Math 164 (Spring 2008)

Solutions to Homework #8

<u>Section 6.1</u>:

9, 10, 11, 12, 13, 14, 15, 16, 17, 18,
19, 23, 27, 29, 30, 31, 33

Remark:

For #27(d), convert the histogram into a table resembling that in #19, and then use it to compute the mean.

#9 (a) Continuous, $r \geq 0$ (b) Discrete, $x = 0,1,2,\cdots$
(c) Discrete, $x = 0,1,2,\cdots$ (d) Continuous, $t > 0$.

#10 (a) Discrete, $x = 0,1,2,\cdots$ (b) Continuous, $d > 0$
(c) Discrete, $x = 0,1,2,\cdots$ (d) Continuous, $s > 0$

#11 Yes **#12** Yes **#13** No, because P(50)<0 **#14** Yes

#15 No, because $\sum P(x) \neq 1$ **#16** No, because $\sum P(x) \neq 1$

#17 $P(4) = 0.3$ **#18** $P(2) = 0.15$

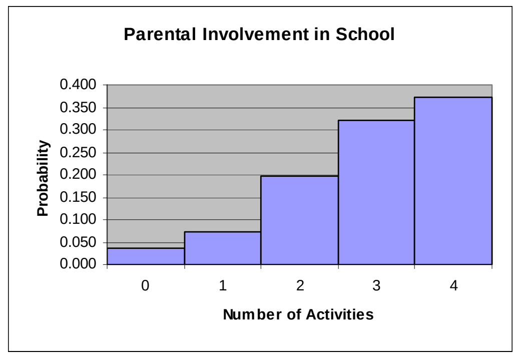
#19 (a) $P(x) \geq 0$ for every x, and $\sum P(x) = 1$. So it's a discrete probability distribution.

(b) (See the accompanying histogram.)

(c) $\mu_X = \sum [x \cdot P(x)] = 2.924 \approx 2.9$

This means that, considering the experiment where we randomly pick one parent and determine the number of activities the parent is involved in, we expect that, in the long run, the average of the observed numbers is close to 2.9 if the experiment is repeated an enormous number of times.

x	$P(x)$	$x \cdot P(x)$
0	0.035	0.000
1	0.074	0.074
2	0.197	0.394
3	0.320	0.960
4	0.374	1.496
		2.924



(d) Approach 1: (Use the table to the right.)

$$\sigma_X^2 = \sum [x^2 P(x)] - \left(\sum xP(x)\right)^2$$

$$= 9.726 - (2.924)^2 = 1.176224 \approx 1.2$$

x	$P(x)$	$x \cdot P(x)$	$x^2 \cdot P(x)$
0	0.035	0.000	0.000
1	0.074	0.074	0.074
2	0.197	0.394	0.788
3	0.320	0.960	2.880
4	0.374	1.496	5.984
		2.924	9.726

Approach 2: (Use the table below.)

$$\sigma_X^2 = \sum (x - \mu_X)^2 = 1.176224 \approx 1.2$$

x	$P(x)$	$x \cdot P(x)$	μ_X	$x - \mu_X$	$(x-\mu_X)^2$	$(x-\mu_X)^2 \cdot P(x)$
0	0.035	0.000	2.924	-2.924	8.549776	0.29924216
1	0.074	0.074	2.924	-1.924	3.701776	0.273931424
2	0.197	0.394	2.924	-0.924	0.853776	0.168193872
3	0.320	0.960	2.924	0.076	0.005776	0.00184832
4	0.374	1.496	2.924	1.076	1.157776	0.433008224
		2.924				1.176224

(e) $\sigma_X = \sqrt{\sigma_X^2} = \sqrt{1.176224} \approx 1.084539 \approx 1.1$ ← **Make sure you use the unrounded value 1.176224 for σ_X^2 in this computation!!!!!! Don't use the rounded value 1.2 !!**

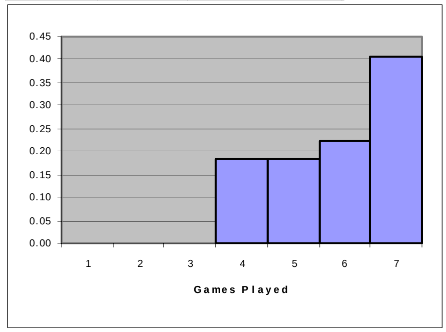
(f) $P(3) = 0.32$ (g) $P(3) + P(4) = 0.320 + 0.374 = 0.694$

#23 **Note: The answer given in the back of the book cannot be reconciled with the problem. The frequency of $x = 4$ in the problem should be 15, not 16.**

(a) See the table to the right.

