Math 164 (Spring 2008) **Homework #12 (Due Monday, April 21)**

<u>Section 8.1</u>: (Second round)

19, 21, 22, 25, 27, 28

#19 (a) The Z score corresponding to $X = 260$ is $Z = \frac{X-\mu}{\sigma} = \frac{260-266}{16} = -0.375 \approx -0.38$

so $P(X < 260) = P(Z < -0.38) = 0.3520$.

(b) $n = 20$, $\mu_{\bar{x}} = \mu = 266$, $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{16}{\sqrt{20}} = 3.5777$

The Z score corresponding to $\bar{x} = 260$ is $Z = \frac{\bar{x}-\mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{260-266}{3.5777} \approx -1.68$

So $P(\bar{x} \le 260) = P(Z \le -1.68) = 0.0465$

(c) $n = 50$, $\mu_{\bar{x}} = \mu = 266$, $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{16}{\sqrt{50}} = 2.2627417$

The Z score corresponding to $\bar{x} = 260$ is $Z = \frac{\bar{x}-\mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{260-266}{2.2627417} \approx -2.65$

So $P(\bar{x} \le 260) = P(Z \le -2.65) = 0.0040$

(d) If $\mu = 266$ is true, then we have just seen that the probability for seeing $\bar{x}$ to be as low as 260 or lower would be only 0.0040, i.e. 0.4%, way below 5%, making this observed event highly unusual. In light of this observed event, we begin to doubt whether the population mean is really 266. Perhaps it is smaller than 266.

(e) $n = 15$, $\mu_{\bar{x}} = \mu = 266$, $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{16}{\sqrt{15}} = 4.13118$.

The Z score corresponding to $\bar{x} = 266 - 10 = 256$ is $Z = \frac{\bar{x}-\mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{256-266}{4.13118} \approx -2.42$.

The Z score corresponding to $\bar{x} = 266 + 10 = 276$ is $Z = \frac{\bar{x}-\mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{276-266}{4.13118} \approx 2.42$

So $P(256 \le \bar{x} \le 276) = P(-2.42 \le Z \le -2.42) = 0.9922 - 0.0078 = 0.9844$

#21 (a) The Z score corresponding to $X = 95$ is $Z = \frac{X-\mu}{\sigma} = \frac{95-85}{21.25} \approx 0.47$

so $P(X > 95) = P(Z > 0.47) = 1 - 0.6808 = 0.3192$.

(b) $n = 20$, $\mu_{\bar{x}} = \mu = 85$, $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{21.25}{\sqrt{20}} = 4.751644$

The Z score corresponding to $\bar{x} = 95$ is $Z = \frac{\bar{x}-\mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{95-85}{4.751644} \approx 2.10$

So $P(\bar{x} > 95) = P(Z > 2.10) = 1 - 0.9821 = 0.0179$

(c) $n = 30$, $\mu_{\bar{x}} = \mu = 85$, $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{21.25}{\sqrt{30}} = 3.879701$

The Z score corresponding to $\bar{x} = 95$ is $Z = \frac{\bar{x}-\mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{95-85}{3.879701} \approx 2.58$

So $P(\bar{x} > 95) = P(Z > 2.58) = 1 - 0.9951 = 0.0049$

(d) Increasing n causes $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$ to decrease, thus the same $\bar{x}$ value of 95, which is to the right of the center 85, becomes farther and farther away from the center 85 when the separation is measured in terms of $\sigma_{\bar{x}}$. Thus the area of the tail to the right of 95 becomes smaller and smaller.

(e) Since, for $n = 30$, with the assumption that $\mu = 85$, we have $P(\bar{x} > 95) = 0.0049$, which is very small, observing such an unusual event will make us begin to question whether $\mu = 85$ is true. Perhaps μ is higher than 85.

#22 (a) The Z score corresponding to $X = 75$ is $Z = \frac{X - \mu}{\sigma} = \frac{75 - 81.7}{6.9} \approx -0.97$

so $P(X < 75) = P(Z < -0.97) = 0.1660$.

(b) $n = 5$, $\mu_{\bar{x}} = \mu = 81.7$, $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{6.9}{\sqrt{5}} = 3.085774$

The Z score corresponding to $\bar{x} = 75$ is $Z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{75 - 81.7}{3.085774} \approx -2.17$

so $P(X < 75) = P(Z < -2.17) = 0.0150$

(c) $n = 8$, $\mu_{\bar{x}} = \mu = 81.7$, $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{6.9}{\sqrt{8}} = 2.439518$

The Z score corresponding to $\bar{x} = 75$ is $Z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{75 - 81.7}{2.439518} \approx -2.75$

so $P(X < 75) = P(Z < -2.75) = 0.0030$

(d) Since, for $n = 8$, with the assumption that $\mu = 81.7$, we have $P(X < 75) = 0.0030$, which is very small, observing such an unusual event will make us begin to question whether $\mu = 81.7$ is true. Perhaps μ is lower than 81.7.

