leq i \leq n+m$
- 4 if p is present in f_{i-n} , $n+1 \leq i \leq n+m$

For $p = \overline{x_2}$ the *cube* $c = [2, 0, 2, 4, 3]$.

Why call it a cube?

Given $c = [c_1, c_2, \dots, c_n, c_{n+1}, \dots, c_{n+m}]$

- the first n entries is called the *input part* or *input cube* of c denoted by $I(c)$.
- the last m entries is called the *output part* or *output cube* of c denoted by $O(c)$.
- A variable corresponding to a 2 in $I(c)$ is called *input don't care*.

Given a cube c of a product term p , $I(c)$ represents in compact form the vertices of the n -cube for which p is true.

$$p = \overline{x_2}, \quad I(c) = [2, 0, 2],$$

is the 2-cube (000, 001, 100, 101)

Cover of a n -input, m -output function $F = (f_1, f_2, \dots, f_m)$.

A set of cubes $\mathcal{C} = \{c^1, c^2, \dots, c^k\}$, is a cover of F if and only if $\forall j, 1 \leq j \leq m$,

The set of the input parts of the cubes in $\mathcal{C}$ that have a 4 in $(n + j)^{th}$ contain the ON-set of f_j and does not include any element from OFF-set of f_j .

A cover represents the union of the ON-set and some arbitrary portion of the don't-care set.

Consider a two-output and three-input Boolean function $F = \{f_1, f_2\}$.

Here $m = 2$ and $n = 3$.

$$f_1 = \overline{x_2} + x_1 \overline{x_3}$$

$$f_2 = x_2 + \overline{x_1} \overline{x_3}$$

The *Matrix Representation* for the function is the arrangement of the cube set $\mathcal{C} = \{c^1, c^2, c^3, c^4\}$ corresponding to the four product terms. Each cube form a row of the matrix.

Therefore $M(\mathcal{C})$ is

$$\begin{array}{ccccc} 2 & 0 & 2 & 4 & 3 \\ 1 & 2 & 0 & 4 & 3 \\ 2 & 1 & 2 & 3 & 4 \\ 0 & 2 & 0 & 3 & 4 \end{array}$$

Most of the minimization algorithms use the matrix $M(\mathcal{C})$. $I(M(\mathcal{C}))$ and $O(M(\mathcal{C}))$ are the input and output parts of $M(\mathcal{C})$.

Let $c = \{c_1, \dots, c_{n+m}\}$ and $d = \{d_1, \dots, d_{n+m}\}$ be two cubes. We say that c_j *contains* ($\supseteq$), *does not contain* ($\not\supseteq$) or *strictly contains* ($\supset$) d_j accorsing to the table below.

$$\forall j, 1 \leq j \leq n$$

		d_j		
		0	1	2
c_j	0	$\supseteq$	$\not\supseteq$	$\not\supseteq$
	1	$\not\supseteq$	$\supseteq$	$\not\supseteq$
	2	$\supset$	$\supset$	$\supseteq$

$$\forall j, n + 1 \leq j \leq n + m$$

		d_j	
		3	4
c_j	3	$\supseteq$	$\not\supseteq$
	4	$\supset$	$\supseteq$

$c \mathcal{R} d$, if $c_j \mathcal{R} d_j$, $\forall j, 1 \leq j \leq n + m$,

where $\mathcal{R} = \{\supset, \supseteq, \not\supseteq\}$.

A *minterm* e^i is a cube whose input parts does not contain any 2 and the output part contains $(m - 1)$ 3's and one 4 in position i .

A *minterm* does not contain any other cube.

Any cube c can be *decomposed* into a set of minterms.

$c = [2, 2, 1, 4, 4]$ can be *decomposed* to

$$m^1 = [0, 0, 1, 4, 3]$$

$$m^2 = [0, 0, 1, 3, 4]$$

$$m^3 = [1, 0, 1, 4, 3]$$

$$m^4 = [1, 0, 1, 3, 4]$$

$$m^5 = [0, 1, 1, 4, 3]$$

$$m^6 = [0, 1, 1, 3, 4]$$

$$m^7 = [1, 1, 1, 4, 3]$$

$$m^8 = [1, 1, 1, 3, 4]$$

A set of cubes $\mathcal{C} = \{c^1, \dots, c^k\}$ *covers* a cube c , ($c \supseteq \mathcal{C}$), if each of the minterms of c is covered by at least one cube of $\mathcal{C}$.

Intersection of two cubes is the intersection of each of its components.

$$\forall j, 1 \leq j \leq n$$

		d_j		
		0	1	2
	0	0	ϕ	0
c_j	1	ϕ	1	1
	2	0	1	2

$$\forall j, n + 1 \leq j \leq n + m$$

		d_j	
		3	4
	3	3	3
c_j			
	4	3	4

A cube is empty if the input part contains at least one ϕ or the output part has only 3's. Two cubes whose intersection is ϕ are said to be *orthogonal*.

Implicants, Prime Implicants and Essential Implicants

A cube p is said to be an implicant of $F = (f, d, r)$ if it has an empty intersection with the cubes of a representation of r .

An implicant is *prime* if any implicant containing it must be equal to it.

A *prime cover* is a cover whose cubes are all prime implicants.

An *essential prime* of F is one which contains a minterm of F not contained in any other prime.

Proposition: A prime p is not essential if and only if there exists a set of primes S distinct from p and covering p .

A cover $\mathcal{C}$ is *irredundant* or *minimal* if no proper subset of it is also a cover. In other words, no proper subset of cubes in $\mathcal{C}$ covers a cube in it.

A set of cubes $\mathcal{C}$ is *minimal with respect to single cube containment* if $a \not\supseteq b$, for any distinct cubes $a, b \in \mathcal{C}$.