A SIMPLE STATIC ANALYSIS OF MOVING ROAD VEHICLE UNDER CROSSWIND

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ABSTRACT

In the article a static model of vehicle for the determination of critical wind speed for overturn, sideslip and rotation is considered. The basic equations are setup on the assumptions of uniform straight line motion of the vehicle. Explicit formulas for critical wind speed for all three possible wind induced accidents are derived. Numerical examples compares present static model with simple dynamic model.

Keywords: road vehicle, crosswind

1 INTRODUCTION

According to Baker ([1]) crosswind accidents may be classified by three types (Figure 1):

- rollover accidents
- sideslip accidents
- rotating accidents

Figure 1: Vehicle accidents in crosswind according to Baker

In the first type of accident, a vehicle is blown over; in the second type a vehicle is blown a significance distance sideways; and in the third kind a vehicle is rotating through a significant
angle around its vertical axis. The proposed criteria for detecting the risk of possible type of accident when a vehicle enters a sudden crosswind are:

- the contact force falls to zero within 0.5 s
- the lateral displacement of the vehicle exceeds 0.5 m within 0.5 s
- the absolute value of yaw angle exceeds $11.5^0$ (0.2 rad) within 0.5 s

In this paper the Baker dynamic criteria for particular accident detection which are a bit artificial will be replaced with static criteria which are the following:

- the contact force falls to zero
- all the wheels reach the friction limit
- a vehicle wheel reach the friction limit

For determine the critical wind speed that may cause a particular type of crosswind accident a simplest possible vehicle model will be used; namely, a vehicle will be considered as a single rigid body with a given mass and dimensions executing straight line motion.

2 BASIC EQUATIONS

Equilibrium equations. Consider the vehicle subject to the uniform crosswind that executes a steady straight line motion. To remain on a straight path under the aerodynamic loads produced by wind, the friction forces should be generated in the contact of vehicle wheels and road (see Figure 2).

![Figure 2: Dimensions, reaction forces and wind induced resultant forces and moments on the vehicle. Note that the drag force, the rolling moment and the pitching moment are assumed to act in negative directions with respect to vehicle RHS vehicle coordinate system.](image)

If a vehicle is treated as a rigid body then the equilibrium conditions of forces and moment with respect to vehicle center of gravity yields six equilibrium equations. These are the equilibrium of forces

\[-F_D - F_{x1} - F_{x2} - F_{x3} - F_{x4} + i_1(T_1 + T_2) + i_2(T_3 + T_4) = 0 \]  
\[F_S - F_{y1} - F_{y2} - F_{y3} - F_{y4} = 0 \]
\[-mg + F_L + F_{z1} + F_{z2} + F_{z3} + F_{z4} = 0 \] (3)

and the equilibrium of moments

\[-M_R + \frac{c}{2} (F_{z2} - F_{z1} + F_{z4} - F_{z3}) - h (F_{y1} + F_{y2} + F_{y3} + F_{y4}) = 0 \] (4)

\[-M_p - a (F_{z1} + F_{z2}) + b (F_{z3} + F_{z4}) + h (F_{x1} + F_{x2} + F_{x3} + F_{x4}) = 0 \] (5)

\[M_y - a (F_{y1} + F_{y2}) + b (F_{y3} + F_{y4}) + \frac{c}{2} (F_{x2} - F_{x1} + F_{x4} - F_{x3}) \]

\[+i_1 \frac{c}{2} (T_1 - T_2) + i_2 \frac{c}{2} (T_3 - T_4) = 0 \] (6)

where \( F_D \) is drag force, \( F_S \) is side force, \( F_L \) is lift force, \( M_R \) is rolling moment, \( M_p \) is pitching moment, \( M_y \) is yawing moment, \( T_1, T_2, T_3, T_4 \) are wheels traction forces and \( i_1 \) and \( i_2 \) takes value 1 or 0, depends if axle is driven or not.

For given aerodynamics load the above equations constitute a set of six equations for sixteen unknowns: twelve reaction forces and four traction forces. The nine additional equations have to be supplied by constraint and constitutive assumptions.

