Math 170 (Spring 2008) **Solutions for Assignment #10**

<u>Section 5.1</u>:

Prob. 7: $I = Prt = (\$3850)(0.09)(8/12) = \$231.00$

Prob. 8: $I = Prt = (\$1974)(0.063)(7/12) = \$72.54$

Prob. 11: $I = Prt = (\$2930.42)(0.119)(123/360) = \$119.15$ (assuming a 360-day year)

Prob. 15: $A = P(1+i)^n = (\$1000)[1+0.07/1]^{10\times1} = (\$1000)(1.967151357) = \$1967.15$

Prob. 17:
$A = P(1+i)^n = (\$15{,}000)[1+0.06/2]^{11\times2} = (\$15{,}000)(1.916103409) = \$28{,}741.55$

Prob. 18: $A = P(1+i)^n = (\$6500)(1+0.12/4)^{6\times4} = (\$6500)(2.032794106) = \$13{,}213.16$

Prob. 21: $P = \dfrac{A}{(1+i)^n} = \dfrac{\$27{,}159.68}{(1+0.123/1)^{11\times1}} = \dfrac{\$27{,}159.68}{3.582426748} = \$7{,}581.36$

Prob. 23: $P = \dfrac{A}{(1+i)^n} = \dfrac{\$2000}{(1+0.11/2)^{8\times2}} = \dfrac{\$2000}{20355262699} = \$849.16$

Prob. 24: $P = \dfrac{A}{(1+i)^n} = \dfrac{\$8800}{(1+0.10/4)^{5\times4}} = \dfrac{\$8800}{1.63861644} = \$5{,}370.38$

Prob. 29: $r_e = \left(1+\dfrac{0.08}{4}\right)^4 - 1 = 0.08243216 \approx 0.08243$, i.e. 8.243%

Prob. 31: $r_e = \left(1+\dfrac{0.1008}{2}\right)^2 - 1 = 0.10334016 \approx 0.10334$, i.e. 10.334%

Prob. 33:
$A = P(1+rt) = (\$725{,}896.15)[1+(0.127)(34/365)] = (\$725{,}896.15)(1.011830137) = \$734{,}483.60$

Prob. 35: $A = 7\times\$5104 = \$35{,}728$, so

$$P = \frac{A}{1+rt} = \frac{\$35{,}728}{1+(0.0642)(7/12)} = \frac{\$35{,}728}{1.03745} = \$34{,}438.29$$

Prob. 37: $A = P(1+rt)$, so $1+rt = \dfrac{A}{P} = \dfrac{\$10{,}000}{\$5{,}988.02} = 1.670001102$. Thus

$rt = 0.670001102$. But $t = 10$, so $r = \dfrac{0.670001102}{10} \approx 0.067$, i.e. 6.7%.

Prob. 41: $P = \dfrac{A}{(1+i)^n} = \dfrac{\$2{,}900{,}000}{(1+0.08/12)^{5\times12}} = \dfrac{\$2{,}900{,}000}{1.489845708} = \$1{,}946{,}510.29$

Prob. 43: $A = P(1+i)^n = (\$10{,}000)(1+0.06/2)^{3\times2} = \$11{,}940.52$

Prob. 45: From $r_e = \left(1+\frac{r}{m}\right)^m - 1$, one gets $1+r_e = \left(1+\frac{r}{m}\right)^m$. Taking m th on both sides, one gets $\sqrt[m]{1+r_e} = 1+\frac{r}{m}$. Subtract 1 from both sides, $\sqrt[m]{1+r_e} - 1 = \frac{r}{m}$. Then multiply both sides by m to get $m\left(\sqrt[m]{1+r_e}-1\right) = r$. Thus we see that $r = m\left(\sqrt[m]{1+r_e}-1\right)$. Apply this to each of the 5 deals (all with quarterly compounding, thus $m = 4$):

- 6 mo term with 1.30% APY:
  $r = m\left(\sqrt[m]{1+r_e}-1\right) = 4\left(\sqrt[4]{1+0.013}-1\right) = 4(1.003234275-1) \approx 0.0129$, i.e. 1.29%.
- 1 yr term with 2.17% APY:
  $r = m\left(\sqrt[m]{1+r_e}-1\right) = 4\left(\sqrt[4]{1+0.0217}-1\right) \approx 0.0215$, i.e. 2.15%.
- 18 mo term with 2.27% APY:
  $r = m\left(\sqrt[m]{1+r_e}-1\right) = 4\left(\sqrt[4]{1+0.0227}-1\right) \approx 0.0225$, i.e. 2.25%.

