

Financial Math

Simple interest:

$$I = Prt$$

$$A = P + I$$

I: Interest (\$)

P: Money invested (principal) (\$)

r: Annual interest rate (decimal)

t: time (years)

A: Amount (\$) (at the end of period)

Compound Interest:

$$A = P(1+i)^n$$

$$P = \frac{A}{(1+i)^n}$$

P: Money invested (principal) (\$)

A: Amount (at the end of compounding period) (\$)

i: interest rate for each compounding period (decimal)

n: number of compounding periods

Example:

Calculate the amount of \$2000 for 5 years at 8% compounded quarterly.

$$i = \frac{0.08}{4} = 0.02$$

$$n = 4 \times 5 = 20$$

$$A = P(1+i)^n = 2000 (1+0.02)^{20} = 2972$$

Annuities

$$A = \frac{R[(1+i)^n - 1]}{i}$$

$$P = \frac{R[1 - (1 + i)^{-n}]}{i}$$

$$R = \frac{Pi}{1 - (1 + i)^{-n}}$$

$$R = \frac{Ai}{(1 + i)^n - 1}$$

R: Regular deposit made at the end of each period for n periods (\$)

A: Amount of annuity at the end of nth period) (\$)

P: Present value of the annuity (\$)

Example:

Calculate the amount of annuity for \$200 deposited every 6 months by 6% for 3 years.

$$A = \frac{R[(1 + i)^n - 1]}{i} = \frac{200[(1 + 0.03)^6 - 1]}{0.03} = 1294$$

Mortgages

Mortgage interest rate is compounded semi-annually (6 months) by Canadian laws. But people want to make payment monthly.

Example:

Stephanie buys a house. She needs a mortgage of \$ 200, 000. Mortgage rate: 8%, mortgage period: 20 years. Find the monthly payment.

i: monthly rate (decimal)

Mortgage rate (decimal) = 0.08

Number of payments: n

$$\text{Semi-annual rate: } \frac{0.08}{2} = 0.04$$

$$(1+i)^6 = 1+0.04 = 1.04$$

$$i = 0.0065581969 \approx 0.0065582$$

$$n = 20 \times 12 = 240$$

$$P = \frac{R[1 - (1+i)^{-n}]}{i}$$

solve for R

$$R = \frac{Pi}{1 - (1+i)^{-n}} = \frac{(200000)(0.0065581969)}{1 - (1 + 0.0065581969)^{-240}} = 1656.72$$

The monthly payment is \$1656.72