**Constraint equation.** Let \((x_i, y_i, z_i)\) are coordinates of centre of \(i\)th wheels where \(z_i\) is displacement of wheel center from its equilibrium position in vertical direction. Since vehicle is rigid all the coordinates of wheels centre must all the time lay on the same plane which is given by say \(Ax + By + Cz + D = 0\). Substituting coordinates of wheels centre \(x_i = x_2 = a\), \(x_3 = x_4 = -b\), \(y_1 = y_3 = -c/2\) and \(y_2 = y_4 = c/2\) into the plane equation one obtain a homogeneous system of four linear equations for parameters of the equation. For nontrivial solution the determinant of the system must be equal to zero. This yield condition

\[z_1 - z_2 + z_4 - z_3 = 0 \] (7)

If we assume that each vertical force is proportional to the displacement and all the wheels suspension has same stiffness then constraint equation (7) leads to vertical forces constraint equation

\[F_{z1} - F_{z2} + F_{z4} - F_{z3} = 0 \] (8)

The unilateral contact between wheels and road demands that

\[F_{zj} \geq 0 \quad (j = 1, 2, 3, 4) \] (9)

If \(F_{zj} = 0\) then \(j\)th wheel loose contact.

For static consideration the reaction side force on a vehicle wheel is restricted by well known Coulomb friction law

\[\sqrt{(T_j - F_{zj})^2 + F_{zj}^2} \leq \mu F_{zj} \quad (j = 1, 2, 3, 4) \] (10)

where \(\mu\) is static friction coefficient. When inequality holds then a wheel is stick with the road, when equality holds then wheel wheels sliding just begins. Note also that unilateral contact between wheel and road implicate that if \(F_{zj} = 0\) then also \(F_{zj} = 0\).

**Constitutive equations.** The rolling resistances are given by (12)

\[F_{zj} = f_R F_{zj} \quad (j = 1, 2, 3, 4) \] (11)

where rolling resistance coefficient \(f_R\) is assumed to be a constant. Also it is assumed that the traction forces have a form purposed by Baker (11)
where \( q \) is an unknown traction parameter. The purposed form is a bit artificial but takes into account that if wheel lose contact then there is no traction force.

\[
T_j = q F_j \quad (j = 1, 2, 3, 4)
\]

\( \) Figure 3: Absolute (true) wind and relative (apparent) wind

**Aerodynamic forces and moments.** When a vehicle is moving with velocity \( v_0 \) and the true (ambient, absolute) wind is aligned to the direction of a vehicle moving at angle \( \beta_w \) then the apparent (relative) wind that acts on the vehicle has velocity \( v_a \) and is aligned to the direction of the vehicle moving by angle \( \psi_w \). The angles \( \beta_w \) and \( \psi_w \) are taken to be positive in the compass sense (Figure 3). From the figure one may see that vector equation \( v_0 + v_a = v_w \) holds. From this it follows that when true wind speed and angle are given then the apparent wind speed and its angle are

\[
v_a^2 = (v_0 + w \cos \beta_w)^2 + (w \sin \beta_w)^2
\]

\[
\psi_w = \arctan \frac{w \sin \beta_w}{v_0 + w \cos \beta_w}
\]

It is seen from the above equations that the apparent wind speed will be greatest when true wind direction is perpendicular to the direction in which the vehicle is traveling.

Now, the result of the interaction of a vehicle and air are normal pressure and shear stresses on the vehicle surface ([2]), and these produce the aerodynamic forces and moments (Figure 2). The total aerodynamic force and moment acting on a vehicle may be obtained by integrating the stresses over the vehicle surface. This leads to a complex air flow around the vehicle so theoretical formulas are replaced by semi-empirical formulas of the form

\[
F_D = C_D A \frac{\rho v_a^2}{2} \quad F_S = C_S A \frac{\rho v_a^2}{2} \quad F_L = C_L A \frac{\rho v_a^2}{2}
\]

\[
M_R = C_R A h \frac{\rho v_a^2}{2} \quad M_p = C_p A h \frac{\rho v_a^2}{2} \quad M_y = C_y A h \frac{\rho v_a^2}{2}
\]

where \( C_D, C_S, C_L, C_R, C_P \) and \( C_Y \) are aerodynamic load coefficients, \( A \) is the characteristic area of the vehicle, which is usually taken as the projection of front vehicle area and \( h \) is characteristic length which is usually taken as distance between road and vehicle center of mass.

