Course Outline

1. Linear Functions (Chapter 1)
2. Linear Algebra (Chapter 2)
    - Solving a system of linear equations.
    - Matrices and their sum, difference, product, along with inverse matrices.
    - Determinants.
3. Linear Programming (Chapters 3 and 4)
4. Mathematics of Finance (Chapter 5)
5. Logic (Chapter 6)
6. Set, Probability Theory (Chapter 7)
7. Combinatorics (the Art of Counting) (Chapter 8)
8. Introduction to Statistics (Chapter 9)

This is a very basic tool used everywhere!

Invented during the World War II, kept secret until 1947. Heavily used nowadays in business management, economics, and many other fields.

A very practical subject

This chapter stands on its own.

These three chapters lay a good foundation for later studies in statistics.

Chapter 1: Linear Functions

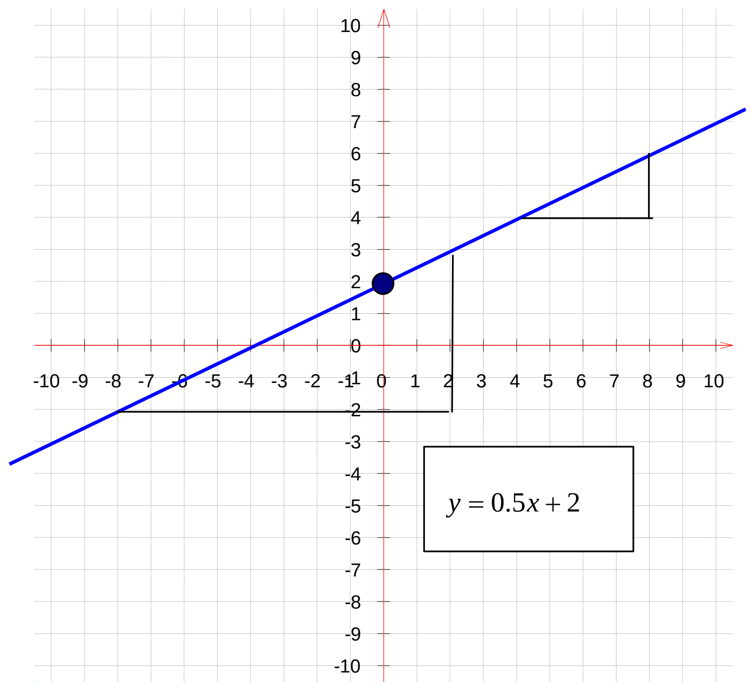
Two quantities are related to each other. The simplest case: linear relation.

Slope of a Line: Pick two points on the line, say (x_1, y_1) and $(x_2\ \ y_2)$, the slope is

$$m = \frac{change\ in\ y}{change\ in\ x} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Slope-intercept Form: Every non-vertical line can be described by a linear equation in the slope-intercept form

$$y = mx + b$$



<u>Ex</u>: Find the equation in slope-intercept form of the line passing through (0, 2) with a slope of 0.5.

Sol:

$$y = 0.5x + 2$$

<u>Ex</u>: Write the linear equation $x - 2y = -4$ in slope-intercept form.

Sol: Simply "solve for y in terms of x"!!

$$-2y = -x - 4$$

$$\frac{-2y}{-2} = \frac{-x}{-2} - \frac{4}{-2}$$

$$y = 0.5x + 2$$

<u>Ex</u>: Find the equation in slope-intercept form of the horizontal line passing through $(0, 5)$.

Sol:

$y = 0x + 5$, **i.e.** $y = 5$

Point-slope Form: The line of slope m that passes through the point (x_1, y_1) can be described by the equation

$$y - y_1 = m(x - x_1)$$

Ex: Find a linear equation for the line passing through the point $(2, 3)$ with a slope of $1/2$.

