Math 164 (Spring 2008) **Solutions to Homework #9**

<u>Section 6.2</u>: (Second round)

17--28 all (even and odd), 29, 31, 33, 35, 37, 39, 43, 45, 49

Remark: In #43(c), the condition $np(1-p)\geq 10$ of p. 308 is not quite satisfied. But nevertheless let's still assume that the distribution is fairly bell-shaped and apply the Empirical Rule.

#17 $P(3)={}_{10}C_3\,(0.4)^3(1-0.4)^{10-3}=\dfrac{(10)(9)(8)}{(1)(2)(3)}\cdot(0.4)^3(0.6)^7=0.214990848\approx 0.2150$

You may also look up Table II to get the same result.

#18 You may find it useful to recall that ${}_{15}C_{12}={}_{15}C_3$. So

$P(12)={}_{15}C_{12}\,(0.85)^{12}(1-0.85)^{15-12}={}_{15}C_3\cdot(0.85)^{12}(0.15)^3=\dfrac{(15)(14)(13)}{(1)(2)(3)}\cdot(0.85)^{12}(0.15)^3$

$\cong 0.218429998\approx 0.2184$

Table II gives the same result.

#19 Since $p=0.99$ doesn't appear across the top of Table II, we can't use that table.

You may find it useful to recall that ${}_{40}C_{38}={}_{40}C_2$. So

$P(38)={}_{40}C_{38}\,(0.99)^{38}(1-0.99)^{40-38}={}_{40}C_2\,(0.99)^{38}(0.01)^2=\dfrac{(40)(39)}{(1)(2)}\cdot(0.99)^{38}(0.01)^2$

$\cong 0.03523926\approx 0.0352$

#20 Since $p=0.02$ doesn't appear across the top of Table II, we can't use that table.

$P(3)={}_{50}C_3\,(0.02)^3(1-0.02)^{50-3}=\dfrac{(50)(49)(48)}{(1)(2)(3)}\cdot(0.02)^3(0.98)^{47}\cong 0.06066967\approx 0.0607$

#21 $P(3)={}_8C_3\,(0.35)^3(1-0.35)^{8-3}=\dfrac{(8)(7)(6)}{(1)(2)(3)}\cdot(0.35)^3(0.65)^5\cong 0.27858578\approx 0.2786$

Table II gives the same result.

#22 Since $n=17$ doesn't appear in Table II, we can't use that table.

You may find it useful to recall that ${}_{20}C_{17}={}_{20}C_3$. So

$P(17)={}_{20}C_{17}\,(0.6)^{17}(1-0.6)^{20-17}={}_{20}C_3\,(0.6)^{17}(0.4)^3=\dfrac{(20)(19)(18)}{(1)(2)(3)}\cdot(0.6)^{17}(0.4)^3$

$\cong 0.01234969\approx 0.0123$

#23 $P(X\leq 3)$ can be found in Table III (not Table II !!!) to be 0.9144. Or, if we compute in directly,

$P(X\leq 3)=P(0)+P(1)+P(2)+P(3)$

$={}_9C_0\,(0.2)^0(1-0.2)^{9-0}+{}_9C_1\,(0.2)^1(1-0.2)^{9-1}+{}_9C_2\,(0.2)^2(1-0.2)^{9-2}+{}_9C_3\,(0.2)^3(1-0.2)^{9-3}$

$=1\cdot 1\cdot(0.8)^9+9\cdot(0.2)^1(0.8)^8+\dfrac{(9)(8)}{(1)(2)}(0.2)^2(0.8)^7+\dfrac{(9)(8)(7)}{(1)(2)(3)}(0.2)^3(0.8)^6$

$=0.13421773+0.30198989+0.30198989+0.17616077$

≈ 0.9144

#24 Note that $P(X<5)=P(0)+P(1)+P(2)+P(3)+P(4)=P(X\leq 4)$. Use Table III, with $n=10$, $p=0.65$, $x=4$, to find $P(X\leq 4)$ to be 0.0949.

#25 Note that $P(X>3)=1-P(X\leq 3)$. Use Table III with $n=7$, $p=0.5$, $x=3$ to find $P(X\leq 3)$ to be 0.5000. Thus $P(X>3)=1-P(X\leq 3)=1-0.5000=0.5000$.