3 **SOLUTION**

The fifteen equations, namely six equilibrium equations (1)-(6), constraint equation (8) and eight constitutive equations (11) and (12), include seventeen unknowns, namely twelve reaction
forces, four traction forces and traction parameter $q$. The system is clearly indeterminate and if Coulomb conditions (10) for each wheel are included then it become overdeterminate. Consequently from the system one cannot determine all the unknowns. However, by the inspection of the system, one may see that it is complete if only resultant side force for each vehicle axis is included as unknowns. In this case the solution of the system is the following expressions for vertical reaction forces

$$F_{z1} = \frac{1}{2} \frac{b(mg - F_L)}{a+b} - \frac{1}{2} \frac{hF_s + M_R}{c} - \frac{1}{2} \frac{hF_D + M_P}{a+b}$$  \hspace{1cm} (16)$$

$$F_{z2} = \frac{1}{2} \frac{b(mg - F_L)}{a+b} + \frac{1}{2} \frac{hF_s + M_R}{c} - \frac{1}{2} \frac{hF_D + M_P}{a+b}$$  \hspace{1cm} (17)$$

$$F_{z3} = \frac{1}{2} \frac{a(mg - F_L)}{a+b} - \frac{1}{2} \frac{hF_s + M_R}{c} + \frac{1}{2} \frac{hF_D + M_P}{a+b}$$  \hspace{1cm} (18)$$

$$F_{z4} = \frac{1}{2} \frac{a(mg - F_L)}{a+b} + \frac{1}{2} \frac{hF_s + M_R}{c} + \frac{1}{2} \frac{hF_D + M_P}{a+b}$$  \hspace{1cm} (19)$$

the following expressions for the resultant lateral force on each vehicle axis

$$F_{y1} + F_{y2} = \frac{bF_s + M_L}{a+b} - q \frac{hF_s + M_R}{a+b}$$  \hspace{1cm} (20)$$

$$F_{y3} + F_{y4} = \frac{aF_s - M_L}{a+b} + q \frac{hF_s + M_R}{a+b}$$  \hspace{1cm} (21)$$

where

$$q' = \frac{i_1 + i_2}{2} q - f_R$$  \hspace{1cm} (22)$$

and the traction parameter is

$$q = \frac{(a+b)[F_D + f_R (mg - F_L)]}{(ib + i_a)(mg - (i_2 - i_1)(hF_s + M_P))}$$  \hspace{1cm} (23)$$

By using expressions for aerodynamic forces and moments (15) the traction parameter may be also written as

$$q = \frac{(a+b)[f_R gm + (C_D - f_R C_L)\frac{\rho A v^2_s}{2}]}{(ib + i_a)mg - [(i_1 - i_2)h(C_P + C_D) + (ib + i_a)C_L]\frac{\rho A v^2_s}{2}}$$  \hspace{1cm} (24)$$

Note that vertical reactions are different from those given by Baker ([1]). Namely present static vertical reactions (16) contain side force $F_s$ while in dynamical case the wheels side forces are proportional to sideslip angle which is assumed to be zero at overturning condition. Observe also that

$$F_{z1} + F_{z4} = F_{z2} + F_{z3} = \frac{1}{2} (mg - F_L)$$  \hspace{1cm} (25)$$

This means that antisymetrical vehicles wheels support half of the vehicle weight reduced by lift force.
The conditions for overturning, rotation and sideslip will now be treated separately. It what follows we will assume that wind blows from vehicle right so windward wheels are 1 and 3, and leeward wheels are 2 and 4. This assumption implies that \(F_s, M_R \neq 0\).

**Overturning.** First indication of possible vehicle overturn is that one of its wheels loses contact with the ground. If all values of aerodynamics forces and moments are positive then, as it is seen from (16)-(19), the minimal vertical force is on wheel 1. If drag force and pitch moments turns its sign then the minimal vertical force is on wheel 3. By substituting expressions for aerodynamic loads (15) into (16) and (18) we find the critical apparent wind speed for one wheel to lose contact

\[
v_{a,\text{overturn}_1} = \frac{2mg}{\rho A} \left[ \frac{bc}{h(a+b)(C_s + C_R) + hc(C_D + C_P) + bcC_L} \right]
\]

(26)

\[
v_{a,\text{overturn}_3} = \frac{2mg}{\rho A} \left[ \frac{ac}{h(a+b)(C_s + C_R) + hc(C_D + C_P) + acC_L} \right]
\]

(27)

This formula become Baker condition for overturn if one set \(C_s = 0\).