Sol:

$$y - 3 = \frac{1}{2} \cdot (x - 2)$$

This can be simplified to

$$2(y-3) = x - 2$$
$$2y - 6 = x - 2$$
$$-x + 2y = 6 - 2$$
$$-x + 2y = 4$$

Note that if we bring this to the slope-intercept form by "solving for y in terms of x, we would get

$$2y = x + 4$$

$y = \frac{1}{2}x + 2$, i.e. $y = 0.5x + 2$. It is the same line graphed in the opening example.

Ex: Find a linear equation for the line passing through the point $(-8, -2)$ and the point $(10, 7)$.

Sol: Find the slope first.

$$m = \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - (-2)}{10 - (-8)}$$
$$= \frac{9}{18} = \frac{1}{2}$$

Thus the line has a slope of ½ and it passes through, say, $(-8, -2)$. Therefore it can be described by an equation in point-slope form as

$$y - (-2) = \frac{1}{2} \cdot (x - (-8))$$
$$y + 2 = \frac{1}{2} \cdot (x + 8)$$
$$y = \frac{1}{2} \cdot (x + 8) - 2$$

$y = \frac{1}{2}x + 4 - 2$, i.e. $y = \frac{1}{2}x + 2$ It is still the same line we graphed!!

Parallel: Two lines are parallel precisely when they share the same slope.

<u>Ex</u>: Find an equation describing the line passing through the point $(-2, 1)$ parallel to the line $x - 2y = 8$

Sol: First find the slope of the line $x - 2y = 8$ by solving for y in terms of x to bring it into the slope-intercept form.

$$-2y = -x + 8$$

$$y = \frac{1}{2}x - 4 \qquad \text{(divide by } -2\text{)}$$

So the slope of $x - 2y = 8$ is ½. Since the line being sought is parallel to this line, it should also enjoy a slope of ½. Thus the slope-intercept form of it should be

$$y = \frac{1}{2}x + b$$

Determine b by plugging in $(-2, 1)$:

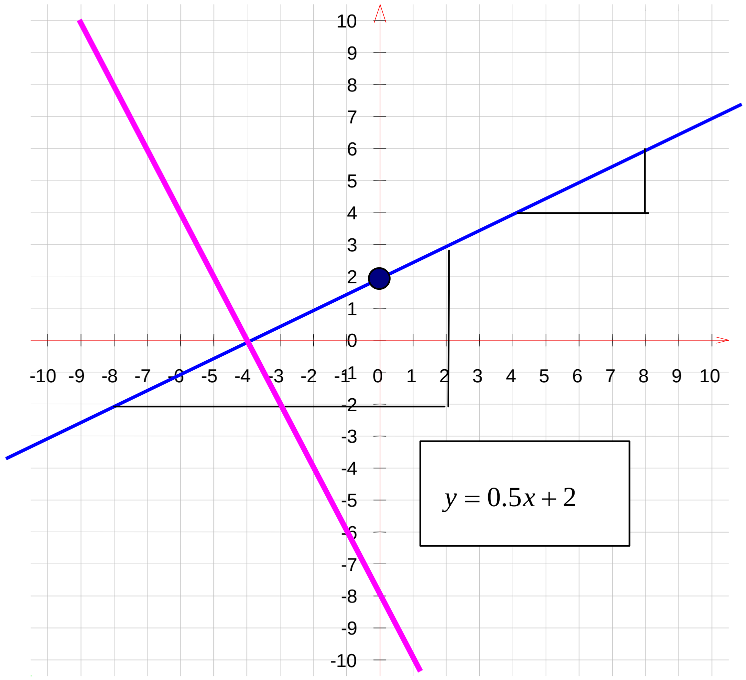
$1 = \frac{1}{2}(-2) + b$, i.e. $1 = -1 + b$, so solve for b to get $b = 2$.

