Section 1.1

Section 1.1, Prob. 36:

(A) The slope of the line $2x+3y=6$ is $-2/3$ (because the equation can be brought to the slope-intercept form as $y=-\frac{2}{3}x+2$.) So we should have

$\frac{2-(-1)}{k-4}=-\frac{2}{3}$, i.e. $\frac{3}{k-4}=-\frac{2}{3}$, thus $(3)(3)=-2(k-4)$, i.e. $9=-2k+8$, which we solve to obtain $k=-1/2$.

Another approach: Since the slope is $-2/3$, the line in slope-intercept form is $y=-\frac{2}{3}x+b$. Since the point $(4,-1)$ satisfies this equation, we have $-1=-\frac{2}{3}(4)+b$. Solving for b, we get $-1+\frac{8}{3}=b$, i.e. $b=\frac{5}{3}$. So the line is $y=-\frac{2}{3}x+\frac{5}{3}$. But the point $(k,2)$ also satisfies the equation. So $2=-\frac{2}{3}k+\frac{5}{3}$. Solve this for k: $6=-2k+5$, which gives $k=-1/2$.

(B) The line $5x-2y=-1$ has its slope-intercept form as $y=\frac{5}{2}x+\frac{1}{2}$. Thus its slope is $\frac{5}{2}$. Recall the formula $m_1m_2=-1$ for two perpendicular lines. This says then that the line perpendicular to $5x-2y=-1$ should have a slope that is the "opposite reciprocal" of $\frac{5}{2}$. Namely it should have a slope of $-\frac{2}{5}$. Then, following the idea from (A), we set up the equation $\frac{2-(-1)}{k-4}=-\frac{2}{5}$, i.e. $\frac{3}{k-4}=-\frac{2}{5}$. Solve it to get k: $(3)(5)=-2(k-4)$, so $k=-7/2$.

Section 1.1, Prob. 61:

(a) (See the solution in the book for the graph.)

(b) The slope is $m=\frac{5.770-3.599}{11-1}=\frac{2.171}{10}=0.2171$. So the equation is of the form $y-3.599=0.2171(x-1)$, i.e. $y=0.2171x-0.2171+3.599$, namely $y=0.2171x+3.3819$. The slope 0.2171 indicates that the federal debt was increasing at a rate of 0.2171 trillion dollars per year. (Note that we are specific in the units of measurement for the slope!!!!) I will leave the graphing to you…

(c) The year 2002 corresponds to $x=2002-1990=12$. Substitute this in $y=0.2171x+3.3819$, we get $y=0.2171(12)+3.3819=5.9871$. This means the predicted federal debt for the year 2002 is 5.9871 trillion dollars. This predicted value was less than the actual value of 6.199 trillion dollars.

Section 1.1, Prob. 66: (Life Expectancy.) Let x be the years since 1900. The life expectancy y at birth follows a linear equation in x and y that is satisfied by two data points: $(0, 46)$ and $(100, 76.9)$. The slope is $\frac{76.9-46}{100-0}=0.309$, with y-intercept 46. So the equation is $y=0.309x+46$. Similarly, the life expectancy at age 65 can be described by a linear equation in x and y that is satisfied by two data points: $(0, 76)$ and $(100, 82.9)$. The slope is $\frac{82.9-76}{100-0}=0.069$, with y-intercept 76. So the equation is $y=0.069x+76$. In order for the two life expectancies to equal to each other, we will have $0.309x+46=0.069x+76$. Solve it to get $x=125$. At $x=125$, the (common) value of y is $y=0.309x+46=0.309(125)+46=84.625\approx 85$. So the maximum life expectancy for humans is about 85 years according to this theory.

<u>Section 1.1, Prob. 68</u>: (Immigration)

x: the number of years after 1974.

y : the number of immigrants

$y = mx + b$

In the year 1974: $x = 0$, we have $y = 86{,}821$.

In the year 2000: $x = 26$, we have $y = 217{,}753$.

(a) $m = \dfrac{217{,}753 - 86{,}821}{26 - 0} = 5035.85$ $\qquad b = 86{,}821$

Answer: $y = 5035.85x + 86{,}821$

(b) For the year 2010, we have $x = 2010 - 1974 = 36$ we predict $y = 5035.85(36) + 86{,}821 = 268{,}112$

<u>Section 1.1, Prob. 70</u>: (Older College Students)

(a) t: the number of years since 1970.

p: the percentage of students that are 35 or older.

Assume $p = mt + b$. We know that

In the year 1970: $t = 0$, we have $p = 9.6$.

In the year 2001: $t = 31$, we have $p = 19.2$.