The (26)-(27) give the limit apparent wind speed for one wheel lost contact. In literature ([3]) however the condition for overturn usually demands that resultant vertical force on wheels on windward lose contact with the road. From (16) and (18) this condition is

\[
F_{z1} + F_{z3} = \frac{1}{2} (mg - F_L) - \frac{hF_s + M_R}{c} = 0
\]

(28)

By substituting expressions for aerodynamic loads (15) into (28) yield the well known critical apparent wind speed for overturn

\[
v_{a,\text{overturn}} = \frac{2mg}{\rho A} \left[ \frac{c}{2h(C_s + C_R) + cC_L} \right]
\]

(29)

Observe that the limit speed for overturn (29) does not depend on drag force and pitch and yaw moment neither on dimension of vehicle wheelbase. Also in any case a wheel will lose contact before the vehicle reach overturn condition that is \(v_{a,\text{overturn}_1} < v_{a,\text{overturn}}\) as expected.

**Rotation.** Since wheels on each vehicle axis are assumed to be rigid connected the sliding of wheels on axis will be reached when both wheels satisfy equality (10). From (20)-(21) and (16)-(19) the condition for wheels on axle to slip are therefore

\[
bF_s + M_y - q'(hF_s + M_R) = \mu_1 \left[ b(mg - F_L) - (hF_D + M_P) \right]
\]

(30)

\[
aF_s - M_y + q'(hF_s + M_R) = \mu_2 \left[ a(mg - F_L) + (hF_D + M_P) \right]
\]

where

\[
\mu_1 = \sqrt{\mu^2 - (i_1q - f_R)^2} \quad \mu_2 = \sqrt{\mu^2 - (i_2q - f_R)^2}
\]

(31)

By substituting expressions for aerodynamic loads (15) yield critical apparent wind speed
Note that these equations are valid only if all the vertical reaction forces are positive and \(|q - f_R| \leq \mu\).

**Sideslip.** When all the vehicle’s wheels reaction side forces simultaneously reaches its maximal values permitted by friction then the vehicle is just to beginning to slide. In this case

\[ F_{y1} + F_{y2} + F_{y3} + F_{y4} = \mu_1 (F_{z1} + F_{z2}) + \mu_2 (F_{z3} + F_{z4}) \]  

(34)

and this by (15) leads to critical apparent wind speed

\[ v_{a, slip} = \sqrt{\frac{2mg}{\rho A} \left( \frac{\mu_1}{b} \right) \left[ bC_s + h\left( C_y - q(C_s + C_R) \right) + \mu_1 [bC_L + h(C_D + C_P)] \right]} \]  

\[ v_{a, slip} = \sqrt{\frac{2mg}{\rho A} \left( \frac{\mu_1}{a} \right) \left[ aC_s - h\left( C_y - q(C_s + C_R) \right) + \mu_2 [aC_L - h(C_D + C_P)] \right]} \]  

(33)

\[ v_{a, slip} = \sqrt{\frac{2mg}{\rho A} \left( \frac{\mu_1}{b} + \mu_2 \ln b \right)} \]  

(35)

Note than when \(q - f_R\) is small then \(\mu_1 \approx \mu_2 \approx \mu\) and the above equation reduce to

\[ v_{a, sideslip} \approx \sqrt{\frac{2mg}{\rho A} \left( \frac{\mu}{C_s + \mu C_L} \right)} \]  

(36)

It is seen from this expression that in simplified version only side force and lift force contribute to critical apparent wind speed.

**Discussion.** If we take most critical situation when wind blows perpendicular to vehicle path then by using (13) all the derived formulas for critical apparent wind speed have the form

\[ v^2_w + v^2_0 = \frac{2mg}{\rho A} f \left( C_D, C_s, C_L, C_R, C_P, C_y, a, b, c, h, \mu, f_R \right) \]  

(37)

When aerodynamic coefficients are known one may from the formula calculate the limit vehicle speed for given wind speed for particular type of accident. If for example one takes all the aerodynamic coefficients as constant then the safe driving is restricted to circle. This is however unrealistic since the formula restrict vehicle speed even in the case when there is no wind.