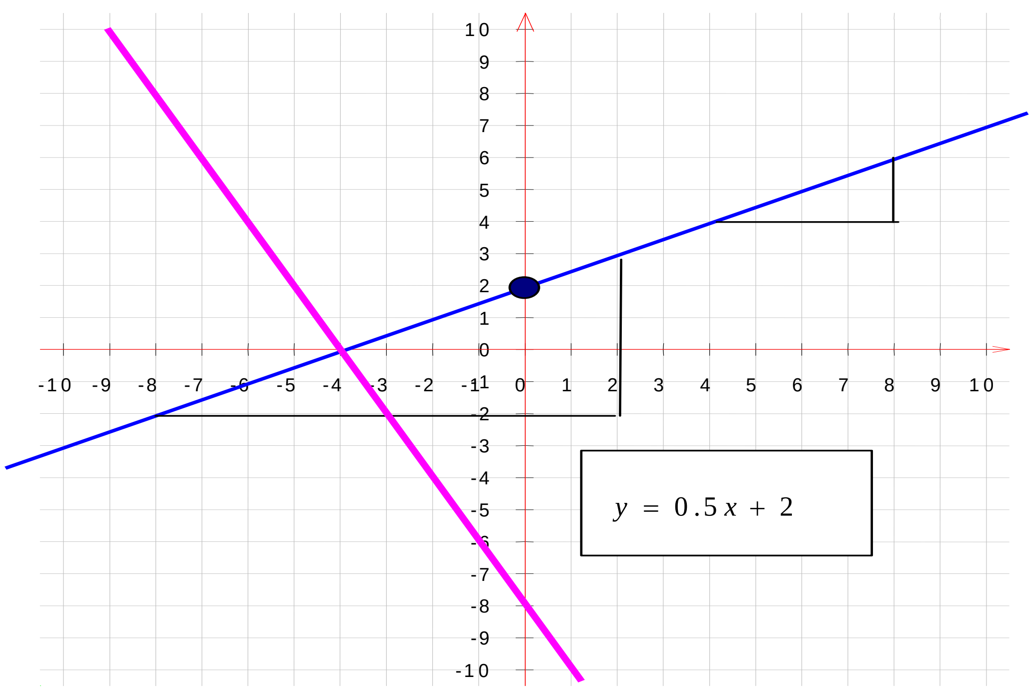
Therefore the line being sought is

$y = \frac{1}{2}x + 2$, i.e. $y = 0.5x + 2$

Perpendicular: Two lines with slope m_1 and m_2 respectively are perpendicular precisely if m_1 and m_2 satisfies $m_1 m_2 = -1$.

Warning: This statement is true only if the x-axis and y-axis use the same scale.





Some Application Problems from the Book

<u>Ex</u>: (1.1, Prob. 69) [Cohabitation] The number of unmarried couples in the U.S who are living together has been rising at a roughly linear rate in recent years. The number of cohabiting adults was 1.1 million in 1977 and 5.5 million in 2000.

(a) Write and equation expressing the number of cohabiting adults (in millions) y, in terms of the number of years after 1977, x. ($y = 0.19x + 1.1$)

(b) Use your result in part (a) to predict the number of cohabiting adults in the year 2010. (About 7.4 million)

<u>Ex</u>: (1.1, Prob. 71) [Consumer Price Index] The CPI is a measure of the change in the cost of goods over time. If 1977 is used as the base year of comparison (CPI = 100 in 1977), then the CPI of 252.3 in 2000 would indicate that an item that cost $1.00 in 1977 would cost $2.52 in 2000. The CPI has been increasing at an approximately linear rate for the past 30 years.

(a) Use this information to determine a linear function for this data, letting x be the years since 1977. ($y = 6.62x + 100$)

(b) Based on your function, what was the CPI in 1995? Compare this estimate with the actual CPI of 224.7. (219.16, which is less than the actual CPI.)

