

Math 170 (Spring 2008) **Solutions for Assignment #7**

Section 3.2: 2, 4, 6, 7, 8, 9, 10, 12

Section 3.3: 9, 11, 13

Prob. 2: $z = 6x - y$

(1, 2): $z = 4$ (1, 5): $z = 1$ (6, 8): $z = 28$ (9, 1): $z = 53$

The maximum value of 53 occurs at (9, 1). The minimum value of 1 occurs at (1, 5)

Prob. 4: $z = 0.35x + 1.25y$

(0, 0): $z = 0$, (0, 15) $z = 18.75$,

(6, 18) $z = 24.60$, (10, 9) $z = 14.75$, (12, 0) $z = 4.20$

The maximum of 24.60 occurs at (6, 18). The minimum of 0 occurs at (0, 0).

Prob. 6:

(a)

	(0, 10)	(2, 4)	(5, 2)	(15, 0)
$z = 4x + y$	10	12	22	60

The maximum value doesn't exit!!! (Because we could move as far to the northeast as possible to raise z as much as we desired!!)

The minimum value of 10 occurs at (0, 10)

(b)

	(0, 10)	(2, 4)	(5, 2)	(15, 0)
$z = 5x + 6y$	60	34	37	75

The maximum value doesn't exit!!! (Because we could move as far to the northeast as possible to raise z as much as we desired!!)

The minimum value of 34 occurs at (2, 4)

(c) c

	(0, 10)	(2, 4)	(5, 2)	(15, 0)
$z = x + 2y$	20	10	9	15

The maximum value doesn't exit!!! (Because we could move as far to the northeast as possible to raise z as much as we desired!!)

The minimum value of 9 occurs at (5, 2)

(d)

	(0, 10)	(2, 4)	(5, 2)	(15, 0)
$z = x + 6y$	60	26	17	15

The maximum value doesn't exit!!! (Because we could move as far to the northeast as possible to raise z as much as we desired!!)

The minimum value of 15 occurs at (15, 0)

Prob. 7: Minimize $z = 5x + 2y$

Subject to:

$$\begin{cases} 2x + 3y \geq 6 \\ 4x + y \geq 6 \\ x \geq 0 \\ y \geq 0 \end{cases}$$

Note that the maximum of $z = 5x + 2y$ doesn't exit.

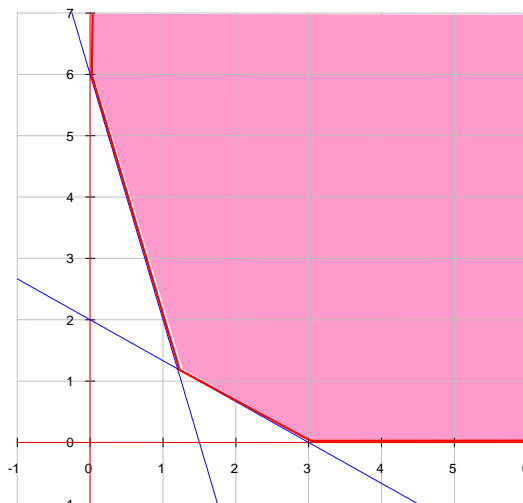
But the minimum does. We locate the corners as

(0, 6) with $z = 12$

(1.2, 1.2) with $z = 8.4$

(3, 0) with $z = 15$.

So the minimum is 8.4, which occurs at (1.2, 1.2).



Prob. 8:Minimize $z = x + 3y$

Subject to:

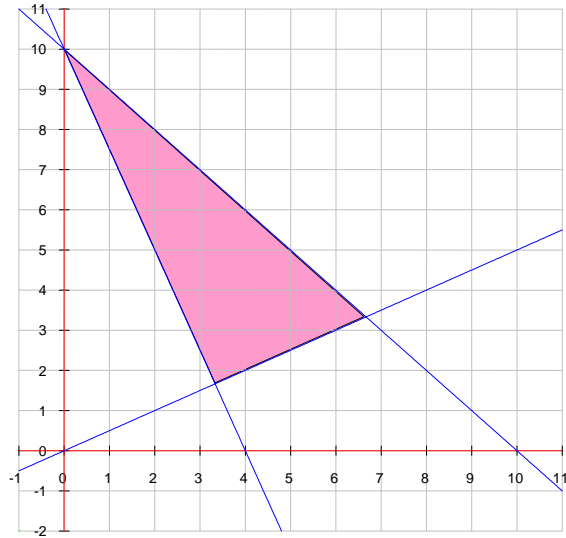
$$\begin{cases} x + y \leq 10 \\ 5x + 2y \geq 20 \\ -x + 2y \geq 0 \\ x \geq 0 \\ y \geq 0 \end{cases}$$