#25 (a) Without the knowledge of the shape of the population distribution, we have to have a sample size n at least 30 in order to be sure that the sampling distribution is approximately normal.

(b) $n = 40$, $\mu_{\bar{x}} = \mu = 11.4$, $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{3.2}{\sqrt{40}} = 0.505964$

The Z score corresponding to $\bar{x} = 10$ is $Z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{10 - 11.4}{0.505964} \approx -2.77$

so $P(X < 10) = P(Z < -2.77) = 0.0028$

#27 (a) Because $n = 50 \geq 30$.

(b) $n = 50$, $\mu_{\bar{x}} = \mu = 3$, $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{\sqrt{3}}{\sqrt{50}} = 0.244949$

(c) The Z score corresponding to $\bar{x} = 3.6$ is $Z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{3.6 - 3}{0.244949} \approx 2.45$

so $P(X \geq 3.6) = P(Z \geq 2.45) = 1 - 0.9929 = 0.0071$. This is unusual. Because of this, we begin to question whether the assumption that $\mu = 3$ is true. Perhaps it is higher than 3.

#28 (a) Because $n = 40 \geq 30$.

(b) $n = 40$, $\mu_{\bar{x}} = \mu = 20$, $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{\sqrt{20}}{\sqrt{40}} = 0.707107$

(c) The Z score corresponding to $\bar{x} = 21.1$ is $Z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{22.1 - 20}{0.707107} \approx 2.97$

so $P(X \geq 22.1) = P(Z \geq 2.97) = 1 - 0.9985 = 0.0015$. This is unusual. Because of this, we begin to question whether the assumption that $\mu = 20$ is true. Perhaps it is higher than 20.

Section 8.2:

7, 8, 9, 10, 15, 16, 19, 20

#7 $n = 500$, $0.05N = (0.05)(25{,}000) = 1250$, so $n \le 0.05N$.

$p = 0.4$, $np(1-p) = (500)(0.4)(0.6) = 120$, so $np(1-p) \ge 10$.

Thus the sampling distribution of $\hat{p}$ is approximately normal, with

$$\mu_{\hat{p}} = p = 0.4, \text{ and } \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{(0.4)(0.6)}{500}} = 0.021908902 \approx 0.022.$$

#8 $n = 300$, $0.05N = (0.05)(25{,}000) = 1250$, so $n \le 0.05N$.

$p = 0.7$, $np(1-p) = (300)(0.7)(0.3) = 63$, so $np(1-p) \ge 10$.

Thus the sampling distribution of $\hat{p}$ is approximately normal, with

$$\mu_{\hat{p}} = p = 0.7, \text{ and } \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{(0.7)(0.3)}{300}} = 0.0264575 \approx 0.026.$$

#9 $n = 1000$, $0.05N = (0.05)(25{,}000) = 1250$, so $n \le 0.05N$.

$p = 0.103$, $np(1-p) = (1000)(0.103)(1-0.103) = 92.391$, so $np(1-p) \ge 10$.

Thus the sampling distribution of $\hat{p}$ is approximately normal, with

$$\mu_{\hat{p}} = p = 0.103, \text{ and } \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{(0.103)(0.897)}{1000}} = 0.009612024 \approx 0.010$$

#10 $n = 1010$, $0.05N = (0.05)(25{,}000) = 1250$, so $n \le 0.05N$.

$p = 0.84$, $np(1-p) = (1010)(0.84)(1-0.84) = 135.744$, so $np(1-p) \ge 10$.

Thus the sampling distribution of $\hat{p}$ is approximately normal, with

$$\mu_{\hat{p}} = p = 0.84, \text{ and } \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{(0.84)(1-0.84)}{1010}} = 0.0115356 \approx 0.012$$

#15 (a) $n = 100$, $0.05N = (0.05)(5000) = 250$, so $n \le 0.05N$

$p = 0.30$, $np(1-p) = (100)(0.30)(0.70) = 21$, so $np(1-p) \ge 10$

Thus the sampling distribution of $\hat{p}$ is approximately normal, with

$$\mu_{\hat{p}} = p = 0.30, \text{ and } \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{(0.30)(0.70)}{100}} = 0.045825757 \approx 0.046$$

(b) For $\hat{p} = \frac{37}{100} = 0.37$, the Z score is $Z = \frac{\hat{p} - \mu_{\hat{p}}}{\sigma_{\hat{p}}} = \frac{0.37 - 0.30}{0.045825757} \approx 1.53$

So $P(X > 37) = P(\hat{p} > 0.37) = P(Z > 1.53) = 1 - 0.9370 = 0.0630$.

Yes, this is unusual.