(c) How is the annual CPI changing? (It increases by 6.62 per year)

<u>Ex</u>: (1.1, Prob. 65) [Ponies Trotting] A 1991 study found that the peak vertical force on a trotting Shetland pony increased linearly with the pony's speed, and that when the force reached a critical level, the pony switched from a trot to a gallop. For on pony, the critical force was 1.16 times its body weights. It experienced a force of 0.75 times its body weight at a speed of 2 meters per second, and a force of 0.93 times its body weight at 3 meters per second. At what speed did the pony switch from a trot to a gallop? (Approximately 4.3 m/sec)

Supply and Demand

Example: Vinyl siding

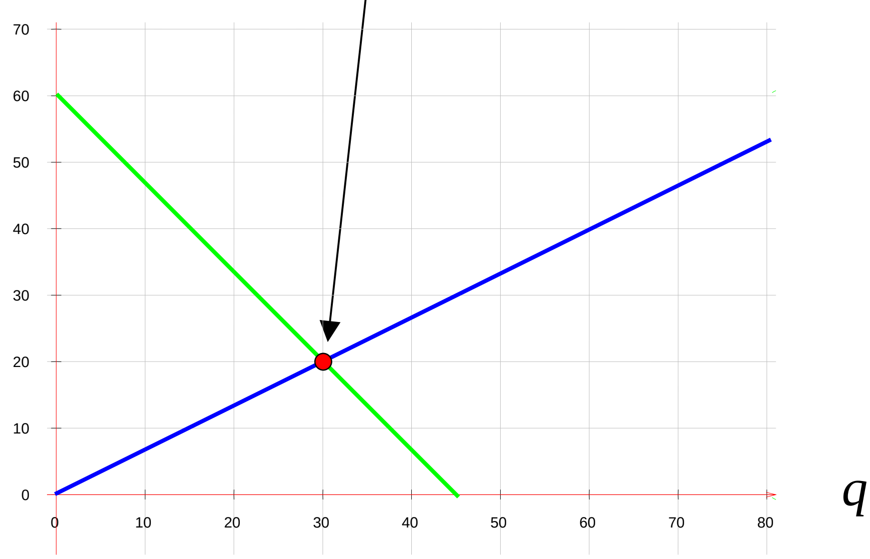
(Unite) price p in dollars per square yard,
quantity q in thousands of square yards.

Demand Curve: $$p = D(q) = 60 - \frac{4}{3}q$$

Supply Curve: $$p = S(q) = \frac{2}{3}q$$

Economists tend to think of the price p as the independent variable.
But they tend to use the VERTICL scale to represent the price p.

Here we honor the tradition of using the vertical scale for p.
But we write the relation between q and p in a way that makes p more like the dependent variable, rather than the independent variable.



Well, it doesn't matter. All we want to achieve is to find (q, p) such that the two equations are satisfied:

$$\begin{cases} p = 60 - \frac{4}{3}q \\ p = \frac{2}{3}q \end{cases}$$

If follows that $60 - \frac{4}{3}q = \frac{2}{3}q$, i.e. $60 = 2q$, therefore it must be that $q = 30$. But then, since $p = \frac{2}{3}q$, it must be that $p = \frac{2}{3}(30) = 20$. This means:

