

Solutions for Homework #3

1. Use the **Gauss-Jordan Method** to solve the following system.

(Warning: You get **NO CREDIT** if you use any other method!!!!!!)

$$(A) [2.2, \#29] \begin{cases} 4x + 4y - 4z = 24 \\ 2x - y + z = -9 \\ x - 2y + 3z = 1 \end{cases}$$

Sol:

$$\begin{array}{ccc} \begin{bmatrix} 4 & 4 & -4 & | & 24 \\ 2 & -1 & 1 & | & -9 \\ 1 & -2 & 3 & | & 1 \end{bmatrix} & R_1/4 \begin{bmatrix} 1 & 1 & -1 & | & 6 \\ 2 & -1 & 1 & | & -9 \\ 1 & -2 & 3 & | & 1 \end{bmatrix} & \begin{array}{l} -R_2 + 2R_1 \\ -R_3 + R_1 \end{array} \begin{bmatrix} 1 & 1 & -1 & | & 6 \\ 0 & 3 & -3 & | & 21 \\ 0 & 3 & -4 & | & 5 \end{bmatrix} \\ \\ R_2/3 \begin{bmatrix} 1 & 1 & -1 & | & 6 \\ 0 & 1 & -1 & | & 7 \\ 0 & 3 & -4 & | & 5 \end{bmatrix} & R_1 - R_2 \begin{bmatrix} 1 & 0 & 0 & | & -1 \\ 0 & 1 & -1 & | & 7 \\ 0 & 0 & 1 & | & 16 \end{bmatrix} & R_2 + R_3 \begin{bmatrix} 1 & 0 & 0 & | & -1 \\ 0 & 1 & 0 & | & 23 \\ 0 & 0 & 1 & | & 16 \end{bmatrix} \end{array}$$

Answer: $x = -1$, $y = 23$, $z = 16$

$$(B) [2.2, \#31] \begin{cases} 3x + 5y - z = 0 \\ 4x - y + 2z = 1 \\ 7x + 4y + z = 1 \end{cases}$$

Sol:

$$\begin{array}{ccc} \begin{bmatrix} 3 & 5 & -1 & | & 0 \\ 4 & -1 & 2 & | & 1 \\ 7 & 4 & 1 & | & 1 \end{bmatrix} & \begin{array}{l} -3R_2 + 4R_1 \\ -3R_3 + 7R_1 \end{array} & \begin{bmatrix} 3 & 5 & -1 & | & 0 \\ 0 & 23 & -10 & | & -3 \\ 0 & 23 & -10 & | & -3 \end{bmatrix} \\ \\ 23R_1 - 5R_2 \begin{bmatrix} 69 & 0 & 27 & | & 15 \\ 0 & 23 & -10 & | & -3 \\ 0 & 0 & 0 & | & 0 \end{bmatrix} & R_1/69 & \begin{bmatrix} 1 & 0 & \frac{9}{23} & | & \frac{5}{23} \\ 0 & 1 & -\frac{10}{23} & | & -\frac{3}{23} \\ 0 & 0 & 0 & | & 0 \end{bmatrix} \\ R_3 - R_2 & R_2/23 & \end{array}$$

This is in row-reduced echelon form already. So we can read off the answer:

Answer: $\left(\frac{5}{23} - \frac{9}{23}z, -\frac{3}{23} + \frac{10}{23}z, z\right)$,

$$(C) [2.2, \#38] \begin{cases} x + 3y - 2z - w = 9 \\ 2x + 4y + 2w = 10 \\ -3x - 5y + 2z - w = -15 \\ x - y - 3z + 2w = 6 \end{cases}$$