Remark: This is not surprising. For example, having 3 successes is the same as having 4 failures. But since $p=0.5$, the probability for 4 failures equals the probability for 4 successes, thus we see that $P(3) = P(4)$. Likewise, $P(2) = P(5)$, $P(1) = P(6)$, and $P(0) = P(7)$. It follows that $P(0) + P(1) + P(2) + P(3) = P(7) + P(6) + P(5) + P(4)$. But we know $P(0) + P(1) + P(2) + P(3) + P(4) + P(5) + P(6) + P(7) = 1$. It thus follows that both $P(0) + P(1) + P(2) + P(3)$ and $P(7) + P(6) + P(5) + P(4)$. are equal to 0.5. I.e. both $P(X \le 3)$ and $P(X > 3)$ are equal to 0.5 for the case $n = 7$, $p = 0.5$.

#26 Note that $P(X \ge 12) = 1 - P(X < 12) = 1 - P(X \le 11)$.

Use Table III with $n = 20$, $p = 0.7$, $x = 12$ to find $P(X \le 11) = 0.1133$.

So $P(X \ge 12) = 1 - P(X \le 11) = 1 - 0.1133 = 0.8867$

#27 Use Table III with $n = 12$, $p = 0.35$, $x = 4$ to find $P(X \le 4) = 0.5833$.

#28 Note that $P(X \ge 8) = 1 - P(X < 8) = 1 - P(X \le 7)$.

Use Table III with $n = 11$, $p = 0.75$, $x = 7$ to find $P(X \le 7) = 0.2867$.

So $P(X \ge 8) = 1 - P(X \le 7) = 1 - 0.2867 = 0.7133$

#29 (a) $n = 6$, $p = 0.3$. You may look up Table II to get the probability distribution table, shown to the right.

x	$P(x)$
0	0.1176
1	0.3025
2	0.3241
3	0.1852
4	0.0595
5	0.0102
6	0.0007

(b) The computation for μ_X and σ_X is shown in the table below.

x	$P(x)$	$x \cdot P(x)$	μ_X	$x - \mu_X$	$(x - \mu_X)^2 \cdot P(x)$
0	0.1176	0.0000	1.7995	-1.7995	0.38081235
1	0.3025	0.3025	1.7995	-0.7995	0.19335808
2	0.3241	0.6482	1.7995	0.2005	0.01302890
3	0.1852	0.5556	1.7995	1.2005	0.26691029
4	0.0595	0.2380	1.7995	2.2005	0.28811091
5	0.0102	0.0510	1.7995	3.2005	0.10448064
6	0.0007	0.0042	1.7995	4.2005	0.01235094
	0.9998	1.7995			1.25905211

Thus $\mu_X \cong 1.7995 \approx 1.8$ (Note that $\sum P(x)$ is not exactly equal to 1, because the data from Table II have been rounded. If we had used very accurate data, $\sum P(x)$ would have been exactly 1 and μ_X would have been exactly 1.8!!)

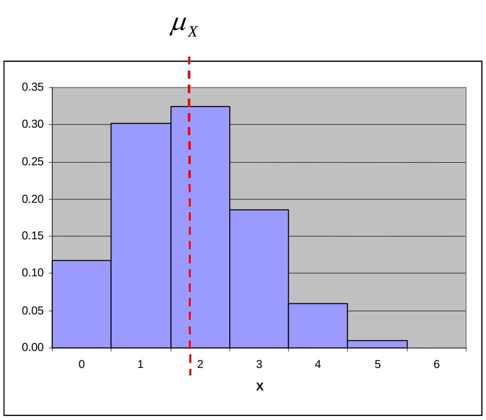
And $\sigma_X = \sqrt{1.7995} \cong 1.1221 \approx 1.1$

(c) $\mu_X = np = (6)(0.3) = 1.8$ (exactly!)

$\sigma_X = \sqrt{np(1-p)} = \sqrt{(6)(0.3)(0.7)}$

$\cong 1.12249722 \approx 1.1$

(d) See the accompanying histogram. This is skewed to the right.