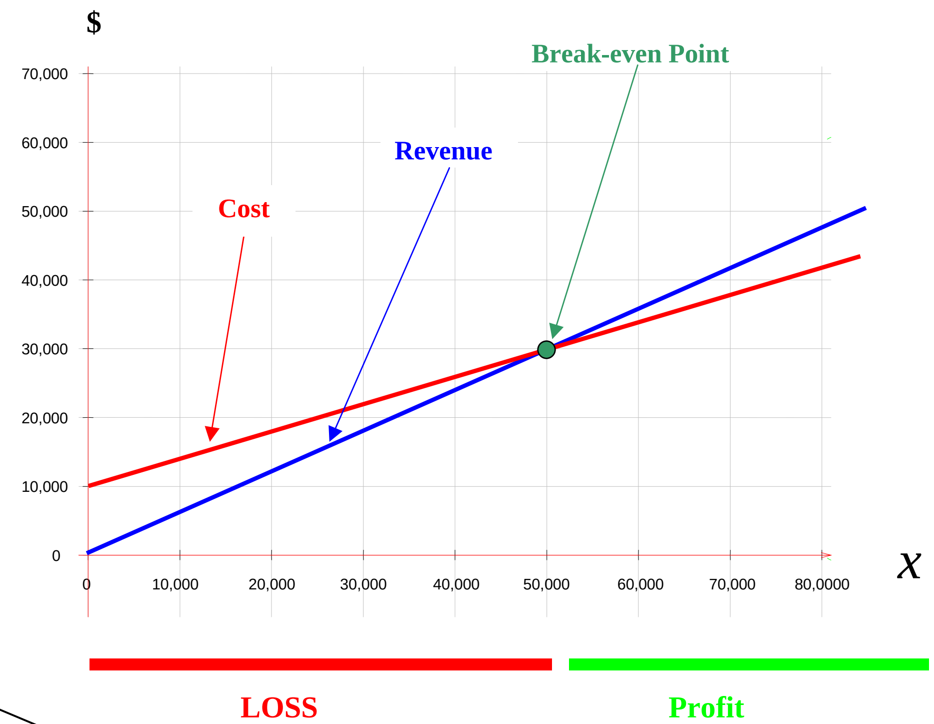
Equilibrium price is $20 per square yards,
Equilibrium quantity is 30,000 square yards.

Cost and Revenue, Break-Even Analysis

A linear model for the cost:
$C(x) = mx + b$

m (the 0.4 in our case) is called the **Marginal Cost** (cost for each additional unit produced)

b (the 10,000 here) is called the **Fixed Cost**.



x : the number of burritos produced.

Cost: $C(x) = 0.4x + 10{,}000$

Revenue: $R(x) = 0.6x$

Profit: $P(x) = R(x) - C(x)$

(A) Find the profit if 60,000 burritos are sold.

$$P(60{,}000) = R(60{,}000) - C(60{,}000) = [0.6(60{,}000)] - [0.4(60{,}000) + 10{,}000]$$

$$= 36{,}000 - 34{,}000 = 2{,}000$$

Answer: $2,000

(B) What about when 30,000 burritos are sold?

$$P(30{,}000) = R(30{,}000) - C(30{,}000) = [0.6(30{,}000)] - [0.4(30{,}000) + 10{,}000]$$

$$= 18{,}000 - 22{,}000 = -4{,}000$$

Answer: There would be a ***loss*** of $4,000.

(C) ***Break-event:*** When the cost matches the revenue, i.e. when profit is 0.

$$10{,}000 + 0.4x = 0.6x$$

i.e. $10{,}000 = 0.2x$

i.e. $x = 50{,}000$

Conclusion: 50,000 burritos have to be sold in order to “break even”.

Application Problems on Functions

Ex: (1.2, Prob. 9) Write a linear cost function. Identify all variables used:
"A trailer-hauling service charges $45 plus $2 per mile.

Sol: $C(x) = 2x + 45$

x is the distance traveled in miles.

$C(x)$ means the cost for hauling the trailer for x miles.

Ex: (1.2, essentially Prob. 25) In deciding whether to set up a new manufacturing plant, company analysts have decided that a linear function is a reasonable estimation for the total cost $C(x)$ in dollars to produce x items. They estimate the cost to produce 10,000 items is $547,500, and the cost to produce 50,000 items is $737,500. Find a formula for $C(x)$, and identify the fixed cost and the marginal cost. Determine the cost to produce 100,000 items. What does the marginal cost mean to the manager?

($C(x) = 500{,}000 + 4.75x$. Fixed cost: $500,000. Marginal cost: $4.75. The cost for producing 100,000 items is $975,000. The marginal cost tells us that each additional item cost $4.75 to produce.)