Sol:

$$\begin{bmatrix} 1 & 3 & -2 & -1 & | & 9 \\ 2 & 4 & 0 & 2 & | & 10 \\ -3 & -5 & 2 & -1 & | & -15 \\ 1 & -1 & -3 & 2 & | & 6 \end{bmatrix} \quad R_2/2 \quad \begin{bmatrix} 1 & 3 & -2 & -1 & | & 9 \\ 1 & 2 & 0 & 1 & | & 5 \\ -3 & -5 & 2 & -1 & | & -15 \\ 1 & -1 & -3 & 2 & | & 6 \end{bmatrix}$$

$$\begin{matrix} -R_2 + R_1 \\ R_3 + 3R_1 \\ -R_4 + R_1 \end{matrix} \begin{bmatrix} 1 & 3 & -2 & -1 & | & 9 \\ 0 & 1 & -2 & -2 & | & 4 \\ 0 & 4 & -4 & -4 & | & 12 \\ 0 & 4 & 1 & -3 & | & 3 \end{bmatrix} \quad \begin{matrix} R_3/4 \\ R_4/4 \end{matrix} \begin{bmatrix} 1 & 3 & -2 & -1 & | & 9 \\ 0 & 1 & -2 & -2 & | & 4 \\ 0 & 1 & -1 & -1 & | & 3 \\ 0 & 4 & 1 & -3 & | & 3 \end{bmatrix}$$

$$\begin{matrix} R_1 - 3R_2 \\ R_3 - R_2 \\ R_4 - 4R_2 \end{matrix} \begin{bmatrix} 1 & 0 & 4 & 5 & | & -3 \\ 0 & 1 & -2 & -2 & | & 4 \\ 0 & 0 & 1 & 1 & | & -1 \\ 0 & 0 & 9 & 5 & | & -13 \end{bmatrix} \quad \begin{matrix} R_1 - 4R_3 \\ R_2 + 2R_3 \\ -R_4 + 9R_3 \end{matrix} \begin{bmatrix} 1 & 0 & 0 & 1 & | & 1 \\ 0 & 1 & 0 & 0 & | & 2 \\ 0 & 0 & 1 & 1 & | & -1 \\ 0 & 0 & 0 & 4 & | & 4 \end{bmatrix}$$

$$\begin{matrix} R_4/4 \\ R_1 - R_4 \\ R_3 - R_4 \end{matrix} \begin{bmatrix} 1 & 0 & 0 & 1 & | & 1 \\ 0 & 1 & 0 & 0 & | & 2 \\ 0 & 0 & 1 & 1 & | & -1 \\ 0 & 0 & 0 & 1 & | & 1 \end{bmatrix} \quad \begin{matrix} R_1 - R_4 \\ R_3 - R_4 \end{matrix} \begin{bmatrix} 1 & 0 & 0 & 0 & | & 0 \\ 0 & 1 & 0 & 0 & | & 2 \\ 0 & 0 & 1 & 0 & | & -2 \\ 0 & 0 & 0 & 1 & | & 1 \end{bmatrix}$$

Answer: $x=0, y=2, z=-2, w=1$

$$(D) [2.2, \#39] \begin{cases} x + y - z + 2w = -20 \\ 2x - y + z + w = 11 \\ 3x - 2y + z - 2w = 27 \end{cases}$$

Sol:

$$\begin{bmatrix} 1 & 1 & -1 & 2 & | & -20 \\ 2 & -1 & 1 & 1 & | & 11 \\ 3 & -2 & 1 & -2 & | & 27 \end{bmatrix} \quad \begin{matrix} -R_2 + 2R_1 \\ -R_3 + 3R_1 \end{matrix} \begin{bmatrix} 1 & 1 & -1 & 2 & | & -20 \\ 0 & 3 & -3 & 3 & | & -51 \\ 0 & 5 & -4 & 8 & | & -87 \end{bmatrix}$$

$$\begin{matrix} R_2/3 \\ R_3 - 5R_2 \end{matrix} \begin{bmatrix} 1 & 1 & -1 & 2 & | & -20 \\ 0 & 1 & -1 & 1 & | & -17 \\ 0 & 5 & -4 & 8 & | & -87 \end{bmatrix} \quad \begin{matrix} R_1 - R_2 \\ R_3 - 5R_2 \end{matrix} \begin{bmatrix} 1 & 0 & 0 & 1 & | & -3 \\ 0 & 1 & -1 & 1 & | & -17 \\ 0 & 0 & 1 & 3 & | & -2 \end{bmatrix}$$

$$R_2 + R_3 \begin{bmatrix} 1 & 0 & 0 & 1 & | & -3 \\ 0 & 1 & 0 & 4 & | & -19 \\ 0 & 0 & 1 & 3 & | & -2 \end{bmatrix}$$

Answer: $(-3-w, -19-4w, -2-3w, w)$