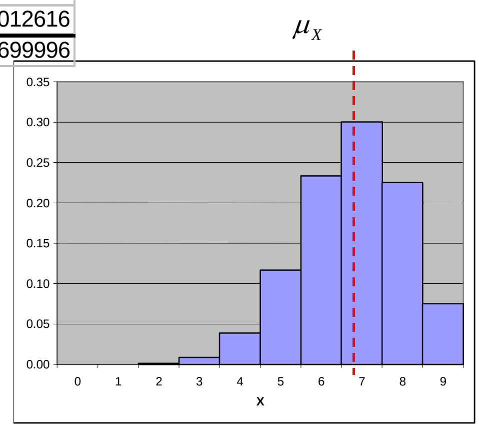


#31 (a) $n = 9$, $p = 0.75$. You may look up Table II to get the probability distribution table, shown to the right.

x	$P(x)$
0	0.0000
1	0.0001
2	0.0012
3	0.0087
4	0.0389
5	0.1168
6	0.2336
7	0.3003
8	0.2253
9	0.0751

(b) The computation for μ_X and σ_X is shown in the table below.

x	$P(x)$	$x \cdot P(x)$	μ_X	$x-\mu_X$	$(x-\mu_X)^2 \cdot P(x)$
0	0.0000	0.0000	6.7502	-6.7502	0.00000000
1	0.0001	0.0001	6.7502	-5.7502	0.00330648
2	0.0012	0.0024	6.7502	-4.7502	0.02707728
3	0.0087	0.0261	6.7502	-3.7502	0.12235680
4	0.0389	0.1556	6.7502	-2.7502	0.29422404
5	0.1168	0.5840	6.7502	-1.7502	0.35778176
6	0.2336	1.4016	6.7502	-0.7502	0.13147009
7	0.3003	2.1021	6.7502	0.2498	0.01873873
8	0.2253	1.8024	6.7502	1.2498	0.35191861
9	0.0751	0.6759	6.7502	2.2498	0.38012616
	1.0000	6.7502			1.68699996



Thus $\mu_X \cong 6.7502 \approx 6.75$

and $\sigma_X = \sqrt{1.68699996} \cong 1.29884563 \approx 1.3$

(c) $\mu_X = np = (9)(0.75) = 6.75$ (exactly!)

$$\sigma_X = \sqrt{np(1-p)} = \sqrt{(9)(0.75)(0.25)}$$

$$\cong 1.29903811 \approx 1.3$$

(d) See the accompanying histogram. This is skewed to the left.

#33 (a) $n = 10$, $p = 0.5$. You may look up Table II to get the probability distribution table, shown to the right.

x	$P(x)$
0	0.0010
1	0.0098
2	0.0439
3	0.1172
4	0.2051
5	0.2461
6	0.2051
7	0.1172
8	0.0439
9	0.0098
10	0.0010

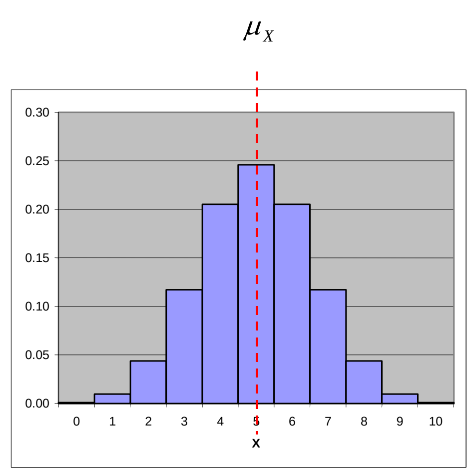
(b) The computation for μ_X and σ_X is shown in the table below.

x	$P(x)$	$x \cdot P(x)$	μ_X	$x-\mu_X$	$(x-\mu_X)^2 \cdot P(x)$
0	0.0010	0.0000	5.0005	-5.0005	0.02500500
1	0.0098	0.0098	5.0005	-4.0005	0.15683920
2	0.0439	0.0878	5.0005	-3.0005	0.39523171
3	0.1172	0.3516	5.0005	-2.0005	0.46903443
4	0.2051	0.8204	5.0005	-1.0005	0.20530515
5	0.2461	1.2305	5.0005	-0.0005	0.00000006
6	0.2051	1.2306	5.0005	0.9995	0.20489495
7	0.1172	0.8204	5.0005	1.9995	0.46856563
8	0.0439	0.3512	5.0005	2.9995	0.39496831
9	0.0098	0.0882	5.0005	3.9995	0.15676080
10	0.0010	0.0100	5.0005	4.9995	0.02499500
	1.0001	5.0005			2.50160025

Thus $\mu_X \cong 5.005 \approx 5.0$

and $\sigma_X = \sqrt{2.50160025} \cong 1.58164479 \approx 1.6$

The data taken from Table II are rounded values. If precise values had been used, μ_X would have been exactly 5.