(c) For $\hat{p} = \frac{18}{100} = 0.18$, the Z score is $Z = \frac{\hat{p} - \mu_{\hat{p}}}{\sigma_{\hat{p}}} = \frac{0.18 - 0.30}{0.045825757} \approx -2.62$

So $P(X \le 18) = P(\hat{p} \le 0.18) = P(Z \le -2.62) = 0.0044$.

Yes, this is unusual.

#16 (a) $n = 1136$. Since the total number of college students N in the U.S. is definitely greater than $1136 / 0.05 = 22{,}720$, $n = 1136$ is less than 5% of N

$p = 0.60$, $np(1-p) = (1136)(0.60)(0.40) = 272.64$, so $np(1-p) \ge 10$

Thus the sampling distribution of $\hat{p}$ is approximately normal, with

$\mu_{\hat{p}} = p = 0.60$, and $\sigma_{\hat{p}} = \sqrt{\dfrac{p(1-p)}{n}} = \sqrt{\dfrac{(0.60)(0.40)}{1136}} = 0.014535047 \approx 0.015$

(b) For $\hat{p} = \dfrac{665}{1136} = 0.58538732$, the Z score is

$$Z = \frac{\hat{p} - \mu_{\hat{p}}}{\sigma_{\hat{p}}} = \frac{0.58538732 - 0.60}{0.014535047} \approx -1.01$$

So $P(X \le 665) = P(\hat{p} \le 0.58538732) = P(Z \le -1.01) = 0.1562$.

No, this is not unusual.

(c) For $\hat{p} = \dfrac{725}{1136} = 0.638204225$, the Z score is

$$Z = \frac{\hat{p} - \mu_{\hat{p}}}{\sigma_{\hat{p}}} = \frac{0.638204225 - 0.60}{0.014535047} \approx 2.63$$

So $P(X \ge 725) = P(\hat{p} \ge 0.638204225) = P(Z \ge 2.63) = 1 - 0.9957 = 0.0043$.

Yes, this is unusual.

#19 (a) $n = 800$. Since the total number of adults N in the U.S. is definitely greater than $800/0.05 = 16{,}000$, $n = 800$ is less than 5% of N

$p = 0.43$, $np(1-p) = (800)(0.43)(1-0.43) = 196.08$, so $np(1-p) \ge 10$

Thus the sampling distribution of $\hat{p}$ is approximately normal, with

$\mu_{\hat{p}} = p = 0.43$, and $\sigma_{\hat{p}} = \sqrt{\dfrac{p(1-p)}{n}} = \sqrt{\dfrac{(0.43)(1-0.43)}{800}} = 0.017503571 \approx 0.018$

For $\hat{p} = 0.40$, the Z score is $Z = \dfrac{\hat{p} - \mu_{\hat{p}}}{\sigma_{\hat{p}}} = \dfrac{0.40 - 0.43}{0.017503571} \approx -1.71$

So $P(\hat{p} \le 0.40) = P(Z \le -1.71) = 0.0436$

(b) For $\hat{p} = 0.45$, the Z score is $Z = \dfrac{\hat{p} - \mu_{\hat{p}}}{\sigma_{\hat{p}}} = \dfrac{0.45 - 0.43}{0.017503571} \approx 1.14$

So $P(\hat{p} \ge 0.45) = P(Z \ge 1.14) = 1 - 0.8729 = 0.1271$.

No, it is not unusual.

#20 (a) $n = 500$. Since N is about 5,000,000, $0.05N$ is about 250,000, so $n \le 0.05N$.

$p = 0.23$, $np(1-p) = (500)(0.23)(1-0.23) = 88.55$, so $np(1-p) \ge 10$

Thus the sampling distribution of $\hat{p}$ is approximately normal, with

$\mu_{\hat{p}} = p = 0.23$, and $\sigma_{\hat{p}} = \sqrt{\dfrac{p(1-p)}{n}} = \sqrt{\dfrac{(0.23)(1-0.23)}{500}} = 0.018820202 \approx 0.019$

For $\hat{p} = \dfrac{125}{500} = 0.25$, the Z score is $Z = \dfrac{\hat{p} - \mu_{\hat{p}}}{\sigma_{\hat{p}}} = \dfrac{0.25 - 0.23}{0.018820202} \approx 1.06$

So $P(X \ge 125) = P(\hat{p} \ge 0.25) = P(Z \ge 1.06) = 1 - 0.8554 = 0.1446$

(b) For $\hat{p} = 0.20$, the Z score is $Z = \dfrac{\hat{p} - \mu_{\hat{p}}}{\sigma_{\hat{p}}} = \dfrac{0.20 - 0.23}{0.018820202} \approx -1.59$

So $P(\hat{p} \le 0.20) = P(Z \le -1.59) = 0.0559$.

This is low, but not lower than 0.05. So, no, it is not unusual.