Clearly $(0, 10)$ is a corner, with $z = 30$ Another corner $(20/3, 10/3)$, with $z = 50/3 = 16\frac{2}{3}$ results from solving

$$\begin{cases} x + y = 10 \\ -x + 2y = 0 \end{cases}$$

The remaining corner $(10/3, 5/3)$, with $z = 25/3 = 8\frac{1}{3}$ results from solving

$$\begin{cases} 5x + 2y = 20 \\ -x + 2y = 0 \end{cases}$$

So the minimum of z is $25/3 = 8\frac{1}{3}$, which occurs at $(10/3, 5/3)$ **Prob. 9:**Maximize $z = 2x + 2y$

Subject to:

$$\begin{cases} 3x - y \geq 12 \\ x + y \leq 15 \\ x \geq 2 \\ y \geq 5 \end{cases}$$

Corner $(27/4, 33/4)$ with $z = 30$ results from solving

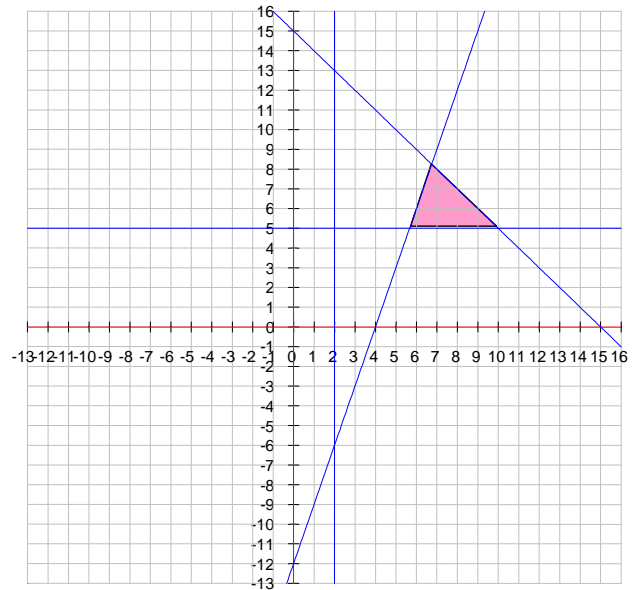
$$\begin{cases} 3x - y = 12 \\ x + y = 15 \end{cases}$$

Corner $(17/3, 5)$ with $z = 64/3 = 21\frac{1}{3}$ results from solving

$$\begin{cases} 3x - y = 12 \\ y = 5 \end{cases}$$

Corner $(10, 5)$ with $z = 30$ results from solving

$$\begin{cases} x + y = 15 \\ y = 5 \end{cases}$$

Thus the maximum value of 30 occurs along the line segment that joins $(27/4, 33/4)$ with $(10, 5)$.

Prob. 10:Maximize $z = 10x + 8y$

Subject to:

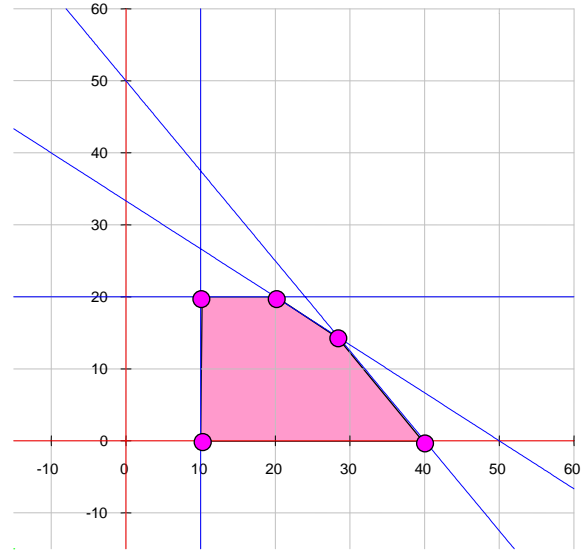
$$\begin{cases} 2x + 3y \leq 100 \\ 5x + 4y \leq 200 \\ x \geq 10 \\ 0 \leq y \leq 20 \end{cases}$$