(c) $\mu_X = np = (10)(0.5) = 5$ (exactly!)

$$\sigma_X = \sqrt{np(1-p)} = \sqrt{(10)(0.5)(0.5)}$$

$$= \sqrt{2.5} \cong 1.58113883 \approx 1.6$$

(d) See the accompanying histogram. This is symmetric.

#35 (a) Check the criteria:

- The experiment is performed a fixed number of times. ($n = 15$)
- The trials are independent. (To be precise, this is a case of selection without replacement. But assuming that the pool of flights from which the 15 flights are selected is huge, the trials are approximately independent.)
- The outcome of each trial is dichotomous. (Either on time or not on time.)
- The probability of success is the same for each trial. ($p = 0.90$)

We have $n = 15$, $X =$ the number of on-time flights among the 15, and $p = 0.90$.

(b) $P(14) = {}_{15}C_{14}(0.90)^{14}(0.10)^1 = 15 \cdot (0.90)^{14}(0.10)^1 = 0.34315189 \approx 0.3432$

(Or use Table II.)

(c)

$$P(X \ge 14) = P(14) + P(15) = {}_{15}C_{14}(0.90)^{14}(0.10)^1 + {}_{15}C_{15}(0.90)^{15}(0.10)^0$$
$$= 15 \cdot (0.90)^{14}(0.10)^1 + 1 \cdot (0.90)^{15} \cdot 1$$
$$= 0.34315189 + 0.20589113$$
$$= 0.54904302 \approx 0.5490$$

(Or you may choose to use Table II to find $P(14)$ and $P(15)$, and then add them together. Alternatively, use Table III to get $P(X \le 13) = 0.4510$, and so $P(X \ge 14) = 1 - P(X \le 13) = 1 - 0.4510 = 0.5490$.)

(d) $P(X < 14) = 1 - P(X \ge 14) = 1 - 0.54904302 = 0.45095698 \approx 0.4510$, where we have used the result of (c).

Alternatively, $P(X < 14) = P(X \le 13) = 0.4510$, using Table III.

(e) $P(12) + P(13) + P(14) = 0.1285 + 0.2669 + 0.3432 = 0.7386$

(Here Table II was consulted to find the three numbers, which we sum together. Alternatively,

$P(12) + P(13) + P(14) = P(X \le 14) - P(X \le 11) = 0.7941 - 0.0556 = 0.7385$.

Of course, you may also choose to use the binomial probability formula for each of the three probabilities and then sum them up.)

#37 $n = 20$, $X =$ the number of households with high-speed Internet , and $p = 0.20$.

(a) $P(5) = {}_{20}C_5(0.20)^5(0.80)^{15} = 0.17455952 \approx 0.1746$, or use Table II.

(b) $P(X \ge 10) = 1 - P(X < 10) = 1 - P(X \le 9) = 1 - 0.9974 = 0.0026$, where Table III was used to find $P(X \le 9)$. Since $0.0026 < 0.05$, so yes, it would be unusual.

(c) $P(X < 4) = P(X \le 3) = 0.4114$, where Table III has been used.

(d)

$$P(2) + P(3) + P(4) + P(5)$$
$$= {}_{20}C_2(0.20)^2(0.80)^{18} + {}_{20}C_3(0.20)^3(0.80)^{17} + {}_{20}C_4(0.20)^4(0.80)^{16} + {}_{20}C_5(0.20)^5(0.80)^{15}$$
$$= \frac{(20)(19)}{(1)(2)} \cdot (0.20)^2(0.80)^{18} + \frac{(20)(19)(18)}{(1)(2)(3)}(0.20)^3(0.80)^{17}$$
$$+ \frac{(20)(19)(18)(17)}{(1)(2)(3)(4)}(0.20)^4(0.80)^{16} + \frac{(20)(19)(18)(17)(16)}{(1)(2)(3)(4)(5)}(0.20)^5(0.80)^{15}$$
$$= 0.735032495 \approx 0.7350$$

Or, if we use Table II, we have

$P(2) + P(3) + P(4) + P(5) = 0.1369 + 0.2054 + 0.2182 + 0.1746 = 0.7351$, very close.

Or,

$P(2) + P(3) + P(4) + P(5) = P(X \le 5) - P(X \le 1) = 0.8042 - 0692 = 0.7350$

#39 $n=25$, $X=$ the number of murders committed with a firearm, and $p=0.669$.

