Math 170 (Spring 2008)

Solutions for Homework #4

Section 2.2: 48, 52, 53, 54, 65

Prob. 48 (Transportation)

(a) x trucks, y vans, z station wagons

$$\begin{cases} 2x + 3y + 3z = 25 \\ 2x + 4y + 5z = 33 \\ 3x + 2y + z = 22 \end{cases}$$

$$\left[\begin{array}{ccc|c} 2 & 3 & 3 & 25 \\ 2 & 4 & 5 & 33 \\ 3 & 2 & 1 & 22 \end{array}\right] \rightarrow \begin{array}{c} \\ R_2 - R_1 \\ -2R_3 + 3R_1 \end{array} \left[\begin{array}{ccc|c} 2 & 3 & 3 & 25 \\ 0 & 1 & 2 & 8 \\ 0 & 5 & 7 & 31 \end{array}\right] \rightarrow \begin{array}{c} R_1 - 3R_2 \\ \\ R_3 - 5R_2 \end{array} \left[\begin{array}{ccc|c} 2 & 0 & -3 & 1 \\ 0 & 1 & 2 & 8 \\ 0 & 0 & -3 & -9 \end{array}\right]$$

$$\rightarrow \begin{array}{c} \\ \\ -R_3/3 \end{array} \left[\begin{array}{ccc|c} 2 & 0 & -3 & 1 \\ 0 & 1 & 2 & 8 \\ 0 & 0 & 1 & 3 \end{array}\right] \rightarrow \begin{array}{c} R_1 + 3R_3 \\ R_2 - 2R_3 \\ \\ \end{array} \left[\begin{array}{ccc|c} 2 & 0 & 0 & 10 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right] \rightarrow \begin{array}{c} R_1/2 \\ \\ \\ \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

Thus $x = 5$, $y = 2$, $z = 3$. **Answer:** 5 trucks, 2 vans, 3 station wagons.

(b)

$$\begin{cases} 2x + 3y + 3z = 25 \\ 2x + 4y + 5z = 33 \end{cases}$$

$$\left[\begin{array}{ccc|c} 2 & 3 & 3 & 25 \\ 2 & 4 & 5 & 33 \end{array}\right] \rightarrow \begin{array}{c} \\ R_2 - R_1 \end{array} \left[\begin{array}{ccc|c} 2 & 3 & 3 & 25 \\ 0 & 1 & 2 & 8 \end{array}\right] \rightarrow \begin{array}{c} R_1 - 3R_2 \\ \\ \end{array} \left[\begin{array}{ccc|c} 2 & 0 & -3 & 1 \\ 0 & 1 & 2 & 8 \end{array}\right]$$

That is

$$\begin{cases} 2x - 3z = 1 \\ y + 2z = 8 \end{cases}, \text{ therefore } \begin{cases} x = \dfrac{1+3z}{2} \\ y = 8 - 2z \end{cases}$$

Since $y \geq 0$, we have $8 - 2z \geq 0$, so $4 \geq z$. Given that z is a nonnegative integer, the only possibilities are $z = 0,1,2,3,4$. We have to rule out $0,2,4$, because such values for z would make $x = \dfrac{1+3z}{2}$ a non-integer. We are left with $z = 1$ and $z = 3$.

Use $x = \dfrac{1+3z}{2}$ and $y = 8 - 2z$ to compute the corresponding x and y, we conclude:

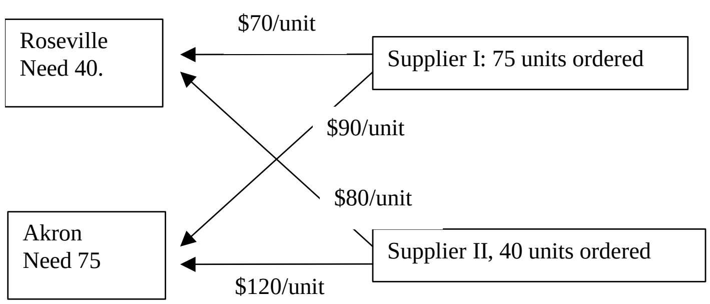
Answer:

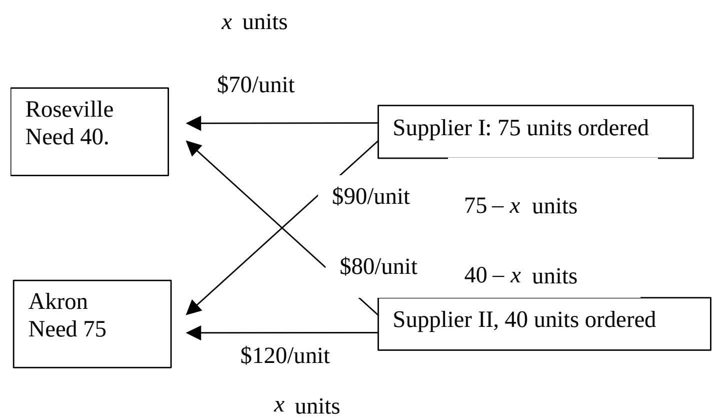
Option 1: 2 trucks, 6 vans, 1 station wagon;

Option 1: 5 trucks, 2 vans, 3 station wagons.

Prob. 52 (Transportation: two plants, two suppliers)

We would like to introduce as few variables as possible. The four routes are shown in the picture below. Let x be the number of units sent from supplier I to Roseville. Then supplier I must send $75-x$ units to Akron, because 75 units are ordered from supplier I. Now, consider the plant at Roseville, which needs 40 units. With x units from supplier I, the remaining $40-x$ needed must come from supplier II. Likewise, the plant at Akron needs 75 units. Since supplier I send $75-x$ to Akron, the remaining x needed by Akron must come from supplier II. So far, as far as supplier II is concerned, we have $40-x$ units sent to Roseville, and x units sent to Akron, therefore 40 units all together are sent from supplier II, which is consistent with the information that 40 units are ordered from supplier II.





Now, let's keep track of the transportation cost, which totals $10,750. We have $70x+90(75-x)+80(40-x)+120x=10{,}750$. Solve it to get $x=40$. We concluded that supplier I sends 40 units to Roseville, and 35 units to Akron, while supplier II sends 0 units to Roseville, and 40 units to Akron.

Prob. 53 (Packaging)

Italian x packages

French y packages

Oriental z packages

$$\begin{cases} 0.3x \phantom{{}+0.6y} + 0.2z = 16{,}200 \\ 0.3x + 0.6y + 0.5z = 41{,}400 \\ 0.4x + 0.4y + 0.3z = 29{,}400 \end{cases}$$

$$\left[\begin{array}{ccc|c} 0.3 & 0 & 0.2 & 16{,}200 \\ 0.3 & 0.6 & 0.5 & 41{,}400 \\ 0.4 & 0.4 & 0.3 & 29{,}400 \end{array}\right] \rightarrow \begin{array}{c} 10R_1 \\ 10R_2 \\ 10R_3 \end{array} \left[\begin{array}{ccc|c} 3 & 0 & 2 & 162{,}000 \\ 3 & 6 & 5 & 414{,}000 \\ 4 & 4 & 3 & 294{,}000 \end{array}\right] \rightarrow$$

$$\begin{array}{c} \\ R_2-R_1 \\ 3R_3-4R_1 \end{array} \left[\begin{array}{ccc|c} 3 & 0 & 2 & 162{,}000 \\ 0 & 6 & 3 & 252{,}000 \\ 0 & 12 & 1 & 234{,}000 \end{array}\right] \rightarrow \begin{array}{c} \\ R_2/3 \\ 3R_3-4R_1 \end{array} \left[\begin{array}{ccc|c} 3 & 0 & 2 & 162{,}000 \\ 0 & 2 & 1 & 84{,}000 \\ 0 & 12 & 1 & 234{,}000 \end{array}\right] \rightarrow$$