- 2 yr term with 2.55% APY
  $r = 4\left(\sqrt[4]{1+0.0255}-1\right) \approx 0.0253$, i.e. 2.53%
- 3yr term with 3.00% APY
  $r = 4\left(\sqrt[m]{1+0.03}-1\right) \approx 0.0297$, i.e. 2.97%

Prob. 51: From the table below shows that it takes about 12 years. You can also use the Rule of 72 to come up with $72/6=12$

n	1.06^n
1	1.0600
2	1.1236
3	1.1910
4	1.2625
5	1.3382
6	1.4185
7	1.5036
8	1.5938
9	1.6895
10	1.7908
11	1.8983
12	2.0122

Prob. 52: The table below shows that it takes about 35 years. You can also use the Rule of 72 to come up with an estimation of $72/2=36$.

n	1.02^n
30	1.8114
31	1.8476
32	1.8845
33	1.9222
34	1.9607
35	1.9999
36	2.0399

<u>Section 5.2</u>:

Prob. 1: $a\,r^{n-1} = 3\times 2^{5-1} = 48$

Prob. 3: $a\,r^{n-1} = (-8)\times 3^{5-1} = -648$

Prob. 5: $a\,r^{n-1} = 1\times(-3)^{5-1} = 81$

Prob. 7: $a\,r^{n-1} = 1024\times(\frac{1}{2})^{5-1} = 64$

Prob. 9: $S_n = \dfrac{a(r^n-1)}{r-1}$, so $S_4 = \dfrac{(1)(2^4-1)}{2-1} = 15$

Prob. 11: $S_n = \dfrac{a(r^n-1)}{r-1}$, so $S_4 = \dfrac{(5)(\left(\frac{1}{5}\right)^4-1)}{\frac{1}{5}-1} = \dfrac{(5)(\frac{1}{625}-1)}{-\frac{4}{5}} = \dfrac{(5)(-\frac{624}{625})}{-\frac{4}{5}} = \frac{156}{25}$

Prob. 13: $S_n = \dfrac{a(r^n-1)}{r-1}$, so $S_4 = \dfrac{(128)(\left(-\frac{3}{2}\right)^4-1)}{-\frac{3}{2}-1} = \dfrac{(128)(\frac{81}{16}-1)}{-\frac{5}{2}} = \dfrac{(128)(\frac{65}{16})}{-\frac{5}{2}} = -208$

Prob. 15: $\dfrac{(1+0.05)^{12}-1}{0.05} = 15.91712652 \approx 15.91713$

Prob. 17: $\dfrac{(1+0.043)^{16}-1}{0.043} = 22.35632926 \approx 22.35633$

Prob. 21: $S = R\left[\frac{(1+i)^n - 1}{i}\right] = 100\left[\frac{(1+0.06)^4 - 1}{0.06}\right] = 100\left[\frac{1.26247696 - 1}{0.06}\right] = 437.46$

Prob. 23:

$$S = R\left[\frac{(1+i)^n - 1}{i}\right] = 46{,}000\left[\frac{(1+0.063)^{32} - 1}{0.063}\right] = 46{,}000\left[\frac{7.064224047 - 1}{0.063}\right] = 46{,}000[96.25752455] = 4{,}427{,}846.13$$

Prob. 25:

$$S = R\left[\frac{(1+i)^n - 1}{i}\right] = 9200\left[\frac{(1+0.10/2)^{7\times 2} - 1}{0.10/2}\right] = 9200\left[\frac{1.979931599 - 1}{0.05}\right] = 9200[19.59863199] = 180{,}307.41$$

Prob. 27:

$$S = R\left[\frac{(1+i)^n - 1}{i}\right] = 800\left[\frac{(1+0.0651/2)^{12\times 2} - 1}{0.0651/2}\right] = 800\left[\frac{2.157079665 - 1}{0.03255}\right] = 800[35.54776236] = 28{,}438.21$$