So, $m = \dfrac{\Delta p}{\Delta t} = \dfrac{19.2 - 9.6}{31 - 0} \approx 0.31$, and $b = 9.6$.

Answer: $p = 0.31t + 9.6$

(b) For the year 2010, $t = 2010 - 1970 = 40$, so $p = 0.31(40) + 9.6 = 22$.

Answer: 22%

(c) Want $p = 31$. Solve for t: $31 = 0.31t + 9.6$, thus $31 - 9.6 = 0.31t$, therefore $t \approx 69$, which means the year $1970 + 69 = 2039$.

Answer: the year 2039.

<u>Section 1.2</u>:

<u>Section 1.2, Prob. 8</u>: $C(x) = 12 + 1x = 12 + x$

$12 is the fixed cost.

$1 is the cost per hour of rental.

x is the number of hours of rental

$C(x)$: the cost in dollars for renting a saw for x hours

<u>Section 1.2, Prob. 10</u>: $C(x) = 0.50 + 0.35x$

$0.50 is the fixed cost.

$0.35 is the cost per half-hour of parking.

x is the number of half-hours of parking (e.g. $x = 2$ if you park for 1 hour.)

$C(x)$: the cost in dollars for parking for x half-hours.

<u>Section 1.2, Prob. 12</u>: The fixed cost is $100. So $C(x) = 100 + mx$. But $C(50) = 1600$, so $100 + 50m = 1600$, therefore $50m = 1500$, and so $m = 30$.

Answer: $C(x) = 100 + 30x$

<u>Section 1.2, Prob. 13</u>: $C(x) = mx + 400$. Know that $C(10)$ equals 650. Thus $10m + 400 = 650$, therefore $m = 25$. So the cost function is $C(x) = 25x + 400$

<u>Section 1.2, Prob. 14</u>: Marginal cost is $90. Thus $C(x) = b + 90x$. But $C(150) = 16{,}000$, therefore $b + 90(150) = 16000$. Solve for b to get $b = 2500$.

Answer: $C(x) = 2500 + 90x$

Section 1.2, Prob. 15: $C(x)=120x+b$. Know that $C(700)$ equals 96,500. Thus $120(700)+b=96500$, therefore b can be solved to be $b=12500$. So the cost function is $C(x)=120x+12{,}500$

Section 1.2, Prob. 18: p is the price in dollars, q is the quantity in hundreds.

We are given $D(q)=16-\frac{5}{4}q$.

(a) $D(0)=16$. So the price is \$16 when there is only a demand of 0 can openers.

(b) $D(4)=16-\frac{5}{4}(4)=11$. I.e. The price is \$11 when 400 can openers are demanded.

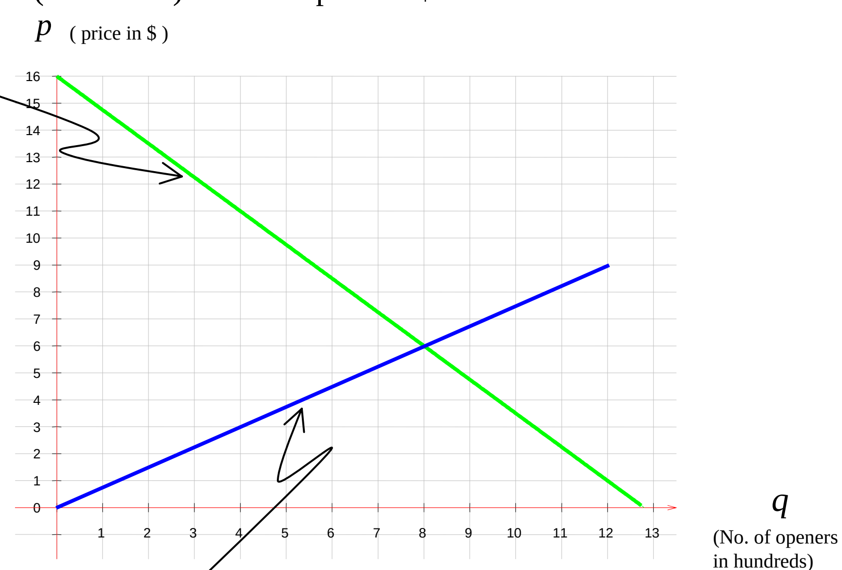
(c) $D(8)=16-\frac{5}{4}(8)=6$. I.e. The price is \$6 when 800 can openers are demanded.

(d) Solve $16-\frac{5}{4}q=8$ for q. Thus $64-5q=32$, $64-32=5q$, so $q=6.4$. This means the demand is 640 (not 6.4 !!!) when the price is \$8.