$$\begin{array}{r} \\ \\ R_3-6R_2 \end{array}\left[\begin{array}{ccc|c} 3 & 0 & 2 & 162{,}000 \\ 0 & 2 & 1 & 84{,}000 \\ 0 & 0 & -5 & -270{,}000 \end{array}\right] \rightarrow \begin{array}{r} \\ \\ -R_3/5 \end{array}\left[\begin{array}{ccc|c} 3 & 0 & 2 & 162{,}000 \\ 0 & 2 & 1 & 84{,}000 \\ 0 & 0 & 1 & 54{,}000 \end{array}\right] \rightarrow$$

$$\begin{array}{r} R_1-2R_3 \\ R_2-R_3 \\ \\ \end{array}\left[\begin{array}{ccc|c} 3 & 0 & 0 & 54{,}000 \\ 0 & 2 & 0 & 30{,}000 \\ 0 & 0 & 1 & 54{,}000 \end{array}\right] \rightarrow \begin{array}{r} R_1/3 \\ R_2/2 \\ \\ \end{array}\left[\begin{array}{ccc|c} 1 & 0 & 0 & 18{,}000 \\ 0 & 1 & 0 & 15{,}000 \\ 0 & 0 & 1 & 54{,}000 \end{array}\right]$$

So $x = 18{,}000$, $y = 15{,}000$, $z = 54{,}000$

Answer: 18,000 packages of Italian style, 15,000 packages of French style, 54,000 packages of Oriental style.

Prob. 54 (Tents)

x two-person tents, y three-person tents, z four-person tents.

$$\begin{cases} 2x + 3y + 4z = 166 \\ 150x + 200y + 250z = 11{,}150 \\ 3x + 5y + 8z = 289 \end{cases}$$

$$\left[\begin{array}{ccc|c} 2 & 3 & 4 & 166 \\ 150 & 200 & 250 & 11{,}150 \\ 3 & 5 & 8 & 289 \end{array}\right] \rightarrow \begin{array}{r} \\ R_2/50 \\ \\ \end{array}\left[\begin{array}{ccc|c} 2 & 3 & 4 & 166 \\ 3 & 4 & 5 & 223 \\ 3 & 5 & 8 & 289 \end{array}\right] \rightarrow \begin{array}{r} \\ -2R_2+3R_1 \\ 2R_3-3R_1 \end{array}\left[\begin{array}{ccc|c} 2 & 3 & 4 & 166 \\ 0 & 1 & 2 & 52 \\ 0 & 1 & 4 & 80 \end{array}\right] \rightarrow$$

$$\begin{array}{r} R_1-3R_2 \\ \\ R_3-R_2 \end{array}\left[\begin{array}{ccc|c} 2 & 0 & -2 & 10 \\ 0 & 1 & 2 & 52 \\ 0 & 0 & 2 & 28 \end{array}\right] \rightarrow \begin{array}{r} R_1/2 \\ \\ R_3/2 \end{array}\left[\begin{array}{ccc|c} 1 & 0 & -1 & 5 \\ 0 & 1 & 2 & 52 \\ 0 & 0 & 1 & 14 \end{array}\right] \rightarrow \begin{array}{r} R_1+R_3 \\ R_2-2R_3 \\ \\ \end{array}\left[\begin{array}{ccc|c} 1 & 0 & 0 & 19 \\ 0 & 1 & 0 & 24 \\ 0 & 0 & 1 & 14 \end{array}\right]$$

So $x = 19$, $y = 24$, $z = 14$

Answer: 19 two-person tents, 24 three-person tents, 14 four-person tents.

Prob. 65 (Toys)

(a) x balls, y dolls, z cars

$$\begin{cases} x + y + z = 100 \\ 2x + 3y + 4z = 295 \\ 12x + 16y + 18z = 1542 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 100 \\ 2 & 3 & 4 & 295 \\ 12 & 16 & 18 & 1542 \end{array}\right] \rightarrow \begin{array}{r} \\ \\ R_3/2 \end{array}\left[\begin{array}{ccc|c} 1 & 1 & 1 & 100 \\ 2 & 3 & 4 & 295 \\ 6 & 8 & 9 & 771 \end{array}\right] \rightarrow \begin{array}{r} \\ R_2-2R_1 \\ R_3-6R_1 \end{array}\left[\begin{array}{ccc|c} 1 & 1 & 1 & 100 \\ 0 & 1 & 2 & 95 \\ 0 & 2 & 3 & 171 \end{array}\right] \rightarrow$$

