Math 170 (Spring 2008) **Solutions for Assignment #11**

<u>Section 5.3</u>: 9, 11, 13, 14, 15, 17, 19, 21, 23, 24, 27, 29, 31, 37(skip Part d), 39, 45

Prob. 9: $P = R\left[\frac{1-(1+i)^{-n}}{i}\right] = (\$1400)\left[\frac{1-(1+0.08)^{-8}}{0.08}\right] = (\$1400)\left[\frac{1-0.5402688845}{0.08}\right] = \$8045.29$

Prob. 11:

$$P = R\left[\frac{1-(1+i)^{-n}}{i}\right] = (\$50{,}000)\left[\frac{1-(1+0.08/4)^{-10\times 4}}{0.08/4}\right] = (\$50{,}000)\left[\frac{1-0.4528904152}{0.02}\right] = \$1{,}367{,}773.96$$

Prob. 13:

$$P = R\left[\frac{1-(1+i)^{-n}}{i}\right] = (\$18{,}579)\left[\frac{1-(1+0.094/2)^{-8\times 2}}{0.094/2}\right] = (\$18{,}579)\left[\frac{1-0.4795711988}{0.047}\right] = \$205{,}724.40$$

Prob. 14:

$$P = R\left[\frac{1-(1+i)^{-n}}{i}\right] = (\$10{,}000)\left[\frac{1-(1+0.04)^{-15}}{0.04}\right] = (\$10{,}000)\left[\frac{1-0.5552645027}{0.04}\right] = \$111{,}183.87$$

Prob. 15:

$$P = R\left[\frac{1-(1+i)^{-n}}{i}\right] = (\$10{,}000)\left[\frac{1-(1+0.06)^{-15}}{0.06}\right] = (\$10{,}000)\left[\frac{1-0.4172650607}{0.06}\right] = \$97{,}122.49$$

Prob. 17: $R = \frac{P}{\left[\frac{1-(1+i)^{-n}}{i}\right]} = \frac{\$2500}{\left[\frac{1-(1+0.08/4)^{-6}}{0.08/4}\right]} = \frac{\$2500}{\left[\frac{1-0.8879713822}{0.02}\right]} = \$446.31$

Prob. 19: $R = \frac{P}{\left[\frac{1-(1+i)^{-n}}{i}\right]} = \frac{\$90{,}000}{\left[\frac{1-(1+0.08)^{-12}}{0.08}\right]} = \frac{\$90{,}000}{\left[\frac{1-0.3971137586}{0.08}\right]} = \$11{,}942.55$

Prob. 21: $R = \frac{P}{\left[\frac{1-(1+i)^{-n}}{i}\right]} = \frac{\$7400}{\left[\frac{1-(1+0.082/2)^{-18}}{0.082/2}\right]} = \frac{\$7400}{\left[\frac{1-0.4851621018}{0.041}\right]} = \$589.31$

Prob. 23: \$7.61

Prob. 24: \$87.10

Prob. 27: If \$1000 is deposited at the end of each year for 8 years at 6% interest compounded annually, then at the end of 8 years there will be a total of $S = R\left[\frac{(1+i)^n-1}{i}\right] = \$1000\left[\frac{(1+0.06)^8-1}{0.06}\right] = \$1000\left[\frac{1.5938480745-1}{0.06}\right] = \$9897.47$. If you deposit a sum of P dollars today at 5% interest rate compounded annually for 8 years, you will have $A = P(1+i)^n = P(1+0.05)^8 = P(1.477455444)$. In order for this A to match \$9897.47, we have to have $P = \frac{\$9897.47}{1.477455555} = \$6699.00$

Prob. 29: $R = \frac{P}{\left[\frac{1-(1+i)^{-n}}{i}\right]} = \frac{\$149{,}560}{\left[\frac{1-(1+0.0775/12)^{-25\times 12}}{0.0775/12}\right]} = \frac{\$149{,}560}{\left[\frac{1-0.1449639341}{0.0064583333}\right]} = \$1{,}129.67$

Prob. 31: $R = \frac{P}{\left[\frac{1-(1+i)^{-n}}{i}\right]} = \frac{\$153{,}762}{\left[\frac{1-(1+0.0845/12)^{-30\times 12}}{0.0845/12}\right]} = \frac{\$153{,}762}{\left[\frac{1-0.0799689882}{0.0070416667}\right]} = \$1{,}176.85$

Prob. 37:

(a) $\$13{,}500/60 = \$225$

(b) $$R=\frac{P}{\left[\frac{1-(1+i)^{-n}}{i}\right]}=\frac{\$10{,}000}{\left[\frac{1-(1+0.059/12)^{-60}}{0.059/12}\right]}=\frac{\$10{,}000}{\left[\frac{1-0.7450699592}{0.0049166667}\right]}=\$192.86$$

(c) In both (a) and (b), you will make 60 payments. Obviously (b) is a better deal, for each payment is only \$192.86.

(d) (This part is not required for the assignment.) From the table below, we see that somewhere between 12% and 13% rate, the resulting monthly payment for (b) will exceed \$225 required. Using a graphing calculator or software (such as GraphCalc), we get a more precise value of 12.5%. Thus as soon as the interest rate is more than 12.5%, the deal in (b) becomes worse than the deal in (a)

i	$\frac{10{,}000}{\left[\frac{1-(1+i)^{-60}}{i}\right]}$
0.12	222.4445
0.13	227.5307

Prob. 39:

(a) The winner receives \$1 million divided by 20 each year, i.e. \$50,000 a year for 20 years. Think of this as an ordinary annuity, we can calculate its present value as follows, assuming 5% interest rate:

$$P=R\left[\frac{1-(1+i)^{-n}}{i}\right]=\$50{,}000\left[\frac{1-(1+0.05)^{-20}}{0.05}\right]=\$50{,}000\left[\frac{1-0.3768894829}{0.05}\right]=\$623{,}110.52$$

(b) Repeat the computation in (a) with 5% replaced by 9%, we get:

$$P=R\left[\frac{1-(1+i)^{-n}}{i}\right]=\$50{,}000\left[\frac{1-(1+0.09)^{-20}}{0.09}\right]=\$50{,}000\left[\frac{1-0.1784308898}{0.09}\right]=\$456{,}427.28$$

(c) Repeat (a) with 20 years replaced by 25 years. Each annual payment is \$1 million divided by 25, i.e. \$40,000

$$P=R\left[\frac{1-(1+i)^{-n}}{i}\right]=\$40{,}000\left[\frac{1-(1+0.05)^{-25}}{0.05}\right]=\$40{,}000\left[\frac{1-0.2953027717}{0.05}\right]=\$563{,}757.78$$

(d) With 9% interest rate and 25 years, each annual payment is \$1 million divided by 25, i.e. \$40,000, and

$$P=R\left[\frac{1-(1+i)^{-n}}{i}\right]=\$40{,}000\left[\frac{1-(1+0.09)^{-25}}{0.09}\right]=\$40{,}000\left[\frac{1-0.1159678356}{0.09}\right]=\$392{,}903.18$$