Prob. 29:

$$S = R\left[\frac{(1+i)^n - 1}{i}\right] = 15{,}000\left[\frac{(1+0.121/4)^{6\times 4} - 1}{0.121/4}\right] = 15{,}000\left[\frac{2.044668737 - 1}{0.03025}\right] = 15{,}000[34.53450371] = 518{,}017.56$$

Prob. 31: **(The four problems starting this one has to do with annuity due, <u>not</u> ordinary annuity!)**

$$S = R\left[\frac{(1+i)^{n+1} - 1}{i}\right] - R = 600\left[\frac{(1+0.06)^{8+1} - 1}{0.06}\right] - 600 = (600)(11.49131598) - 600 = 6{,}294.79$$

Prob. 33: (annuity due)

$$S = R\left[\frac{(1+i)^{n+1} - 1}{i}\right] - R = 20{,}000\left[\frac{(1+0.08)^{6+1} - 1}{0.08}\right] - 20{,}000 = (20{,}000)(8.92280336) - 20{,}000 = 158{,}456.07$$

Prob. 35: (annuity due)

$$S = R\left[\frac{(1+i)^{n+1} - 1}{i}\right] - R = 1000\left[\frac{(1+\frac{0.0815}{2})^{18+1} - 1}{\frac{0.0815}{2}}\right] - 1000 = (1000)(27.87497406) - 1000 = 26{,}874.97$$

Prob. 37: (annuity due)

$$S = R\left[\frac{(1+i)^{n+1} - 1}{i}\right] - R = 100\left[\frac{(1+\frac{0.124}{4})^{36+1} - 1}{\frac{0.124}{4}}\right] - 100 = (100)(67.55994876) - 100 = 6{,}655.99$$

Prob. 39: $S = R\left[\frac{(1+i)^n - 1}{i}\right]$, so $R = \frac{S}{\left[\frac{(1+i)^n - 1}{i}\right]} = \frac{\$10{,}000}{\left[\frac{(1+0.05)^{12} - 1}{0.05}\right]} = \frac{\$10{,}000}{15.91712652} = \$628.25$

Prob. 40: $S = R\left[\frac{(1+i)^n - 1}{i}\right]$, so $R = \frac{S}{\left[\frac{(1+i)^n - 1}{i}\right]} = \frac{\$100{,}000}{\left[\frac{(1+0.08/2)^{9\times 2} - 1}{0.08/2}\right]} = \frac{\$100{,}000}{25.64541288} = \$3{,}899.33$

Prob. 43: $R = \frac{S}{\left[\frac{(1+i)^n - 1}{i}\right]} = \frac{2000}{\left[\frac{(1+0.06)^5 - 1}{0.06}\right]} = \frac{2000}{5.63709296} = 354.79$

Prob. 45: $R = \frac{S}{\left[\frac{(1+i)^n - 1}{i}\right]} = \frac{25{,}000}{\left[\frac{(1+\frac{0.057}{4})^{14} - 1}{\frac{0.057}{4}}\right]} = \frac{25{,}000}{15.37364563} = 1{,}626.16$

Prob. 55: The first 8 years we have an annuity due, with

$$S = R\left[\frac{(1+i)^{n+1}-1}{i}\right] - R = 2435\left[\frac{(1+\frac{0.06}{2})^{16+1}-1}{\frac{0.06}{2}}\right] - 2435 = (2435)(21.76158774) - 2435 = 50{,}554.46615$$

This amount serves as the principal that earn interest in the next 5 years. At the end the total amount will be

$$A = P(1+i)^n = (\$50{,}554.46615)\left[1+\frac{0.06}{2}\right]^{5\times 2} = (\$50{,}554.46615)(1.343916379) = \$67{,}940.98$$

Prob. 59:

$$R = \frac{S}{\left[\frac{(1+i)^n-1}{i}\right]} = \frac{18000}{\left[\frac{(1+\frac{0.05}{4})^{24}-1}{\frac{0.05}{4}}\right]} = \frac{18000}{27.78808403} = 647.769$$