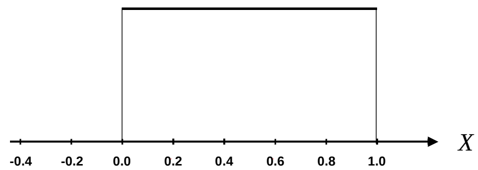
Math 164 (Spring 2008) **Solutions to Homework #11**

Section 7.1:

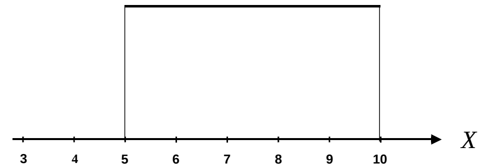
17 [Skip (e).]
18,
23, 24

#17 (a) See the accompanying picture.



(b) $\frac{0.2-0}{1-0}=0.2$ (c) $\frac{0.6-0.25}{1-0}=0.35$ (d) $\frac{1.0-0.95}{1-0}=0.05$

#18 (a) See the accompanying picture.



(b) $\frac{8-6}{10-5}=\frac{2}{5}$ (c) $\frac{8-5}{10-5}=\frac{3}{5}$ (d) $\frac{6-5}{10-5}=\frac{1}{5}$

#23 Graph A: $\mu=10$, $\sigma=2$; Graph B: $\mu=10$, $\sigma=3$

#24 Graph A: $\mu=8$, $\sigma=2$; Graph B: $\mu=14$, $\sigma=2$

Section 7.2:

None. (We did drills on standard normal distribution on Assignment #10 already.)

Section 7.3:

(We did drills on normal distribution on Assignment #10. Here are some applications.)
17, 18, 21, 22, 27, 28

#17 (a) The Z score corresponding to $X=20$ is $Z=\frac{X-\mu}{\sigma}=\frac{20-21}{1}=-1.00$

so $P(X<20)=P(Z<-1.00)=0.1587$.

(b) The Z score corresponding to $X=22$ is $Z=\frac{X-\mu}{\sigma}=\frac{22-21}{1}=1.00$

so $P(X>22)=P(Z>1.00)=1-0.8413=0.1587$

(c) The Z score corresponding to $X=19$ is $Z=\frac{X-\mu}{\sigma}=\frac{19-21}{1}=-2.00$

The Z score corresponding to $X=21$ is $Z=\frac{X-\mu}{\sigma}=\frac{21-21}{1}=0$

so $P(19\le X\le 21)=P(-2.00<Z<0)=0.5000-0.0228=0.4772$

(d) The Z score corresponding to $X=18$ is $Z=\frac{X-\mu}{\sigma}=\frac{18-21}{1}=-3.00$

so $P(X<18)=P(Z<-3.00)=0.0013$.

This is less than 0.05. Thus it is unusual for an egg to hatch in less than 18 days.

#18 (a) The Z score corresponding to $X=80$ is $Z=\frac{X-\mu}{\sigma}=\frac{80-81.7}{6.9}=-0.246377\approx-0.25$

so $P(X>80)=P(Z>-0.25)=1-0.4013=0.5987$.

(b) The Z score corresponding to $X=100$ is $Z=\frac{X-\mu}{\sigma}=\frac{100-81.7}{6.9}\approx 2.65$

so $P(X>100)=P(Z>2.65)=1-0.9960=0.0040$.

(c) The Z score corresponding to $X=60$ is $Z=\frac{X-\mu}{\sigma}=\frac{60-81.7}{6.9}\approx-3.14$

so $P(X<60)=P(Z<-3.14)=0.0008$.

(d) The Z score corresponding to $X=70$ is $Z=\frac{X-\mu}{\sigma}=\frac{70-81.7}{6.9}\approx-1.70$

so $P(X<70)=P(Z<-1.70)=0.0446$.

This is less than 0.05. Thus it's unusual for a resident to work less than 70 hr/wk.

