Chapter 5: Probability

Section 5.1: Probability Rules

Do an experiment that can be repeated if desired. For such an experiment, we have

- **Outcomes (Simple Events, usually denoted by lower case letters, such as e): all the possibilities.**
- **Sample Space S (Perhaps more appropriately called the "Outcome Space"??):**
  The collection of all possible outcomes, i.e. all possible "simple events".
- **Event (usually denoted by a capital letter such as E): a sub-collection of simple events that satisfy certain criterion.**

Example: Roll a die once.

The sample space: $\{1, 2, 3, 4, 5, 6\}$

The event "getting an even number" $\{2, 4, 6\}$

The event "getting a number less than 3": $\{1, 2\}$

The event "getting a number divisible by 3": $\{3, 6\}$

Example: Roll a die twice. The sample space consists of the following 36 simple events.

1 1	2 1	3 1	4 1	5 1	6 1
1 2	2 2	3 2	4 2	5 2	6 2
1 3	2 3	3 3	4 3	5 3	6 3
1 4	2 4	3 4	4 4	5 4	6 4
1 5	2 5	3 5	4 5	5 5	6 5
1 6	2 6	3 6	4 6	5 6	6 6

The event "the sum is 5" consists of the four highlighted simple events.

Probability: a numerical measure of the likelihood that an event will occur.

Notation: $P(E)$: the probability that the event E will occur.

An event of probability 0: impossible event.

An event of probability 1: a certainty.

An "unusual event" an event whose probability is below a certain cut-off agreed upon (usually 0.05, but not always)

Rules of Probabilities:

1. For any event E, we should have $0 \le P(E) \le 1$.
2. If $e_1, e_2, \cdots, e_n$ stand for all the simple events in a sample space, then the sum of their probabilities should be 1: $P(e_1) + P(e_2) + \cdots + P(e_n) = 1$

How to Determine the probabilities?

Classical (Theoretical) Method: If the mechanism of the experiment is fully understood, with *<u>all outcomes (i.e. simple events) equally likely</u>*, then

$$P(E) = \frac{N(E)}{N(S)}$$

Example: Roll a die once.

The sample space: $\{1, 2, 3, 4, 5, 6\}$

The event E “getting an even number” $\{2, 4, 6\}$

$$P(E) = \frac{3}{6} = \frac{1}{2}$$

The event F “getting a number less than 3”: $\{1, 2\}$

$$P(F) = \frac{2}{6} = \frac{1}{3}$$

The event G “getting a number divisible by 3”: $\{3, 6\}$

$$P(G) = \frac{2}{6} = \frac{1}{3}$$

Example: Roll a die twice. The sample space consists of the following 36 simple events.

1 1	2 1	3 1	4 1	5 1	6 1
1 2	2 2	3 2	4 2	5 2	6 2
1 3	2 3	3 3	4 3	5 3	6 3
1 4	2 4	3 4	4 4	5 4	6 4
1 5	2 5	3 5	4 5	5 5	6 5
1 6	2 6	3 6	4 6	5 6	6 6

The event E “the sum is 5” consists of the four highlighted simple events.

$$P(E) = \frac{4}{36} = \frac{1}{9}$$

Empirical Approach: This replies on the following important result:

The Law of Large Numbers: Suppose the probability that event E occurs when the experiment is carried once is $P(E)$. Then if the experiment is repeated an enormous number of times,

$$\frac{number\ of\ trials\ in\ which\ E\ occur}{number\ of\ trials} \quad \text{approaches} \quad P(E)$$

as the number of trials get very big.

In the empirical approach, an experiment is repeated an enormous number of times, with the proportion $\frac{number\ of\ trials\ in\ which\ E\ occur}{number\ of\ trials}$ taken to be a good ***approximation*** of the ultimate true probability $P(E)$.

(If $P(E) = 3/10$, it doesn't mean that event E will occur exactly 30,000 times if you repeat the experiment 100,000 times.

Example: Now do simulations with the applets.

Subjective Probabilities: Resort to personal judgment when all other more reliable methods are out of reach.

Section 5.2: The Addition Rule and Complements

Disjoint Events (a.k.a. Mutually Exclusive Events)

Venn Diagram

Addition Rules for Disjoint Events:

$P(E \text{ or } F) = P(E) + P(F)$ if E and F are disjoint.

General Addition Rule

$$P(E \text{ or } F) = P(E) + P(F) - P(E \text{ and } F)$$

This follows from

$$N(E \text{ or } F) = N(E) + N(F) - N(E \text{ and } F)$$

Divide through by $N(S)$, we get

$$\frac{N(E \text{ or } F)}{N(S)} = \frac{N(E)}{N(S)} + \frac{N(F)}{N(S)} - \frac{N(E \text{ and } F)}{N(S)}$$

which is the same as

$$P(E \text{ or } F) = P(E) + P(F) - P(E \text{ and } F)$$

Complement Rule

E : an event

E^c : The complement of E

(i.e. the negation of E)

$$P(E^c) = 1 - P(E)$$

Example: Randomly draw a card from a standard deck of playing cards. Find the probability for each of the following events:

(a) "Drawing a king"
(b) "Drawing a queen"
(c) "Drawing a king or a queen"
(d) "Drawing a diamond"
(e) "Drawing a king or a diamond"
(f) "Drawing a card that is not a king"

Example: The grades of a calculus class are shown:

Grades	Number of students
F	3
D	2
C	15
B	25
A	5
Total	50

Randomly select a student from this class. Find the probability:

(a) "The student had a grade of C"
(b) "The student had a grade of C or D"
(c) "The student didn't get a C"

Example: The grades of a calculus class are shown:

Grades	Proportion of students
F	0.06
D	0.04
C	0.30
B	0.50
A	0.10
Total	1.00

Randomly select a student from this class. Find the probability:

(a) "The student had a grade of C"
(b) "The student had a grade of C or D"
(c) "The student didn't get a C"

Contingency Table (Two-Way Table)

Example: (p.242, Table 6) Marital status of males and females 18 years old or older in the U.S. in 2003.

		Gender		
		Males (in millions)	Females (in millions)	**Total (in millions)**
Marital Status	Never married	28.6	23.3	**51.9**
	Married	62.1	62.8	**124.9**
	Widowed	2.7	11.3	**14.0**
	Divorced	9.0	12.7	**21.7**
	Total (in millions)	**102.4**	**110.1**	**212.5**

(1) The probability that a randomly selected U.S. resident 18 years old or older is a male.

(2) The probability that a randomly selected U.S. resident 18 years old or older is widowed.

(3) The probability that a randomly selected U.S. resident 18 years old or older is widowed or divorced.

(4) The probability that a randomly selected U.S. resident 18 years old or older is a male or widowed.