Section 5.3: Independence and the Multiplication Rule

Independent Events

E and F are independent if the probability for F to occur is insensitive to whether or not E has occurred.

How to Discern Independence from Probability Table

Prepare a two-way contingency table involving probabilities. For example:

	E	E^c	total
F	0.45	0.30	**0.75**
F^c	0.15	0.10	**0.25**
total	**0.60**	**0.40**	**1.00**

If both columns have exactly the same proportion, then the two events E, F are independent. (This is the case in this example, where both columns have a proportion of 3 to 1)

Whereas in the example

	E	E^c	total
F	0.45	0.20	**0.65**
F^c	0.15	0.20	**0.35**
total	**0.60**	**0.40**	**1.00**

the first column has a proportion of 3 to 1, but the second column has a proportion of 1 to 1. Thus the two events E, F are <u>not</u> independent.

Multiplication Rule for ***Independent*** Events:

$$P(E \text{ and } F) = P(E) \cdot P(F)$$

if E and F are *independent*.

More often, we <u>intuitively</u> determine that E and F are independent, and then use the above formula for computing $P(E \text{ and } F)$

Multiplication Rule for *Independent* n Events:

$$P(E \text{ and } F \text{ and } G \text{ and } \ldots) = P(E) \cdot P(F) \cdot P(G) \cdots$$

if E, F, G, etc. are *independent*.

Example: Roll a die twice. What is the probability that the results are 3-1?

Sol:

E : the first trial gives 3

F : the second trial gives 1

Intuitively we think E and F are independent.

Thus $P(E \text{ and } F) = P(E) \cdot P(F) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$

A Trick for Handling "At Least":

"At least one" is the complement of "None".

Example: Females account for 30% of the population of an industrial city. If four residents are randomly selected

(a) What is the chance that all four of them are female?

(b) What is the chance that at least one of them is a male?

Sol:

(a)

$$P(\textit{female})P(\textit{female})P(\textit{female})P(\textit{female})$$

$$= (0.30)(0.30)(0.30)(0.30) = 0.0081$$

(b)

$$P(\textit{at least one male}) = 1 - P(\textit{none are male})$$

$$= 1 - P(\textit{all are female}) = 1 - 0.0081 = 0.9919$$