(a) It is useful to recall ${}_{25}C_{22}={}_{25}C_3$

$$P(22)={}_{25}C_{22}(0.669)^{22}(0.331)^{3}={}_{25}C_3(0.669)^{22}(0.331)^{3}$$

$$=\frac{(25)(24)(23)}{(1)(2)(3)}\cdot(0.669)^{22}(0.331)^{3}=0.012038889\approx 0.0120$$

(b) Well, to be fair, this one is more suitable for those who do have a calculator that has built-in binomial probability functionality.

$$P(14)+P(15)+P(16)$$

$$={}_{25}C_{14}(0.669)^{14}(0.331)^{11}+{}_{25}C_{15}(0.669)^{15}(0.331)^{10}+{}_{25}C_{16}(0.669)^{16}(0.331)^{9}$$

$$=0.08378286+0.12418088+0.15686747$$

$$=0.36483121\approx 0.3648$$

(c)

$$P(X\geq 22)=P(22)+P(23)+P(24)+P(25)$$

$$={}_{25}C_{22}(0.669)^{22}(0.331)^{3}+{}_{25}C_{23}(0.669)^{23}(0.331)^{2}$$

$$+{}_{25}C_{24}(0.669)^{24}(0.331)^{1}+{}_{25}C_{25}(0.669)^{25}(0.331)^{0}$$

$$=\frac{(25)(24)(23)}{(1)(2)(3)}(0.669)^{22}(0.331)^{3}+\frac{(25)(24)}{(1)(2)}(0.669)^{23}(0.331)^{2}$$

$$+\frac{(25)}{(1)}(0.669)^{24}(0.331)^{1}+1\cdot(0.669)^{25}\cdot 1$$

$$=0.0158$$

which is less than 5%. So, yes, it is unusual to have 22 or more murders that were committed using a firearm.

#43 $n=100$, $X=$ the number of on-time flights among the 100, and $p=0.90$.

(a) $\mu_X=np=(100)(0.90)=90$,

$$\sigma_X=\sqrt{np(1-p)}=\sqrt{(100)(0.90)(0.10)}=3$$

(b) If we randomly select 100 flights, then there tends to be 90 that are on time.

(c) $np(1-p)=(100)(0.90)(0.10)=9$. So the criterion in page 308, $np(1-p)\geq 10$ is not really satisfied. However, we agreed to pretend that it did, and thus proceed to assume that the histogram resembles a bell-shaped curve.

$\mu_X-2\sigma_X=90-2(3)=84$ and $\mu_X+2\sigma_X=90+2(3)=96$.

Thus 80 is outside of the range from 84 to 96. I.e. 80 is more than two standard deviations from the mean. Yes, it is unusual to observe 80 on-time flights in a random sample of 100 flights. (The reason given at the end of the book, namely "$P(80)<0.05$", is inappropriate. Please ignore it.)

#45 $n=100$,

$X=$ the number of households with high-speed connection among 100

$p=0.20$

(a) $\mu_X=np=(100)(0.20)=20$, $\sigma_X=\sqrt{np(1-p)}=\sqrt{(100)(0.20)(0.80)}=4$

(b) If we randomly select 100 households, then there tends to be 20 that have high-speed Internet connection.

(c) $np(1-p) = (100)(0.20)(0.80) = 16$, so the criterion in page 308, $np(1-p) \geq 10$ is satisfied. The histogram thus is fairly bell-shaped.

$\mu_X - 2\sigma_X = 20 - 2(4) = 12$, $\mu_X + 2\sigma_X = 20 + 2(4) = 28$. Since $X = 18$ falls in the range from 12 to 28, it follows that it is <u>not</u> unusual to observe 18 households having a high-speed Internet connection in 100 households.

#49 $n = 400$ and $p = 0.184$.

$\mu_X = np = (400)(0.184) = 73.6$, $\sigma_X = \sqrt{np(1-p)} = \sqrt{(400)(0.184)(0.816)} \approx 7.75$

$np(1-p) = (400)(0.184)(0.816) \approx 60.1$, so the criterion in page 308, $np(1-p) \geq 10$ is satisfied. The histogram thus is fairly bell-shaped.

$\mu_X - 2\sigma_X = 73.6 - 2(7.75) = 58.1$, $\mu_X + 2\sigma_X = 73.6 + 2(7.75) = 89.1$.

Since $X = 86$ falls in the range from 58.1 to 89.1, i.e. $X = 86$ is within two standard deviations of the mean, it follows that it is <u>not</u> unusual to observe 86 patients who experience headaches in a random sample of 400 patients.