(b) See the accompanying histogram.

x	Freq	$P(x)$
4	15	0.185185
5	15	0.185185
6	18	0.222222
7	33	0.407407
sum	81	1



(c) $\mu_X = 5.851852 \approx 5.9$, as shown in the table below. It means that, considering the experiment where a year is randomly selected from 1923 to 2005 and the number of games played in World Series that year is recorded, we expect that, in the long run, the average of the number of games is close to 5.9 if this experiment is to be repeated an enormously number of times.

x	$P(x)$	$x \cdot P(x)$
4	0.185185	0.740741
5	0.185185	0.925926
6	0.222222	1.333333
7	0.407407	2.851852
sum	1	5.851852

(d) Approach 1: (Use the table to the right.)

$$\sigma_X^2 = \sum\left[x^2 P(x)\right] - \left(\sum xP(x)\right)^2$$

$$= 35.55555556 - (5.851852)^2 \approx 1.311385$$

So $\sigma_X = \sqrt{\sigma_X^2} = \sqrt{1.311385} \approx 1.145157 \approx 1.1$

x	$P(x)$	$x \cdot P(x)$	$x^2 \cdot P(x)$
4	0.185185	0.740741	2.962962963
5	0.185185	0.925926	4.629629630
6	0.222222	1.333333	8.000000000
7	0.407407	2.851852	19.962962963
sum	1	5.851852	35.55555556

Approach 2: (Use the table below.)

$$\sigma_X^2 = \sum (x - \mu_X)^2 \approx 1.311385$$

So $\sigma_X = \sqrt{\sigma_X^2} = \sqrt{1.311385} \approx 1.145157 \approx 1.1$

x	$P(x)$	$x \cdot P(x)$	μ_X	$x - \mu_X$	$(x - \mu_X)^2$	$(x - \mu_X)^2 \cdot P(x)$
4	0.185185	0.740741	5.851852	-1.851852	3.429355	0.635066
5	0.185185	0.925926	5.851852	-0.851852	0.725652	0.134380
6	0.222222	1.333333	5.851852	0.148148	0.021948	0.004877
7	0.407407	2.851852	5.851852	1.148148	1.318244	0.537062
	1	5.851852				1.311385

#27 (a) $P(4) = 0.119$

(b) $P(4) + P(5) = 0.119 + 0.103 = 0.222$

(c) $P(6) + P(7) + P(8) = 0.027 + 0.031 + 0.050 = 0.108$

(d) $\mu_X = \sum\left[xP(x)\right] = 3.041 \approx 3.0$

x	$P(x)$	$x \cdot P(x)$
1	0.241	0.241000
2	0.257	0.514000
3	0.172	0.516000
4	0.119	0.476000
5	0.103	0.515000
6	0.027	0.162000
7	0.031	0.217000
8	0.050	0.400000
sum	1.000	3.041

#29

- If the woman survives, the insurance company will gain \$200. The probability for this scenario is 0.999546.
- If the woman dies, the insurance company will have to pay \$250,000 after having gained \$200. This translates into an effect gain of $\$200 - \$250{,}000 = -\$249{,}800$. The probability is $1 - 0.999546 = 0.000454$

$$\mu_X = \sum\left[x \cdot P(x)\right] = (200)(0.999546) + (-249800)(0.000454) = 199.9092 - 113.4092 = 86.50$$

The expected value of this policy to the insurance company is \$86.50. This means in the long run, the company makes an average profit of \$86.50 on every 20-year-old female it insures for a year.

#30

- If the male survives, the insurance company will gain \$350. The probability for this scenario is 0.998611.
- If the male dies, the insurance will have to pay \$250,000 after having gained \$350. This translates into an effective gain of $\$350 - \$250{,}000 = -\$249{,}650$.

  The probability is $1 - 0.998611 = 0.001389$.

$\mu_X = \sum \left[x \cdot P(x) \right] = (350)(0.998611) + (-249653)(0.001389) = 349.513852 - 346.76385 = 2.75$

The expected value of this policy to the insurance company is \$2.75. This means in the long run, the company makes an average profit of \$2.75 on every 20-year-old male it insures for a year.

#31

$\mu_X = \sum \left[x \cdot P(x) \right] = (50{,}000)(0.20) + (10{,}000)(0.70) + (-50{,}000)(0.10)$

$= 10{,}000 + 7{,}000 - 5{,}000 = 12{,}000$

So the answer is \$12,000.

#33

$\mu_X = \sum \left[x \cdot P(x) \right] = (175)(\tfrac{1}{38}) + (-5)(\tfrac{37}{38})$

$= 4.605263 - 4.868421 = -0.26315789 \approx -0.26$

$(1000)(-0.26) = -260$

Answer: The expected value of the game to the player is $-\$0.26$. If the game is played <u>1000 times</u>, the player is expected statistically to lose a total of $(1000)(\$2.60) = \$260$.

Remark: The statement in the problem is a little ambiguous. In the case the player wins, it is not clear whether the player's net gain is \$175 or $\$175 - \$5 = \$170$. The computation here is based on the former.

<u>Section 6.2</u>: (First round—More will be assigned later on.)

7, 8, 9, 10, 11, 12, 13, 14, 15, 16

#7 No (The outcome of each trial is not dichotomous.)
#8 No (The outcome of each trial is not dichotomous.)
#9 Yes.
#10 Yes.
#11 No (The three trials are not independent.)
#12 Yes.
#13 No (The number of trials is not fixed.)
#14 No (The number of trials is not fixed.)
#15 Yes.
#16 No (The 10 trials are not independent.)