In general, the aerodynamic coefficients depend on \(\psi_w\) and this angle depends, as it seen from (14), on vehicle speed, true wind speed and true wind direction. For given true wind speed direction (37) represent a highly transcendent equation for calculation of critical vehicle speed. However, an relatively simple explicit formulas for critical wind speed for sideslip and overturn may be obtained if coefficient of lift force is taken to be constant and the side and roll coefficient are distributed sinusoidally

\[ C_s = C_{s0} \sin \psi_w \quad C_r = C_{r0} \sin \psi_w \quad C_L = C_{L0} \]  

(38)

Note that the condition for overturn and simplified sideslip has identical structure and may be written as

\[ v^* = \sqrt{\frac{2mg}{\rho A} \left( \frac{\mu}{C_{s0} + \mu C_{L0}} \right) \sin \psi_w} \]  

(39)

where
\[ \bar{\mu} = \begin{cases} \mu \text{ sideslip} \\ e/2h \text{ overturn} \end{cases} \]
\[ \bar{C}_{s0} = \begin{cases} C_{s0} \text{ sideslip} \\ C_{s0} + C_{r0} \text{ overturn} \end{cases} \]  

When vehicle is at rest then this formula give the ultimate wind speed at which will overturn or sideslip which is

\[ w \geq \frac{2mg}{\rho A} \frac{\bar{\mu}}{\bar{C}_{s0} + \bar{C}_{L0}} \]  

By using trigonometry identities \( \sin \psi_w = \frac{\tan \psi_w}{\sqrt{1 + \tan^2 \psi_w}} \) and expression (14) one may rewrite (39) to

\[ \bar{\mu}C_{L0}v^*_a + \bar{C}_{s0}w \sin \beta_w - \frac{2mg}{A\rho} \bar{\mu} = 0 \]  

This is quadratic equation for unknown critical apparent wind speed. Now two possibilities have to be considered. If the lift force is zero then the critical apparent wind speed is

\[ v^*_a = \frac{2mg}{A\rho} \frac{\bar{\mu}}{\bar{C}_{s0}w \sin \beta_w} \]  

and when it is not zero then

\[ v^*_a = \frac{2\bar{\mu}C_{L0}}{\sqrt{\left(\bar{C}_{s0}w \sin \beta_w - \sqrt{\left(\bar{C}_{s0}w \sin \beta_w\right)^2 + 8\bar{\mu}^2C_{L0} \frac{mg}{\rho A}}\right)^2 - w^2}} \]

The above expressions may be solved for critical vehicle speed, however the result are relatively complex expressions. These are simplified in special most critical case when wind blows perpendicular to vehicle traveling path. In this case the critical vehicle speed when there is no lift force is

\[ v^*_0 = \sqrt{\frac{2mg}{A\rho} \frac{\bar{\mu}}{\bar{C}_{s0}w} - w^2} \]  

and when one has also lift force the critical vehicle speed is

\[ v^*_0 = \frac{1}{\sqrt{2}\bar{\mu}C_{L0}} \sqrt{\left(\bar{C}_{s0}^2 - 2\bar{\mu}^2C_{L0}^2\right)w^2 - \bar{C}_{s0}w \sqrt{\left(\bar{C}_{s0}w\right)^2 + 8\bar{\mu}^2C_{L0} \frac{mg}{\rho A}} + 4\bar{\mu}^2C_{L0} \frac{mg}{\rho A}} \]  

4 EXAMPLE

As the numerical example we will calculate dependence of critical vehicle speed on true wind speed for two trucks. The data for them are given by Baker ([1],[4]) and are shown in Table 1. Note that the coefficient of friction is calculated from the Baker’s coefficient \( m = 2.5 \) by assuming that sideslip saturation angle is \( 10^0 \). The results of calculations are present as graphs on Figure 4.
Table 1: Vehicle data

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<th>Parameter</th>
<th>Unit</th>
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<th>Case 2</th>
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<td>m</td>
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<td>m</td>
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<td>μ</td>
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</tr>
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<td>0.5(1+sin₃ψ)</td>
</tr>
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</tr>
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<td>2.2sin²ψ</td>
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</table>

Figure 4: Dependence of vehicle speed on wind speed. Baker’s overturn and sideslip correspond dynamical model

5 CONCLUSION

The present static model clearly simplifies real situation of driving in a strong crosswind since many influence factors are ignored. However, as was pointed by Lemay ([3]), luck of relevant data needed for more sophisticated modelling make the static analysis a useful start point for predication of critical vehicle speed in strong crosswind.
REFERENCES