$$\begin{array}{r} R_1-R_2 \\ \\ R_3-2R_2 \end{array}\left[\begin{array}{ccc|c} 1 & 0 & -1 & 5 \\ 0 & 1 & 2 & 95 \\ 0 & 0 & -1 & -19 \end{array}\right] \rightarrow \begin{array}{r} R_1-R_3 \\ R_2+2R_3 \\ \\ \end{array}\left[\begin{array}{ccc|c} 1 & 0 & 0 & 24 \\ 0 & 1 & 0 & 57 \\ 0 & 0 & -1 & -19 \end{array}\right] \rightarrow \begin{array}{r} \\ \\ -R_3 \end{array}\left[\begin{array}{ccc|c} 1 & 0 & 0 & 24 \\ 0 & 1 & 0 & 57 \\ 0 & 0 & 1 & 19 \end{array}\right]$$

So $x = 24$, $y = 57$, $z = 19$ **Answer:** 24 balls, 57 dolls, 19 cars.

(b)

$$\begin{cases} x & + & y & + & z & = & 100 \\ 2x & + & 3y & + & 4z & = & 295 \\ 11x & + & 15y & + & 19z & = & 1542 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 100 \\ 2 & 3 & 4 & 295 \\ 11 & 15 & 19 & 1542 \end{array}\right] \rightarrow \begin{array}{c} \\ R_2 - 2R_1 \\ R_3 - 11R_1 \end{array} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 100 \\ 0 & 1 & 2 & 95 \\ 0 & 4 & 8 & 442 \end{array}\right] \rightarrow \begin{array}{c} \\ \\ R_3/4 \end{array} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 100 \\ 0 & 1 & 2 & 95 \\ 0 & 1 & 2 & 110.5 \end{array}\right]$$

We may stop right here, because we notice that the third contradicts with the second line, i.e. $x + 2y = 110.5$ contradicts with $x + 2y = 95$. So there is no solutions.

(c)

$$\begin{cases} x & + & y & + & z & = & 100 \\ 2x & + & 3y & + & 4z & = & 295 \\ 11x & + & 15y & + & 19z & = & 1480 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 100 \\ 2 & 3 & 4 & 295 \\ 11 & 15 & 19 & 1480 \end{array}\right] \rightarrow \begin{array}{c} \\ R_2 - 2R_1 \\ R_3 - 11R_1 \end{array} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 100 \\ 0 & 1 & 2 & 95 \\ 0 & 4 & 8 & 380 \end{array}\right] \rightarrow \begin{array}{c} \\ \\ R_3/4 \end{array} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 100 \\ 0 & 1 & 2 & 95 \\ 0 & 1 & 2 & 95 \end{array}\right] \rightarrow$$

$$\begin{array}{c} R_1 - R_2 \\ \\ R_3 - R_2 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & -1 & 5 \\ 0 & 1 & 2 & 95 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

So $x = 5 + z$, and $y = 95 - 2z$, with z being arbitrary. However, x, y, z are supposed to be non-negative integers. So, in order for $y \geq 0$, we have to have $95 - 2z \geq 0$, so $2z \leq 95$, so $z = 0,1,2,3,\cdots,47$. Each of these z values would make x a non-negative integer. So we conclude

Answer: 48 solutions.

(d) That's when $z = 0$. So $x = 5 + z = 5 + 0 = 5$, and $y = 95 - 2z = 95 - 0 = 95$. So

Answer: 5 balls, 95 dolls, 0 cars.

(e) That's when $z = 47$. So $x = 5 + z = 5 + 47 = 52$, and $y = 95 - 2z = 95 - 2(47) = 1$. So

Answer: 52 balls, 1 doll, 47 cars.

<u>Section 2.3</u>: 1-31 odd.

(Do them on your own and check the answer in the back of the book.)