#21 (a) The Z score corresponding to $X = 60$ is $Z = \frac{X - \mu}{\sigma} = \frac{60 - 56}{3.2} = 1.25$

so $P(X > 60) = P(Z > 1.25) = 1 - 0.8944 = 0.1056$.

(b) The Z score corresponding to $X = 50$ is $Z = \frac{X - \mu}{\sigma} = \frac{50 - 56}{3.2} = -1.875 \approx -1.88$

so $P(X \le 50) = P(Z \le -1.88) = 0.0301$.

(c) The Z score corresponding to $X = 58$ is $Z = \frac{X - \mu}{\sigma} = \frac{58 - 56}{3.2} = 0.625 \approx 0.63$.

The Z score corresponding to $X = 62$ is $Z = \frac{X - \mu}{\sigma} = \frac{62 - 56}{3.2} = 1.875 \approx 1.88$

so $P(58 \le X \le 62) = P(0.63 \le Z \le 1.88) = 0.9699 - 0.7357 = 0.2342$

(d) The Z score corresponding to $X = 45$ is $Z = \frac{X - \mu}{\sigma} = \frac{45 - 56}{3.2} = -3.4375 \approx -3.44$

so $P(X < 45) = P(Z < -3.44) = 0.0003$

#22 (a) The Z score corresponding to $X = 1500$ is $Z = \frac{X - \mu}{\sigma} = \frac{1500 - 1550}{57} \approx -0.88$

so $P(X < 1500) = P(Z < -0.88) = 0.1894$.

(b) The Z score corresponding to $X = 1650$ is $Z = \frac{X - \mu}{\sigma} = \frac{1650 - 1550}{57} \approx 1.75$

so $P(X > 1650) = P(Z > 1.75) = 1 - 0.9599 = 0.0401$.

(c) The Z score corresponding to $X = 1625$ is $Z = \frac{X - \mu}{\sigma} = \frac{1625 - 1550}{57} \approx 1.32$.

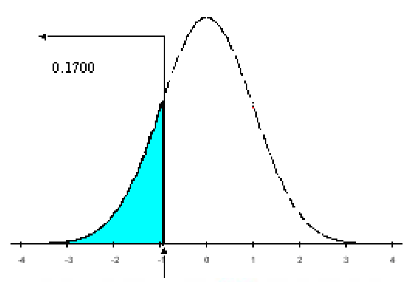
The Z score corresponding to $X = 1725$ is $Z = \frac{X - \mu}{\sigma} = \frac{1725 - 1550}{57} \approx 3.07$

so $P(1625 \le X \le 1725) = P(1.32 \le Z \le 3.07) = 0.9989 - 0.9066 = 0.0923$

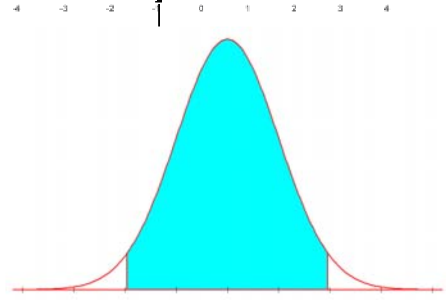
(d) The Z score corresponding to $X = 1400$ is $Z = \frac{X - \mu}{\sigma} = \frac{1400 - 1550}{57} \approx -2.63$

so $P(X > 1400) = P(Z > -2.63) = 1 - 0.0043 = 0.9957$

#27 (a) In the Z-curve shown, the shaded region has an area of 0.1700. Consulting Table IV to find that the border is $Z = -0.95$. This translates into $X = \mu + Z\sigma = 21 + (-0.95)(1) = 20.05 \approx 20$. Thus the 17th percentile for incubation times is about 20 days.