(e) Solve $16-\frac{5}{4}q=10$ for q. Thus $64-5q=40$, $64-40=5q$, so $q=4.8$. This means the demand is 480 (not 4.8 !!!) when the price is \$10.

(f) Solve $16-\frac{5}{4}q=12$ for q. Thus $64-5q=48$, $64-48=5q$, so $q=3.2$. This means the demand is 320 (not 3.2 !!!) when the price is \$12.

(g) Graph:



(h) Solve $\frac{3}{4}q=0$ for q. Thus $q=0$. So 0 can openers are supplied when the price is \$0.

(i) Solve $\frac{3}{4}q=10$ for q. Thus $q=\frac{40}{3}\approx 13.33$. So about 1333 (not 13.33!!) can openers are supplied when the price is \$10.

(j) Solve $\frac{3}{4}q=20$ for q. Thus $q=\frac{80}{3}\approx 26.67$. So about 2667 (not 26.67!!) can openers are supplied when the price is \$20.

(k) Graph:

(l) Solve $16-\frac{5}{4}q=\frac{3}{4}q$ for q: $16=2q$, so $q=8$. Then the price is

$16-\frac{5}{4}q=16-\frac{5}{4}(8)=6$ (or, you may use $\frac{3}{4}q=\frac{3}{4}(8)=6$, same answer).

So the equilibrium price is \$6, with the equilibrium quantity being 800 (not 8!!!).

Section 1.2, Prob. 22: (T-Shirt Cost)

(a) Marginal cost to produce one T-shirt is \$3.50. Therefore the cost for producing x T-shirts is $C(x) = b + 3.50x$. But $C(60) = 300$, so $b + 3.50(60) = 300$. Solve this for b, $b + 210 = 300$, so $b = 90$. Therefore the cost function is $C(x) = 90 + 3.50x$.

(b) Solve $90 + 3.50x = 9x$ for x: $90 = 5.5x$, $x \approx 16.36$. Let's round it up to 17. So she has to produce and sell 17 T-shirts in order to break even.

(c) Solve $[9x] - [90 + 3.50x] = 500$ for x: $5.5x = 590$, thus $x = 107.27$. Let's round it up to 108. So she must produce and sell 108 T-shirts in order to make a profit of \$500.

Section 1.2, Prob. 23: (Publishing Costs)

(a) $C(x) = mx + 525$, with $C(1000)$ equal to 2675. Thus $1000m + 525 = 2675$, therefore $m = 2.15$. So the cost function is $C(x) = 2.15x + 525$

(b) Solve the equation $2.15x + 525 = 4.95x$, so $525 = 2.8x$, thus $x = 187.5$. That is, 188 books have to be sold in order to break even.

(c) Solve the equation $4.95x - (2.15x + 525) = 1000$. So $2.8x = 1000 + 525$, and so $x \approx 544.6$. So 545 books have to be sold before a profit of \$1000 can be made.

Section 1.2, Prob. 24: (Marginal Cost of Coffee)

(a) The cost in dollars is $C(x) = b + mx$ for producing x cups of coffee. We know $C(100) = 11.02$ and $C(400) = 40.12$. The marginal cost m is just the slope, so $m = \dfrac{40.12 - 11.02}{400 - 100} = 0.097$. So the cost function looks like $C(x) = b + 0.097x$. To find b, use the fact that $C(100) = 11.02$, so solve $b + 0.097(100) = 11.02$ for b, therefore $b = 11.02 - 9.7 = 1.32$. **Answer:** $C(x) = 1.32 + 0.097x$

(b) The fixed cost is \$1.32.

(c) $C(1000) = 1.32 + 0.097(1000) = 98.32$. So the total cost for 1000 cups of coffee is \$98.32.

(d) $C(1001) = 1.32 + 0.097(1001) = 98.417$. So the total cost for 1001 cups of coffee is \$98.417.

(e) The marginal cost of the 1001st cup is \$98.417-\$98.32=\$0.097, i.e. 9.7 ¢.

(f) Since it is a *linear* cost function, the cost of producing any addition cup of coffee is 9.7 ¢.

Section 1.2, Prob. 28: (Break-Even Analysis) $C(x) = 12x + 39$, $R(x) = 25x$, both in dollars.

(a) Break-even: $12x + 39 = 25x$, solve for x: $39 = 13x$, so $x = 3$. So the break-even quantity is 3 units.

(b) $P(250) = R(250) - C(250) = [25(250)] - [12(250) + 39] = 3211$. So the profit from 250 units is \$3,211.

(c) Want a profit of \$130. Solve $[25x] - [12x + 39] = 130$ for x. Thus $13x = 130 + 39$, so $x = 13$. Thus for a profit of \$130, 13 units must be produced.