There are five corners:

 $(10, 0)$, $(10, 20)$, $(40, 0)$, $(20, 20)$, $(200/7, 100/7)$, where the last one results from solving

$$\begin{cases} 2x + 3y \leq 100 \\ 5x + 4y \leq 200 \end{cases}$$

We have



	$(10, 0)$	$(10, 20)$	$(40, 0)$	$(20, 20)$	$(200/7, 100/7)$
$z = 10x + 8y$	100	260	400	360	400

So the maximum value is 400 and it occurs along the line segment joining the corner $(40, 0)$ with $(200/7, 100/7)$.

Prob. 12:Maximize $z = 4x + 5y$

Subject to:

$$\begin{cases} 10x - 5y \leq 100 \\ 20x + 10y \geq 150 \\ x + y \geq 12 \\ x \geq 0 \\ y \geq 0 \end{cases}$$

 $(0, 15)$ is clearly a corner point.

Solve

$$\begin{cases} 20x + 10y = 150 \\ x + y = 12 \end{cases}$$

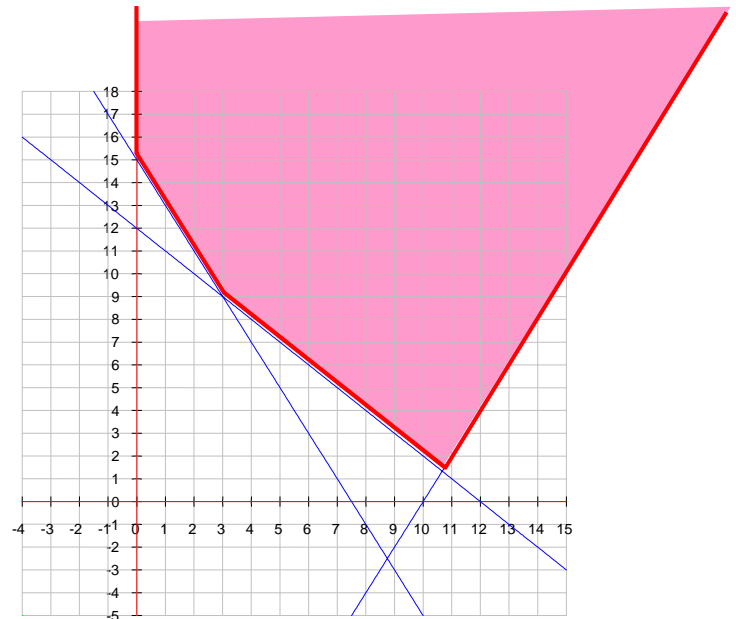
to get the corner $(3, 9)$.

Solve

$$\begin{cases} 10x - 5y = 100 \\ x + y = 12 \end{cases}$$

to get the corner $(32/3, 4/3)$

But the objective function $z = 4x + 5y$ can go up without bound as we move in the north-north-east direction. So the maximum doesn't exist.



Section 3.3:

Prob. 9: x units from Policy A, y units from Policy B.

(a) Minimize $50x + 40y$

Subject to:

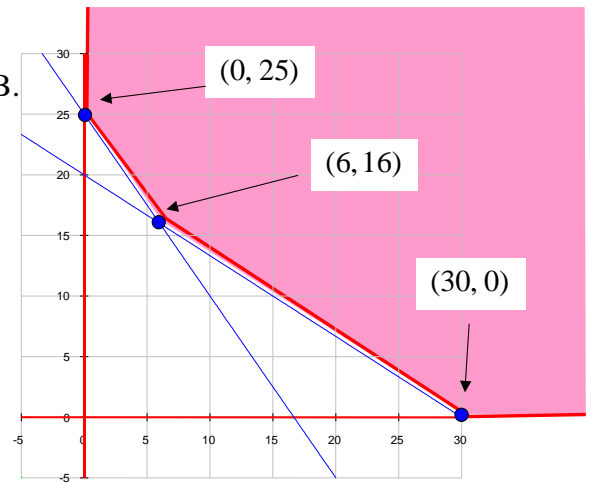
$$\begin{cases} 10,000x + 15,000y \geq 300,000 \\ 180,000x + 120,000y \geq 3,000,000 \end{cases}$$

with $x \geq 0, y \geq 0$.