(b) The Z-curve shown, the shaded region in the middle carries an area of 0.9500 (i.e. 95% of the whole). Each of the two tails therefore has an area of $(1 - 0.9500)/2 = 0.0250$. In particular the left tail has an area of 0.0250. Use Table IV to find the border of the left tail to be $Z = -1.96$. By symmetry, the border of the right tail is $Z = 1.96$. Translate these into X:

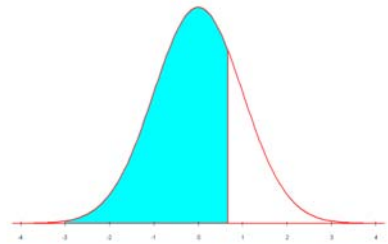


For $Z = -1.96$, $X = \mu + Z\sigma = 21 + (-1.96)(1) = 19.04 \approx 19$

For $Z = 1.96$, $X = \mu + Z\sigma = 21 + (1.96)(1) = 22.96 \approx 23$

Thus the incubation times that make up the middle 95% are between about 19 and 23 days.

#28 (a) In the Z-curve shown, the shaded region has an area of 0.7500 (75%). Consulting Table IV to find that the border is $Z = 0.67$. This translates into $X = \mu + Z\sigma = 81.7 + (0.67)(6.9) \approx 86.3$. Thus the 75th percentile is about 86.3 hours.

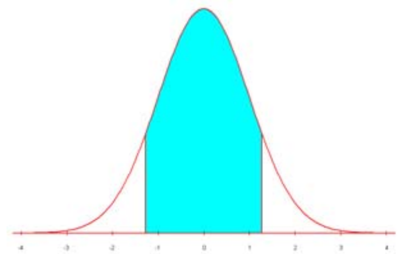


(b) The Z-curve shown, the shaded region in the middle carries an area of 0.8000 (i.e. 80% of the whole). Each of the two tails therefore has an area of $(1-0.8000)/2 = 0.1000$. In particular the left tail has an area of 0.1000. Use Table IV to find the border of the left tail to be $Z = -1.28$. By symmetry, the border of the right tail is $Z = 1.28$. Translate these into X:

For $Z = -1.28$, $X = \mu + Z\sigma = 81.7 + (-1.28)(6.9) \approx 72.9$

For $Z = 1.28$, $X = \mu + Z\sigma = 81.7 + (1.28)(6.9) \approx 90.5$

Thus the number of hours that make up the middle 80% are between about 72.9 and 90.5 days.



Section 8.1:

11, 13, 15, 16, 17, 18

#11 $\mu = 80$, $\sigma = 14$, $n = 49$. Therefore $\mu_{\bar{x}} = \mu = 80$, $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{14}{\sqrt{49}} = 2$

#13 $\mu = 52$, $\sigma = 10$, $n = 21$. Therefore $\mu_{\bar{x}} = \mu = 52$, $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{10}{\sqrt{21}} \approx 2.182179$

#15 $\mu = 80$, $\sigma = 14$, $n = 49$

(a) $n \geq 30$, so the sampling distribution of $\bar{x}$ is approximately normal regardless of whether the population distribution of X is normal. The (approximately) normal distribution of the sample mean $\bar{x}$ has a mean of $\mu_{\bar{x}} = \mu = 80$ and a standard deviation of $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{14}{\sqrt{49}} = 2$.

(b) The Z score corresponding to $\bar{x} = 83$ is $Z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{83-80}{2} = 1.50$

$P(\bar{x} > 83) = P(Z > 1.50) = 1 - 0.9332 = 0.0668$

(c) The Z score corresponding to $\bar{x} = 75.8$ is $Z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{75.8-80}{2} = -2.10$

$P(\bar{x} \leq 75.8) = P(Z \leq -2.10) = 0.0179$

(d) The Z score corresponding to $\bar{x} = 78.3$ is $Z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{78.3-80}{2} = -0.85$.

The Z score corresponding to $\bar{x} = 85.1$ is $Z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{85.1-80}{2} = 2.55$

So $P(78.3 < \bar{x} < 85.1) = P(-0.85 < Z < 2.55) = 0.9946 - 0.1977 = 0.7969$

#16 $\mu = 64$, $\sigma = 18$, $n = 36$

(a) $n \geq 30$, so the sampling distribution of $\bar{x}$ is approximately normal regardless of whether the population distribution of X is normal. The (approximately) normal distribution of the sample mean $\bar{x}$ has a mean of $\mu_{\bar{x}} = \mu = 64$ and a standard deviation of $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{18}{\sqrt{36}} = 3$

(b) The Z score corresponding to $\bar{x} = 62.6$ is $Z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{62.6-64}{3} \approx -0.47$

$P(\bar{x} \leq 62.6) = P(Z \leq -0.47) = 0.3192$

(c) The Z score corresponding to $\bar{x} = 68.7$ is $Z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{68.7 - 64}{3} \approx 1.57$

$P(\bar{x} \ge 68.7) = P(Z \ge 1.57) = 1 - 0.9418 = 0.0582$

(d) The Z score corresponding to $\bar{x} = 59.8$ is $Z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{59.8 - 64}{3} \approx -1.40$.

The Z score corresponding to $\bar{x} = 65.9$ is $Z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{65.9 - 64}{3} \approx 0.63$

So $P(59.8 < \bar{x} < 65.9) = P(-1.40 < Z < 0.63) = 0.7357 - 0.0808 = 0.6549$

#17 $\mu = 64$, $\sigma = 17$, $n = 12$

(a) $n < 30$, we need to make sure that the population distribution is normal in order to use the Central Limit Theorem to claim that the sampling distribution of $\bar{x}$ is normal. Assuming that the population distribution is indeed normal, then the normal distribution of the sample mean $\bar{x}$ has a mean of $\mu_{\bar{x}} = \mu = 64$ and a standard deviation of $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{17}{\sqrt{12}} \approx 4.907477$.

(b) The Z score corresponding to $\bar{x} = 67.3$ is $Z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{67.3 - 64}{4.907477} \approx 0.67$

$P(\bar{x} < 67.3) = P(Z < 0.67) = 0.7486$

(c) The Z score corresponding to $\bar{x} = 65.2$ is $Z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{65.2 - 64}{4.907477} \approx 0.24$.

$P(\bar{x} \ge 65.2) = P(Z \ge 0.24) = 1 - 0.5948 = 0.4052$

#18 $\mu = 64$, $\sigma = 17$, $n = 20$

(a) $n < 30$, we need to make sure that the population distribution is normal in order to use the Central Limit Theorem to claim that the sampling distribution of $\bar{x}$ is normal. Assuming that the population distribution is indeed normal, then the normal distribution of the sample mean $\bar{x}$ has a mean of $\mu_{\bar{x}} = \mu = 64$ and a standard deviation of $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{17}{\sqrt{20}} \approx 3.801316$.