For convenience, we divide the first inequality by 5,000, and the second by 60,000 to get

$$\begin{cases} 2x + 3y \geq 60 \\ 3x + 2y \geq 50 \end{cases}$$

with $x \geq 0, y \geq 0$



	(0, 25)	(6, 16)	(30, 0)
$50x + 40y$	1000	940	1500

So the minimum of 940 occurs at $x = 6, y = 16$.

Answer: 6 units from Policy A, 16 units from Policy B, with a premium of \$940.

(b) The feasible region remains the same. Only the objective function has been changed to $25x + 40y$. We have

	(0, 25)	(6, 16)	(30, 0)
$25x + 40y$	1000	790	750

So the minimum of 750 occurs at $x = 30, y = 0$.

Answer: 30 units from Policy A, 0 units from Policy B, with a premium of \$750.

Prob. 11: x type-1 bolts, y type-2 bolts.

Want to maximize $0.10x + 0.12y$,

Subject to

$$\begin{cases} 0.1x + 0.1y \leq 240 \\ 0.1x + 0.4y \leq 720 \\ 0.1x + 0.02y \leq 160 \end{cases}$$

with $x \geq 0, y \geq 0$.

One corner point is the solution to

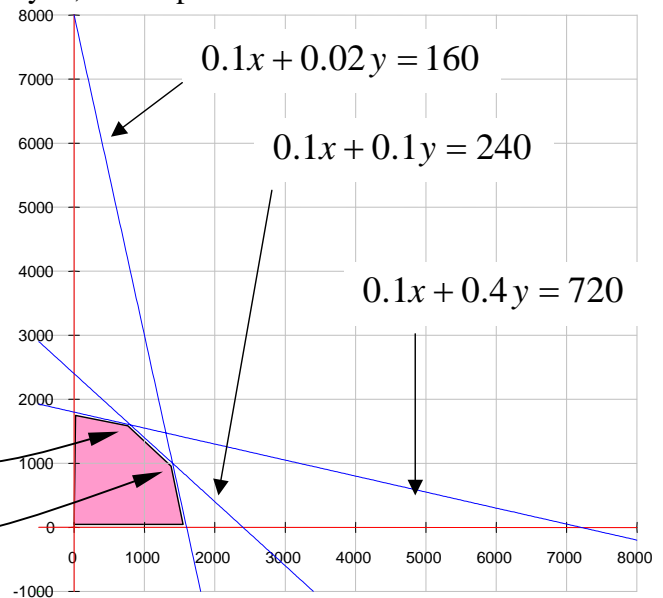
$$\begin{cases} 0.1x + 0.1y = 240 \\ 0.1x + 0.4y = 720 \end{cases}, \text{ i.e. } (800, 1600)$$

Another corner is the solution to

$$\begin{cases} 0.1x + 0.1y = 240 \\ 0.1x + 0.02y = 160 \end{cases}, \text{ i.e. } (1400, 1000)$$

The corner on the positive y -axis is the y -intercept of $0.1x + 0.4y = 720$, i.e. $(0, 1800)$.

The corner on the positive x -axis is the x -intercept of $0.1x + 0.02y = 160$, i.e. $(1600, 0)$



	(0, 0)	(0, 1800)	(800, 1600)	(1400, 1000)	(1600, 0)
$0.10x + 0.12y$	0	216	272	260	160

So the maximum of 272 occurs at $(800, 1600)$.

Thus

(a) Manufacture 800 type-1 bolts and 1600 type-2 bolts.

(b) \$272

(c) As we raise the selling price of type-2 bolts, the level sets of the object function become less and less steep. When they become parallel to the line segment joining $(0, 1800)$ with $(800, 1600)$, the optimal solution begins to change—it will lead to $(0, 1800)$ as the optimal solution. If the price of a type-2 bolt is p dollars, then the objective function will be $z = 0.10x + py$. Solve for y to get $y = -\frac{0.10}{p}x + \frac{z}{p}$.

We see the slope is $-\frac{0.10}{p}$. The slope of the line segment joining $(0, 1800)$ with

$(800, 1600)$ is on the line $0.1x + 0.4y = 720$ and therefore has a slope of -0.25 .

Thus we want $-\frac{0.10}{p} = -0.25$, i.e. $p = 0.40$.

Answer: When the price of type-2 bolts exceeds 40 cents, then it is more profitable to produce 1800 type-2 bolts and no type-1 bolts.