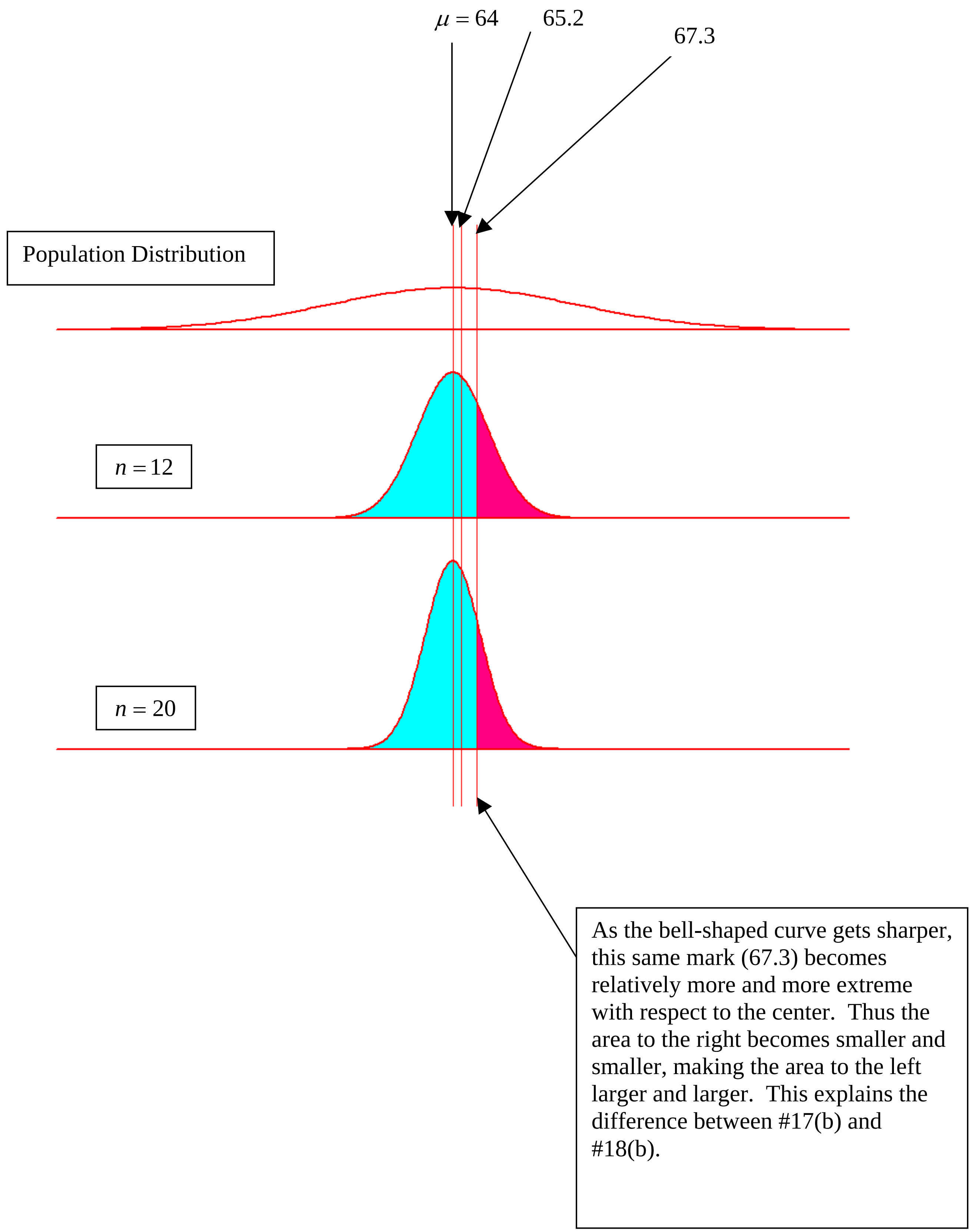
(b) The Z score corresponding to $\bar{x} = 67.3$ is $Z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{67.3 - 64}{3.801316} \approx 0.87$

$P(\bar{x} < 67.3) = P(Z < 0.87) = 0.8078$

(c) The Z score corresponding to $\bar{x} = 65.2$ is $Z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{65.2 - 64}{3.801316} \approx 0.32$.

$P(\bar{x} \ge 65.2) = P(Z \ge 0.32) = 1 - 0.6255 = 0.3745$

(d) Compare (b) (c) with #17 (b) (c), we observe the following: As the sample size increases from 12 to 20, the standard deviation of the sampling distribution of $\bar{x}$ goes down from $\sigma_{\bar{x}} = 4.907477$ to $\sigma_{\bar{x}} = 3.801316$. Thus for each relevant $\bar{x}$ value, such as the 67.3 in (b), the corresponding Z is larger for larger n. It is instructive to line up three graphs together: the population distribution (assumed to be normal), the sampling distribution of $\bar{x}$ when $n = 12$, and the sampling distribution of $\bar{x}$ when $n = 20$. We then mark a relevant $\bar{x}$, and see how the areas of the relevant regions change as n increases. See the accompanying picture.


μ = 64
65.2
67.3
Population Distribution
n = 12
n = 20
As the bell-shaped curve gets sharper, this same mark (67.3) becomes relatively more and more extreme with respect to the center. Thus the area to the right becomes smaller and smaller, making the area to the left larger and larger. This explains the difference between #17(b) and #18(b).